Complete Insecurity of Quantum Protocols for Classical Two-Party Computation

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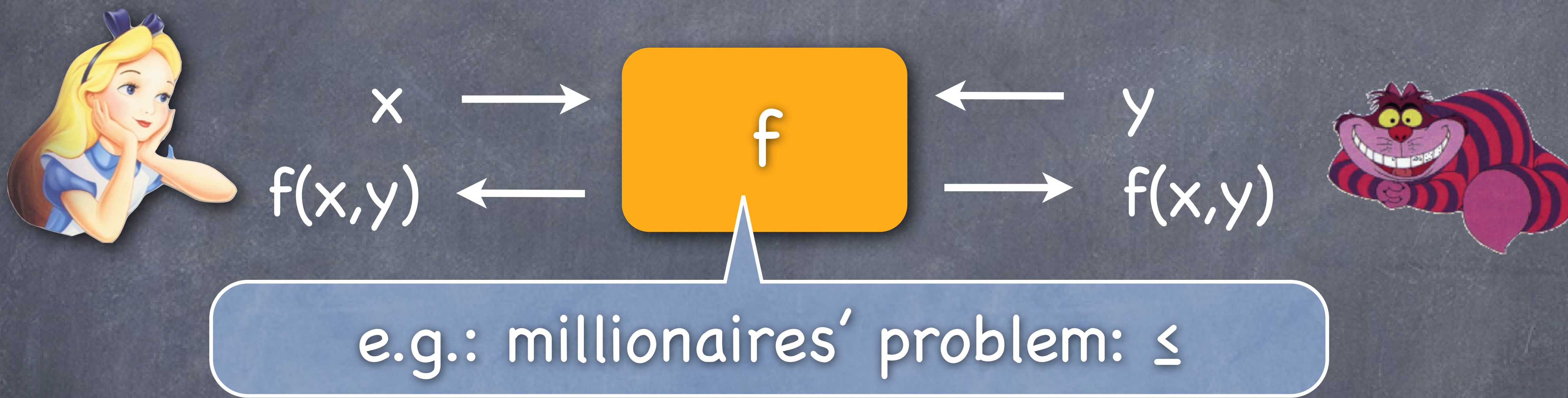
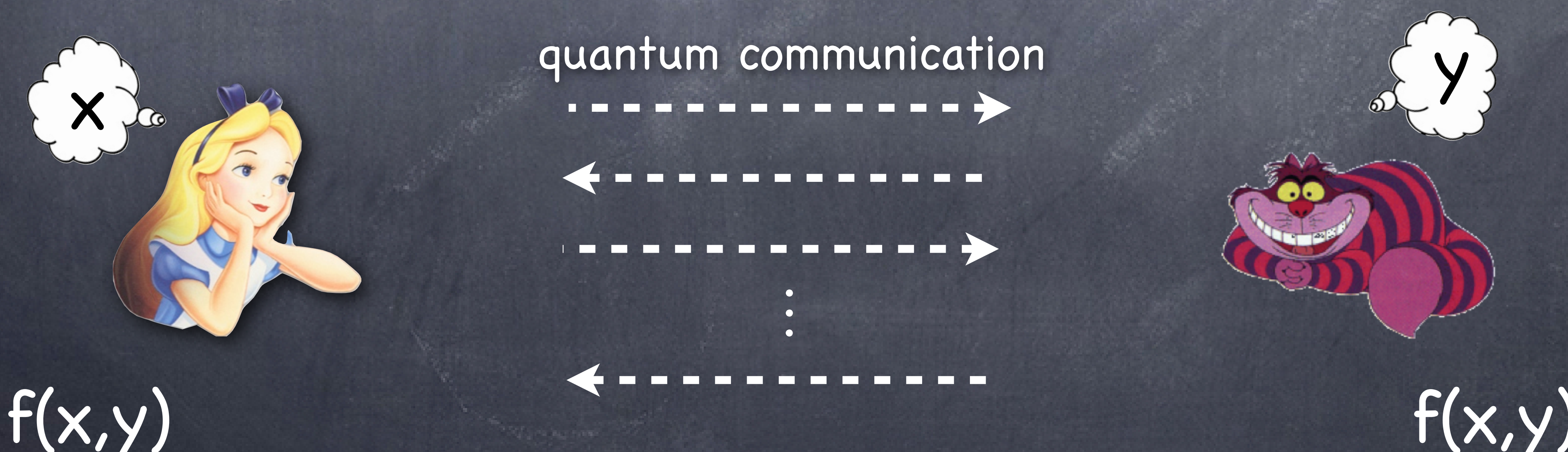
Nederlandse Organisatie voor Wetenschappelijk Onderzoek

Motivation

- ideally: Alice & Bob have a box computing f on private inputs x and y



- reality: Alice and Bob perform a protocol



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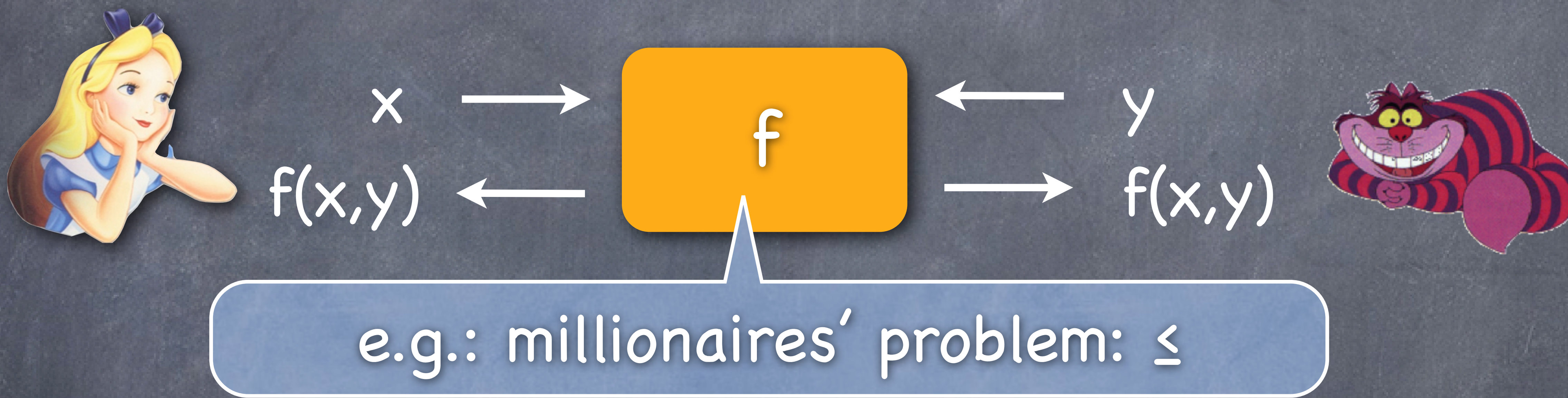
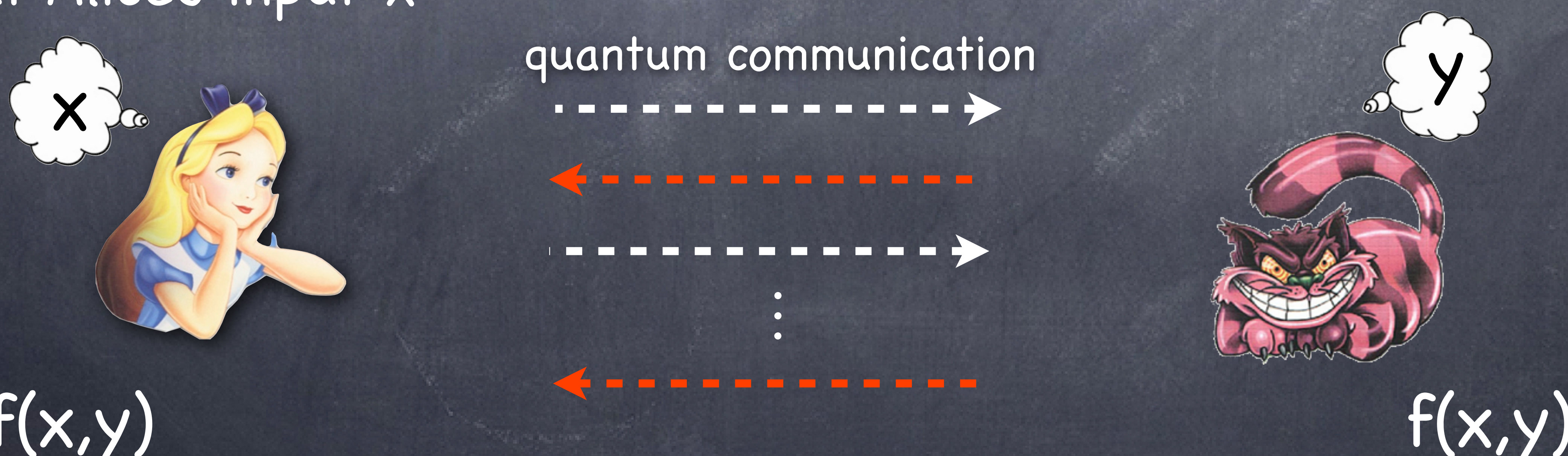


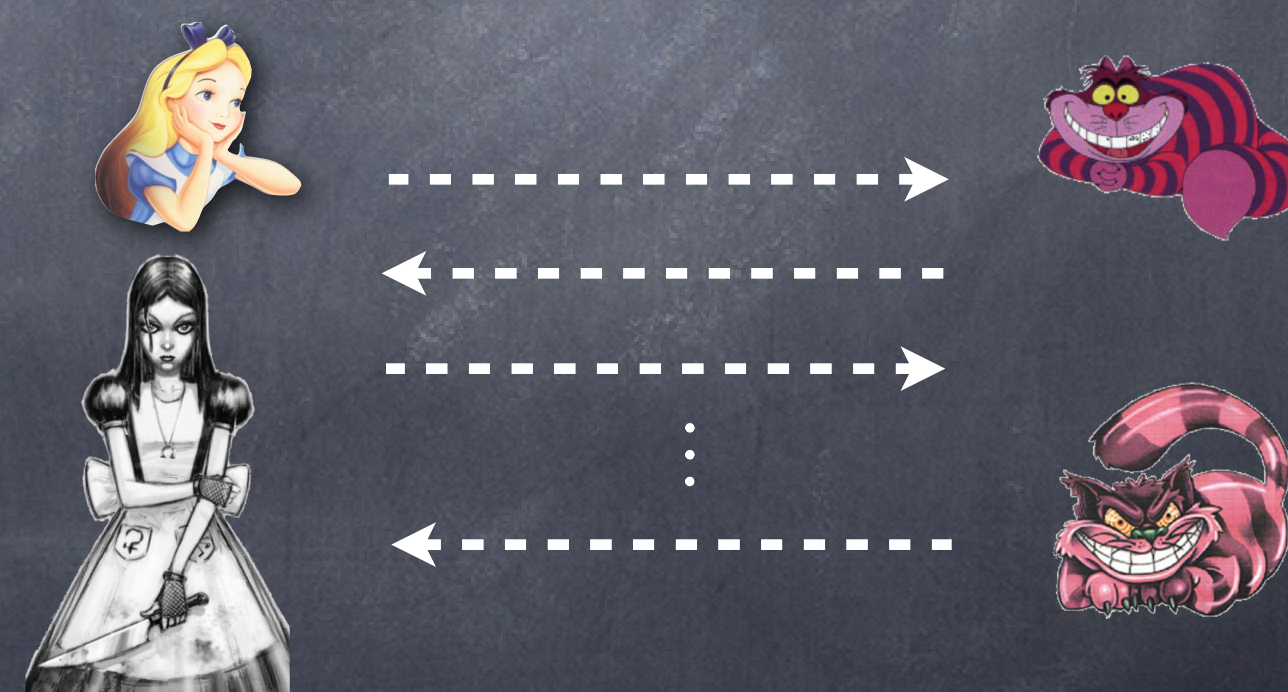
- reality: dishonest Bob might deviate from protocol to learn more about Alice's input x



Secure Function Evaluation

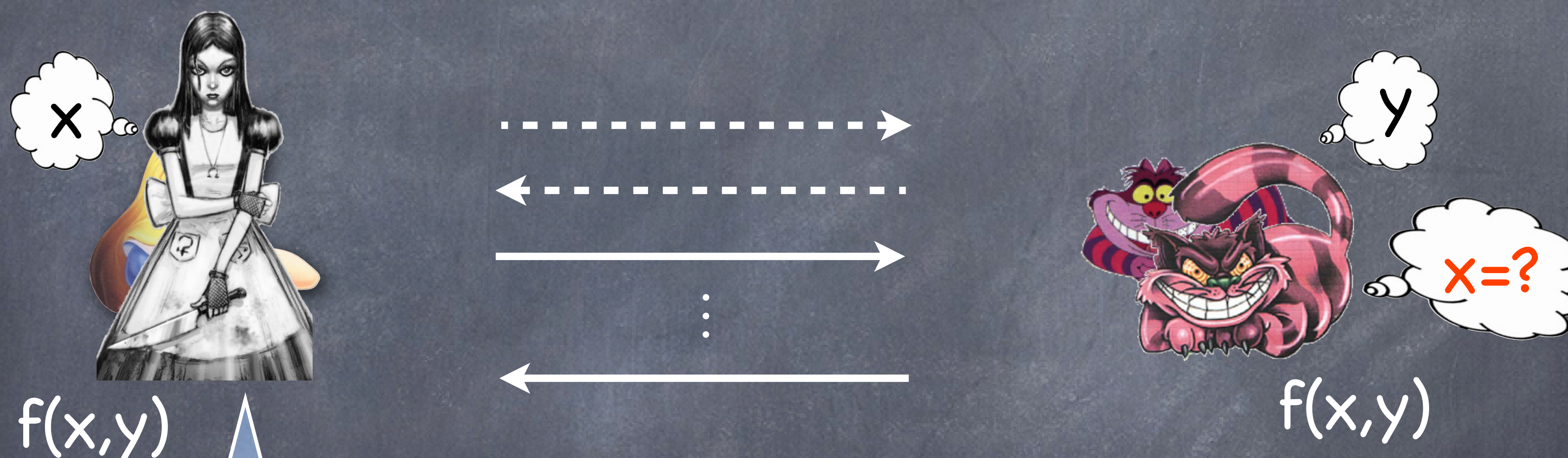
- ideally: Alice & Bob have a box computing f on private inputs x and y

- goal: come up with protocols that are
  - correct
  - secure against dishonest Alice
  - secure against dishonest Bob



Main Impossibility Result

- **Theorem**: If a quantum protocol for the evaluation of f is correct and perfectly secure against Bob, then Alice can completely break the protocol.



after protocol: dishonest Alice can compute $f(x,y)$ not just for one x, but for all x.

- **Theorem:** If a quantum protocol for the evaluation of f is ε-correct and ε-secure against Bob, then Alice can break the protocol with probability $1-O(\varepsilon)$.

History

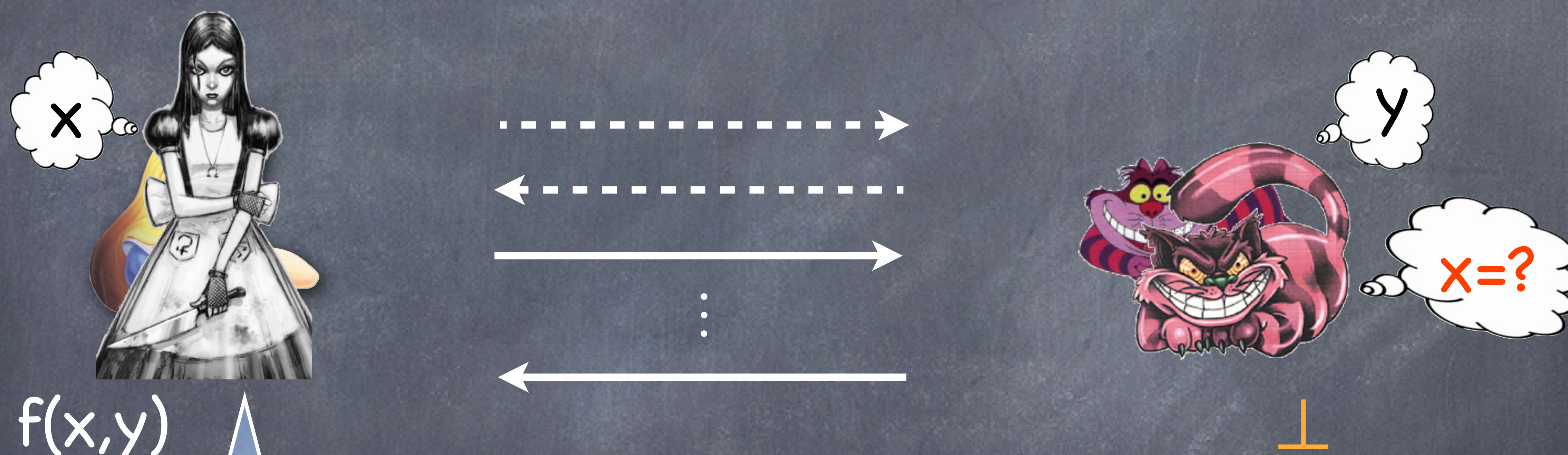
- ~1970: Conjugate Coding [Wiesner]
- 1984: Quantum Key Distribution [Bennett Brassard]
- ~1991: Bit Commitment and Oblivious Transfer?
- 1997: No Bit Commitment [Lo Chau, Mayers]
- 1997: No One-Sided Secure Computation [Lo]
- Really no Quantum Bit Commitment?
- 2007: No BC [D'Ariano Kretschmann Schlingemann Werner]
- 2007: Some Functions are Impossible [Colbeck]
- 2009: Secure Computation has to Leak Information [Salvail Sotakova Schaffner]
- this work: Complete Insecurity of Two-Sided Deterministic Computations

Talk Outline

- explain Lo's impossibility proof
- problem with two-sided computation
- security definition
- impossibility proof
- conclusion

[Lo97] Impossibility Result

- **Theorem**: If a quantum protocol for the evaluation of f is correct and perfectly secure against Bob, then Alice can completely break the protocol.



dishonest Alice can compute f(x,y) not just for one x, but for all x.

- holds only for one-sided computations
- error increases with number of inputs

[Lo97] Impossibility Result

f(x,y) $\quad |\psi^{x,y}\rangle_{AB} \quad \perp$

- only Alice gets output
- wlog measurements are moved to the end, final state is pure
- for dishonest Bob inputting y in superposition, define:

$$|\psi^{x_0}\rangle_{AB} = \sum_y |\psi^{x_0,y}\rangle_{AB_1} |y\rangle_{B_2}$$

- security against dishonest Bob:

$$\mathrm{tr}_A(|\psi^{x_0}\rangle\langle\psi^{x_0}|_{AB}) = \rho_B^{x_0} = \rho_B^{x_1} = \mathrm{tr}_A(|\psi^{x_1}\rangle\langle\psi^{x_1}|_{AB})$$

[Lo97] Impossibility Result

$\vdots$

f(x_0,y), f(x_1,y), ... f(x,y) $\quad |\psi^{x,y}\rangle_{AB} \quad \perp$

- security against dishonest Bob:

$$\mathrm{tr}_A(|\psi^{x_0}\rangle\langle\psi^{x_0}|_{AB}) = \rho_B^{x_0} = \rho_B^{x_1} = \mathrm{tr}_A(|\psi^{x_1}\rangle\langle\psi^{x_1}|_{AB})$$

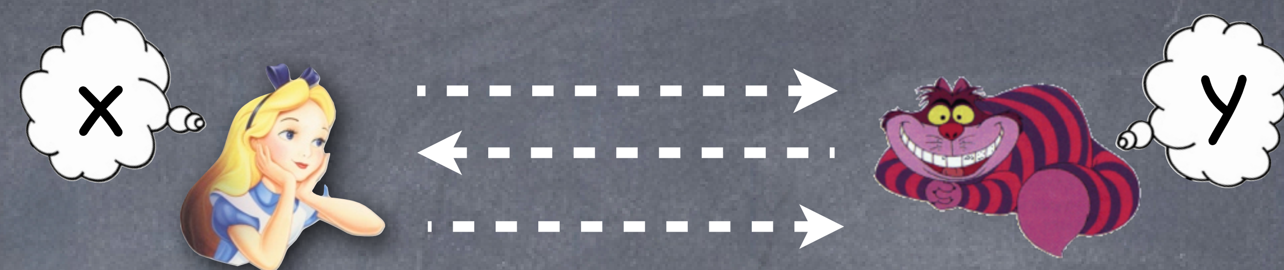
- implies existence of cheating unitary for Alice: (not dep on y)

$$(U_A \otimes \mathbb{I}_B)\,|\psi^{x_0}\rangle_{AB} = |\psi^{x_1}\rangle_{AB}$$

- dishonest Alice starts with input x_0, can read out f(x_0,y), switches to x_1, reads out f(x_1,y) etc.

$$(U_A \otimes \mathbb{I}_B)\,|\psi^{x_0,y}\rangle_{AB} = |\psi^{x_1,y}\rangle_{AB}$$

Two-Sided Comp?



f(x,y) ⋮ f(x,y)

- ~~only~~ Bob& Alice get~~s~~ output
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- implies existence of cheating unitary for Alice: (not dep on y)

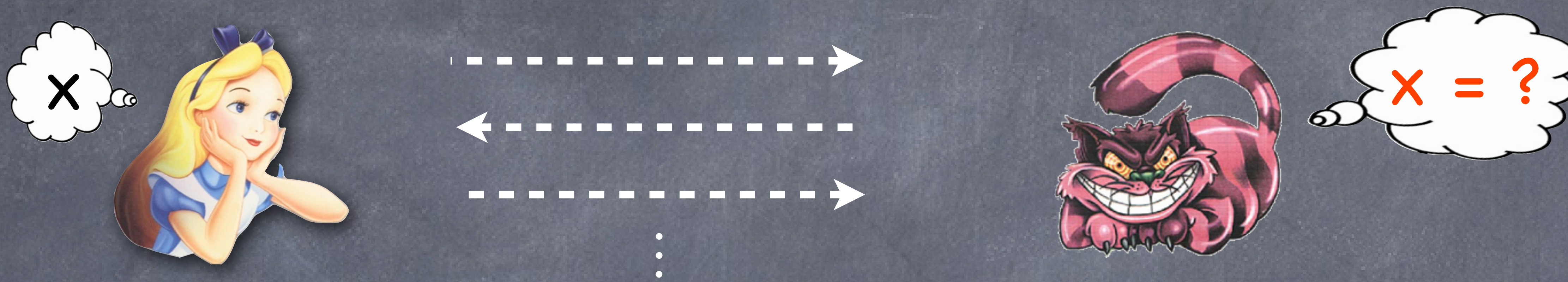
$$(U_A \otimes \mathbb{I}_B)\,|\psi^{x_0}\rangle_{AB} =$$

trouble starts here...

- dishonest Alice starts with input x_0, can read out $f(x_0,y)$, switches to x_1, reads out $f(x_1,y)$ etc.

$$(U_A \otimes \mathbb{I}_B)\,|\psi^{x_0,y}\rangle_{AB} = |\psi^{x_1,y}\rangle_{AB}$$

Security Against Players With Output



f(x,y) $\qquad |\psi^{x,y}\rangle_{AB}$ $\qquad$ f(x,y) $\perp$

- security against dishonest Bob without output:

$$\mathrm{tr}_A(|\psi^{x_0}\rangle\langle\psi^{x_0}|_{AB}) = \rho_B^{x_0} = \rho_B^{x_1} = \mathrm{tr}_A(|\psi^{x_1}\rangle\langle\psi^{x_1}|_{AB})$$

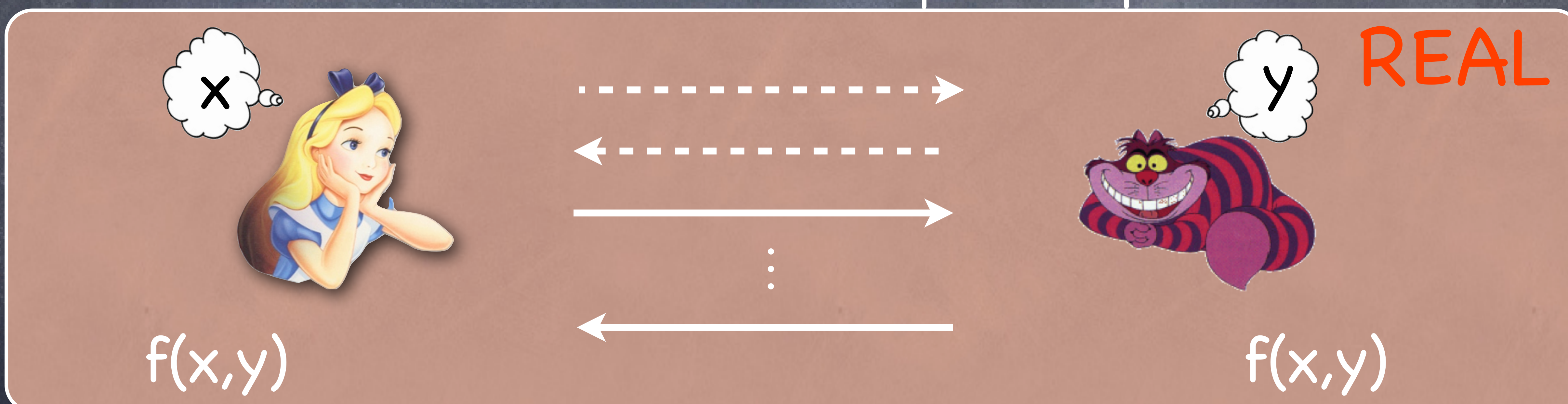
- but given f(x,y) ??? (e.g. in the millionaire's problem)
- precise formalisation of intuitive notion of "not learning more than f(x,y)" is non-trivial

use the real/ideal paradigm

Security Definition

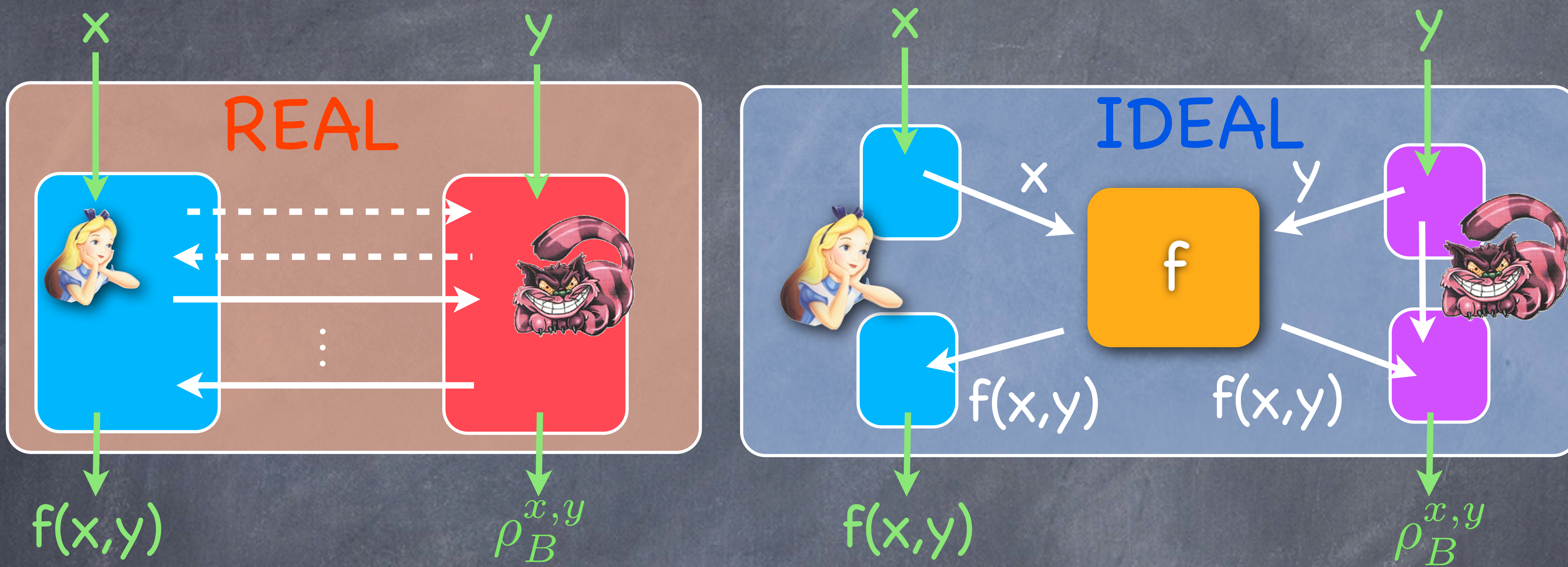
- we want: Alice & Bob interact with the ideal functionality

- we have: Alice & Bob interact in a quantum protocol



security holds if REAL looks like IDEAL to the outside world

More Formal Security Definition

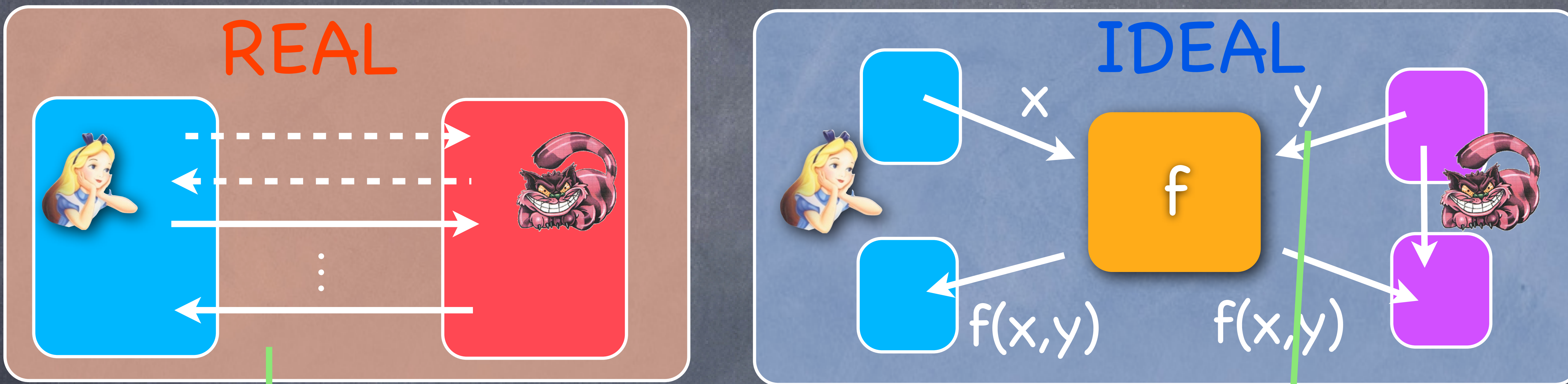


security holds if REAL looks like IDEAL to the outside world

- protocol is secure against dishonest Bob if
  - for every input distribution P(x,y) and $\rho_{XY} = \sum_{x,y} \sqrt{P(x,y)} |x\rangle_A |y\rangle_B$
  - for every dishonest Bob B in the real world,
  - there exists a dishonest Bob B in the ideal world
  - such that $\mathrm{REAL}(\rho_{XY}) = \mathrm{IDEAL}(\rho_{XY})$

Security against Bob => Insecurity against Alice

security holds if REAL looks like IDEAL to the outside world



$|\psi\rangle_{A_p A B B_p}$

state after the real protocol if both parties play "honestly" but purify their actions

$$\xrightarrow{\mathrm{tr}_{A_p}}$$

$$\rho_{ABB_p} = \sigma_{ABB_p} = \mathrm{tr}_Y(\sigma_{ABB_pY})$$

$\downarrow$ purification

$$|\phi\rangle_{ABB_pYP}$$

Security against Bob => Insecurity against Alice

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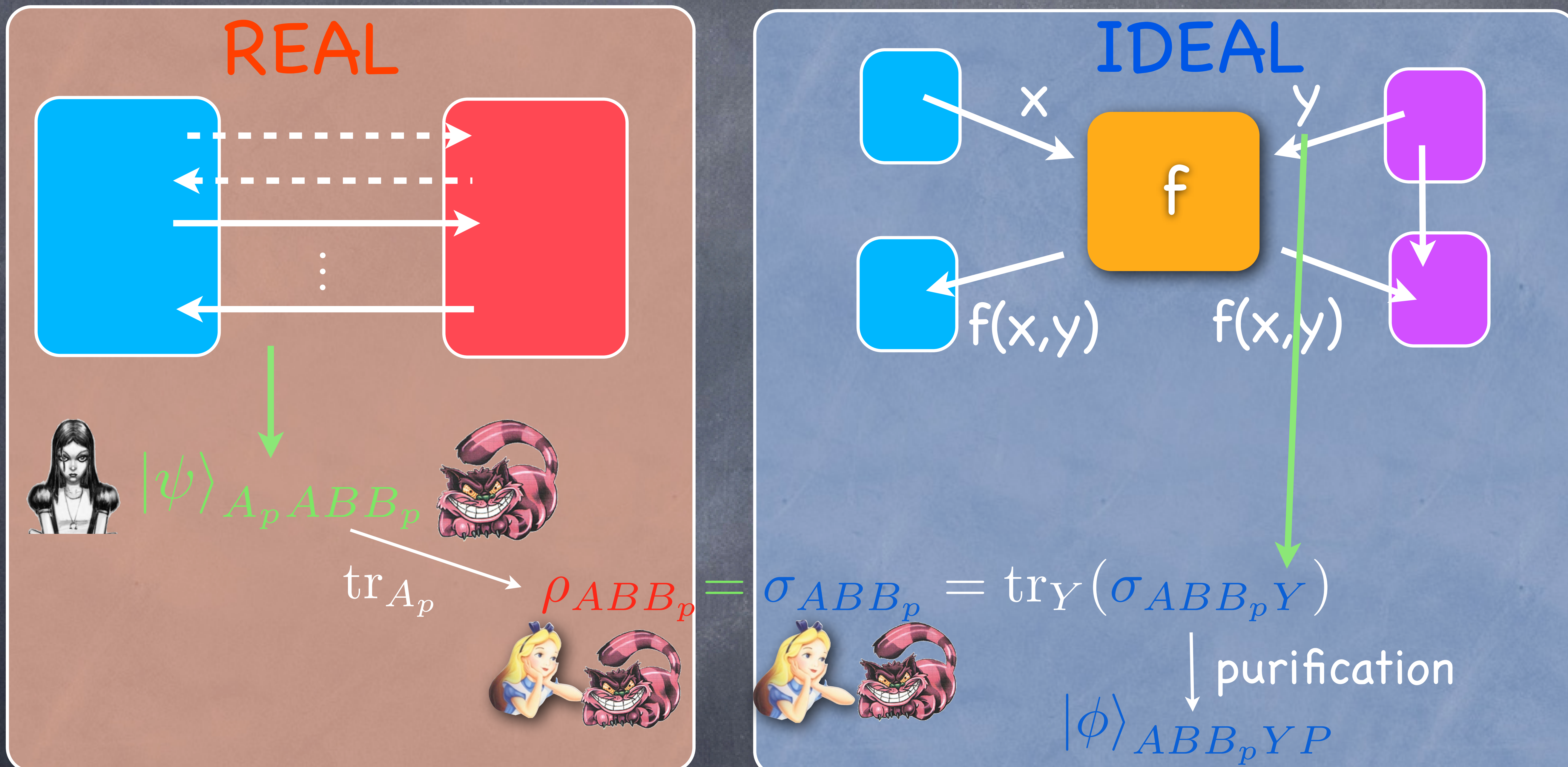
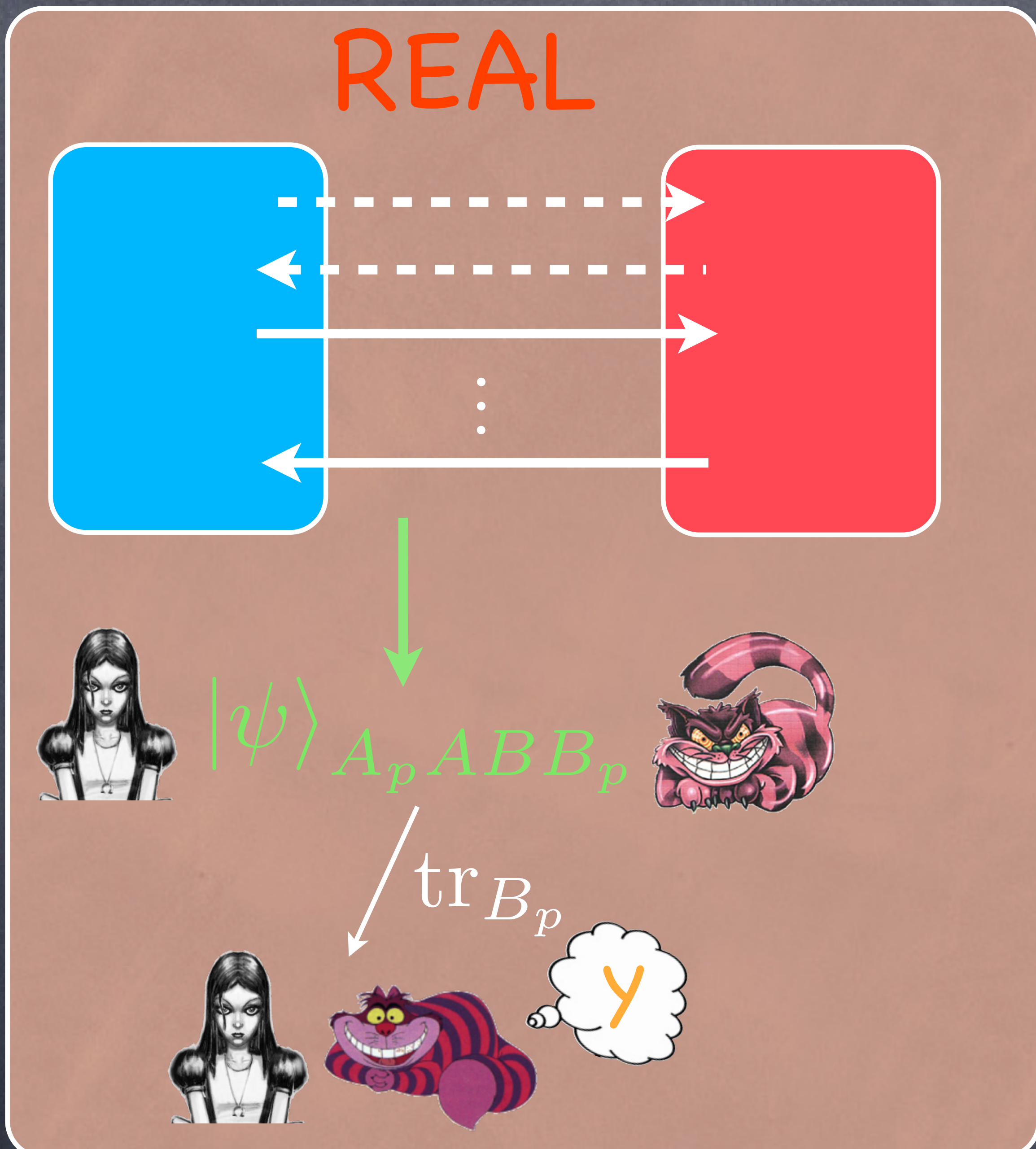
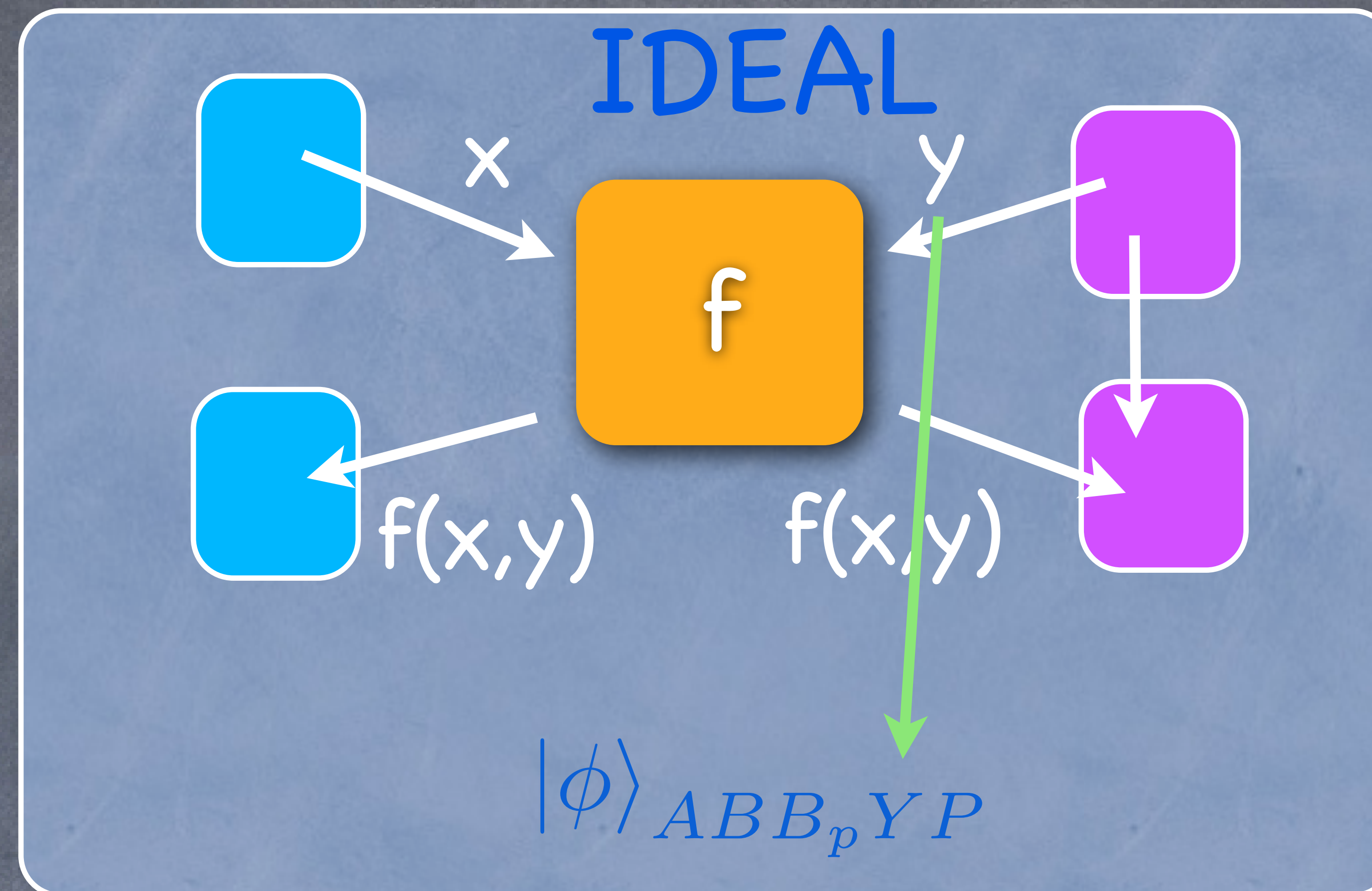


- by Uhlmann's theorem: there exists a cheating unitary U such that $U_{A_p \to YP} |\psi\rangle_{A_pABB_p} = |\phi\rangle_{ABB_pYP}$

Alice's Cheating Strategy





1. plays honest but purified strategy
2. she applies the cheating unitary U
3. measures register Y to obtain y.
4. due to correctness, we can show that for all x: f(x,y) = f(x,y).

$$U_{A_p \to YP} \, |\psi\rangle_{A_pABB_p} = |\phi\rangle_{ABB_pYP}$$

Error Case

- our results also hold for ε-correctness and ε-security

$$\|\mathrm{REAL} - \mathrm{IDEAL}\|_{\diamond} \leq \varepsilon$$

- Alice gets a value y' with distribution $Q(y'|y)$ such that for all x: $\Pr_{y'}[\ f(x,y)=f(x,y')\ ] \geq 1-O(\varepsilon)$,
- in contrast to Lo's proof where the overall error increases linearly with the number of inputs.
- crucial use of von Neumann's minimax theorem

Conclusion & Open Problems

- completes our understanding of why nature does not allow to do two-party secure computation.
- devil lies in details 
- is such a strong security definition necessary for impossibility proof? can it be done with a weaker definition?
- randomized functions?

Thank you!