CWI

An All-But-One Entropic Uncertainty Relation and Application to Password-based Identification

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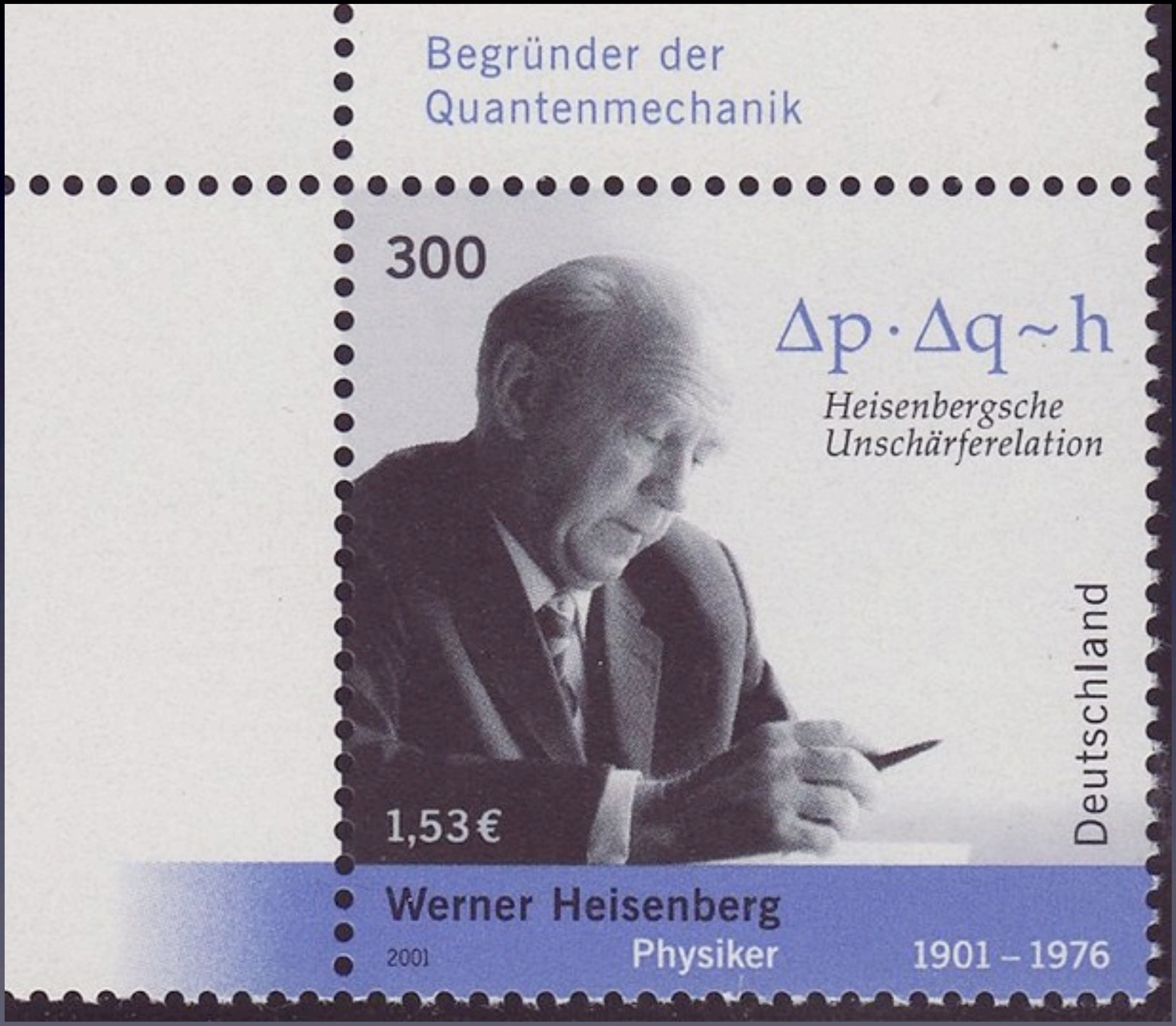

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A more well-known entropic UR:
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Remark

For "good" families, $\lim_{n \to \infty} -\frac{1}{n} \log_2 c \in (0, \frac{1}{2}]$

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$$H(X \mid J = j) + H(X \mid J = k) \geq -2 \log(c)$$

$$\forall\, j \neq k \in [m]$$

where X is the outcome when measuring in $\mathcal{B}_J$

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3. Compatible with single-qubit measurements, hence usable to prove the security of schemes that can be implemented with today's technology

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2. Discuss main application: Password-based Identification

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All-but-One Shannon Entropy Uncert. Relation

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All-but-One Shannon-Entr. UR (follows from Maassen Uffink)	New All-b.-One Min-Entropy UR
$H(X \mid J = j) \geq -\log(c)$ $\forall\, j \neq j'$	$H_{\min}(X \mid J{=}j,\ J'{=}j') \gtrsim -\log(c)$ $\forall\, j \neq j'$

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Recall: For "good" families of bases on an n-qubit space, $-\log(c)$ is linear in n

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The family of (meas.) bases is $\{\mathrm{Comp}, \mathrm{Hadamard}\}$,
on n qubits for which $c = 2^{-n/2}$

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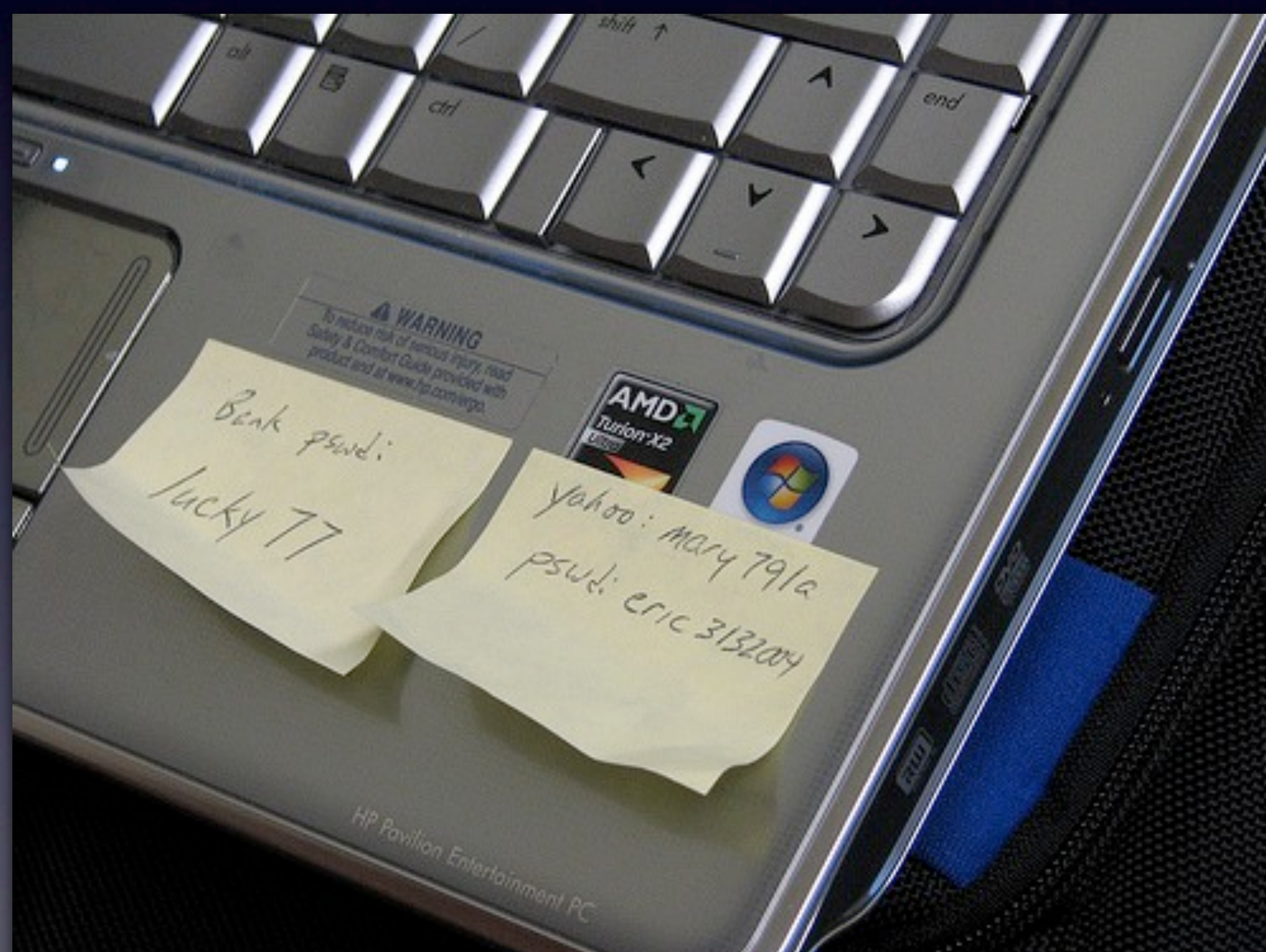

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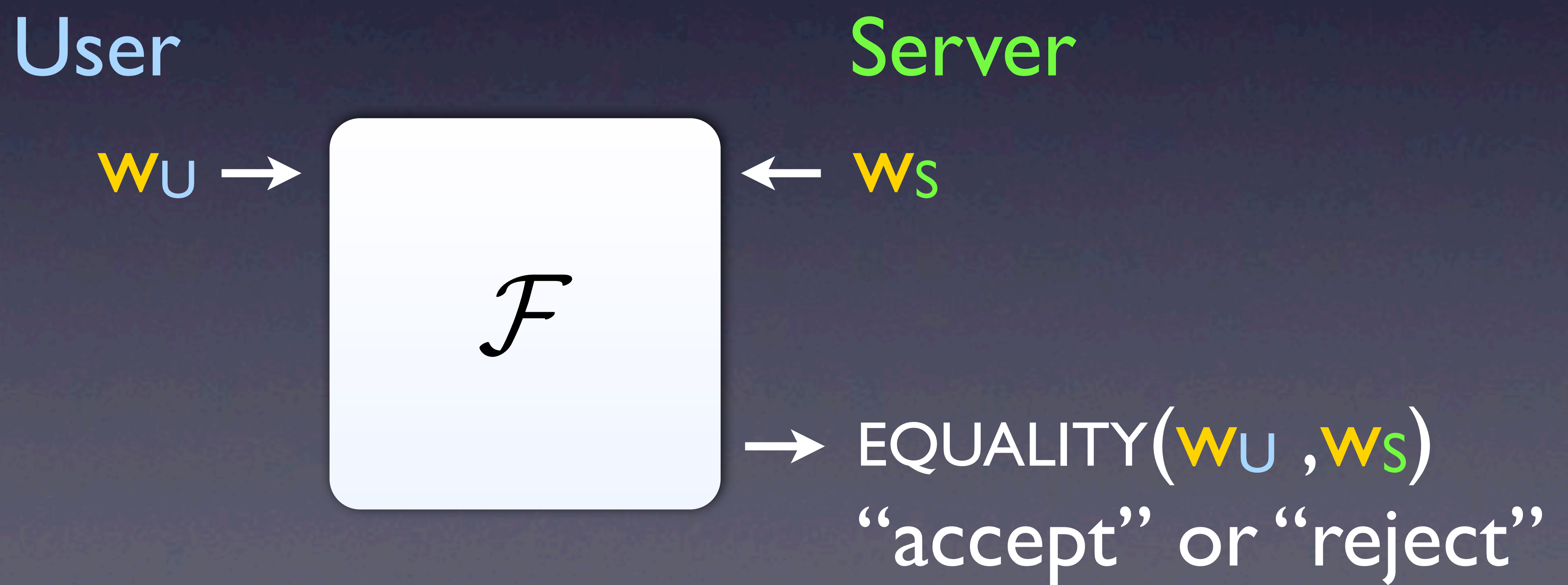

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New: “Single-Qubit Operations Model” (SQOM)

- Malicious party has unbounded quantum storage,
- but is restricted to single-qubit operations and measurements

Existing QID Scheme

QID Scheme of Damgård et al. [DFSS07]

- Unconditionally secure against malicious user
- Secure against malicious server in the BQSM
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Remark

Security proof of new QID scheme in BQSM is based on our uncertainty relation

Thank You