Cryptographic Primitives

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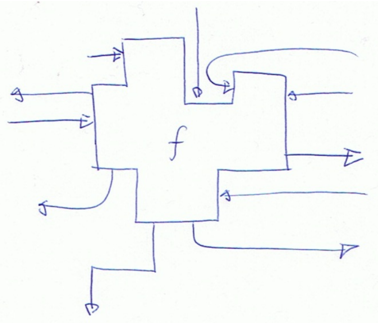
Université de Montréal
McGill University

What are cryptographic primitives?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Focus of This Talk:

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Importance to Quantum Information.

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Bias of the speaker...

Importance to Quantum Information

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Is it secure in the quantum setting?

Importance to Quantum Information

Is it secure in the quantum setting?

Can we do better in the quantum setting?

This Talk: Overview

- Basics of Secure Multi Party Computation
- Oblivious Transfer (OT)
- Bit Commitment (BC)
- Coin Flip (CF)

Secure Multi Party Computation (MPC)

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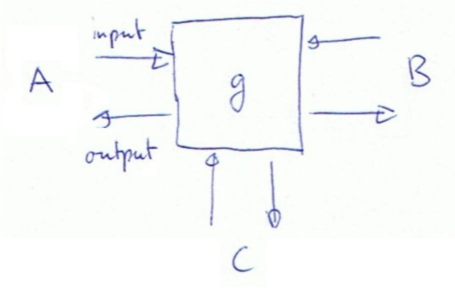
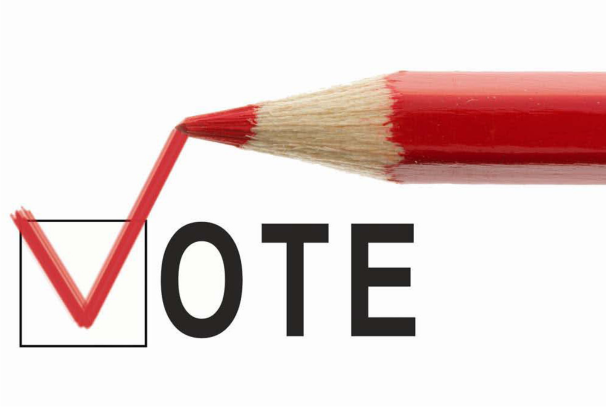
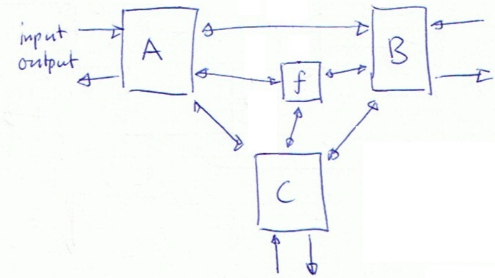
Introduced by [Yao 82]


input
A
g
B
output
C


VOTE

Cryptographic Protocol



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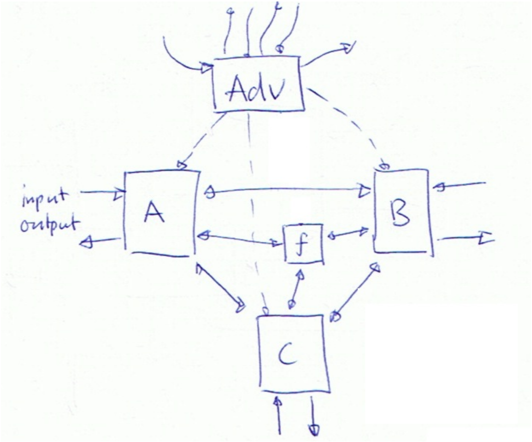

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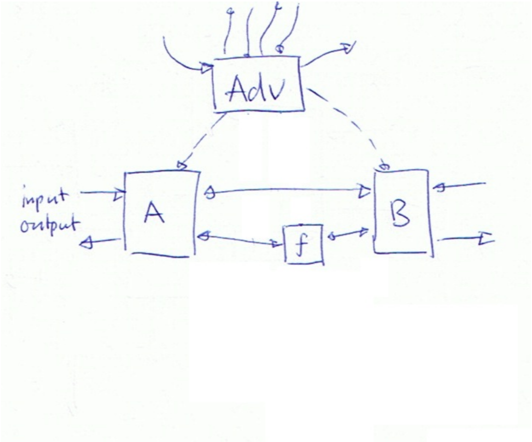

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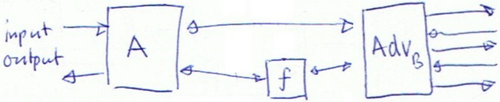

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- (Correctness) If both are honest, the protocol calculates g.
- (Sec. for B) Malicious A should not learn ... , except ... .
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Problems:

- Difficult to formalize.
- Ad hoc. Did we think of everything?
- How to use the primitive?

Security: Real vs. Ideal

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What do we want to achieve?

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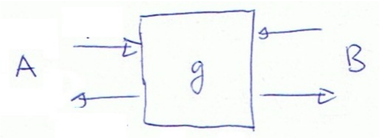

Show: the protocol implements g, **but nothing else**.

Security: Real vs. Ideal

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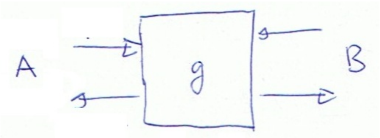
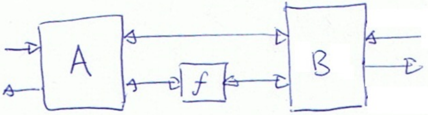


Show: the protocol implements g, **but nothing else**.

Anything the Adv can do in the protocol, he could also do with g.

Security: Real vs. Ideal



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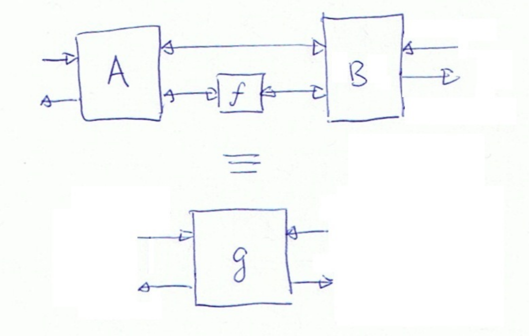

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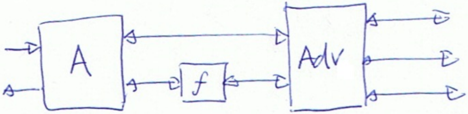

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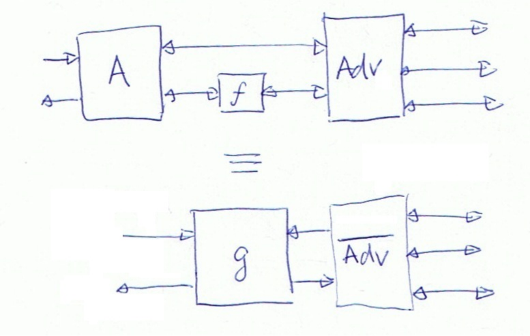

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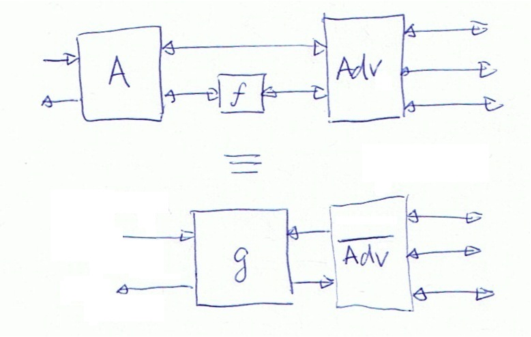

$$\forall\, Adv\ \exists\, \overline{Adv}$$

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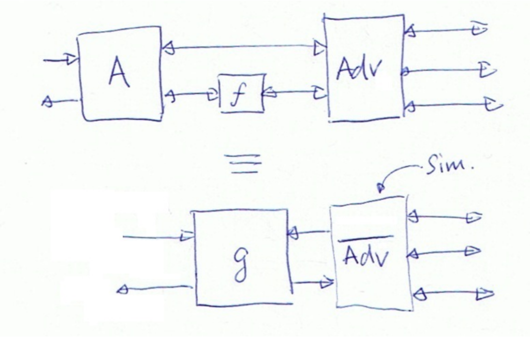

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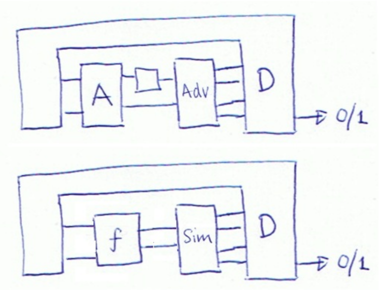

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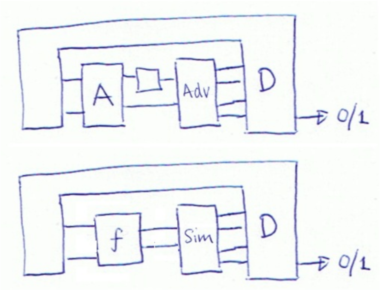

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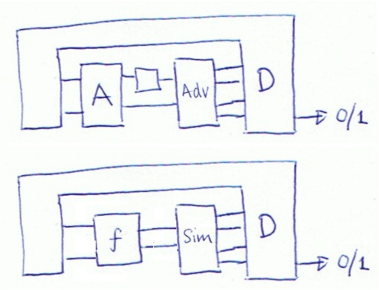

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$$\frac{1}{2}\|\rho_{real} - \rho_{ideal}\|_1 \leq \varepsilon \ .$$

Sequential vs. Universal Composability

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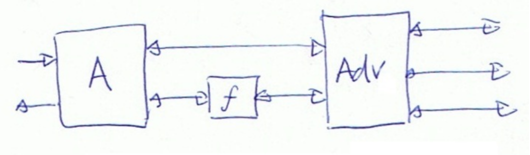

Online

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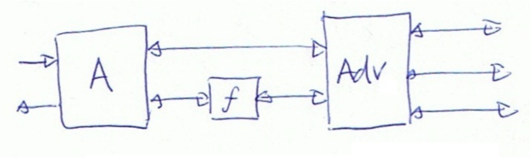

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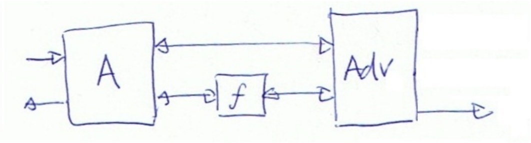

Offline

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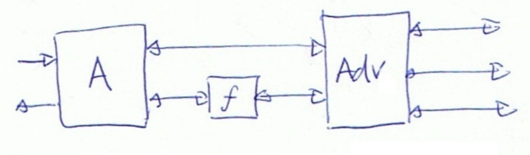
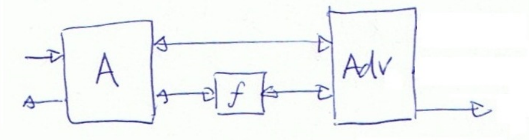
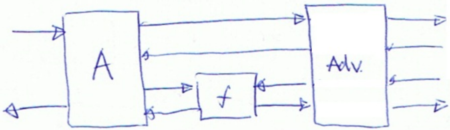


Online / Universal Composability (UC) [Canetti 01]



Offline / Sequential Composability [Beaver 92, Canetti 96]

Dummy Adversary



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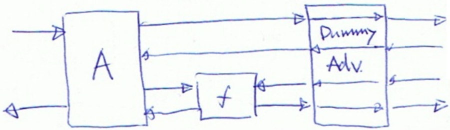

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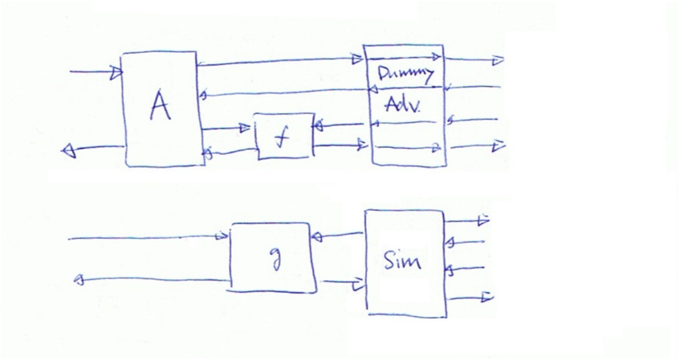

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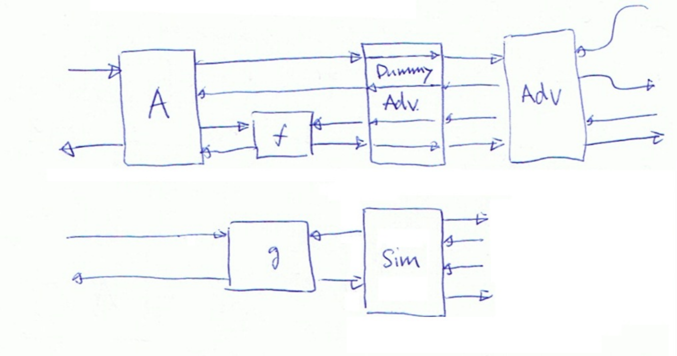

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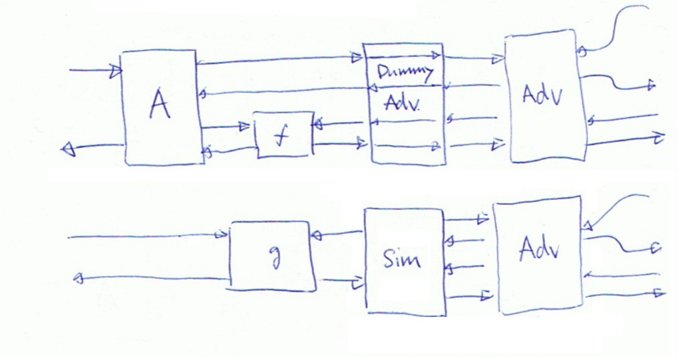

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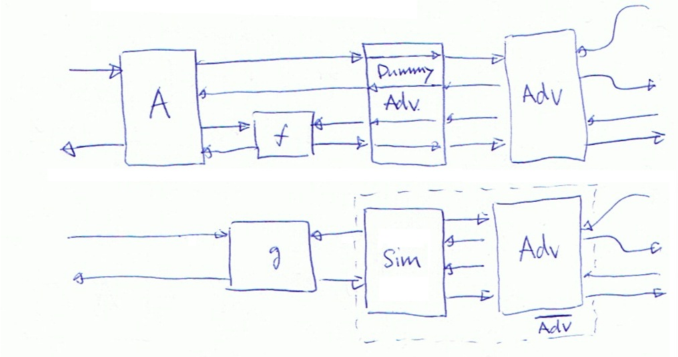

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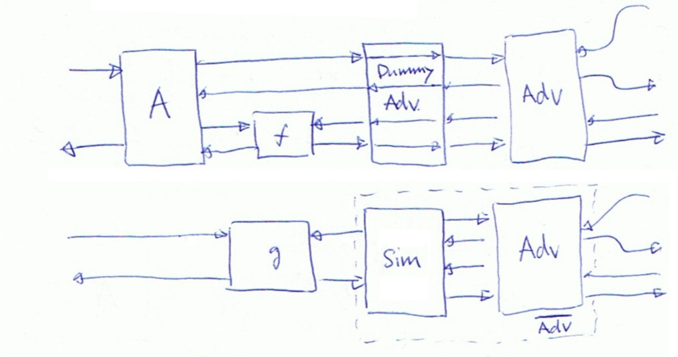

Sec. against dummy $\Rightarrow$ Sec. against **any** Adv!

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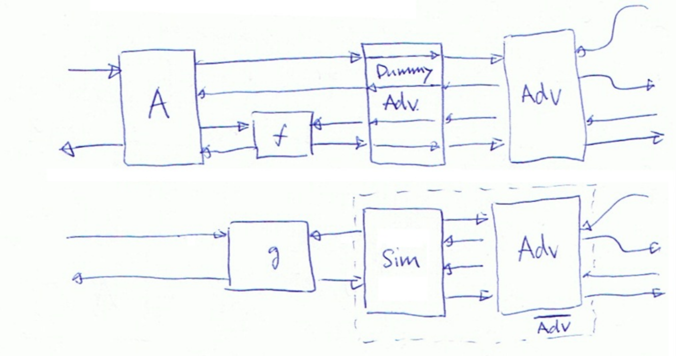

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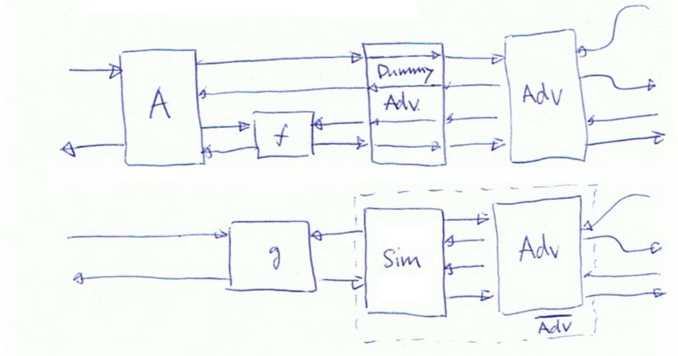

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Quantum Lifting Theorem: [Unruh10]

Classical UC implies Quantum UC.

The Semi-Honest Adversary

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- Follows the protocol.
- Remembers everything.

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Attention: Also the simulator must be semi-honest!

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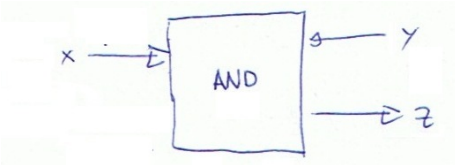

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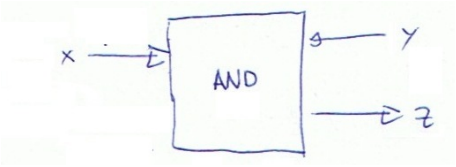

Malicious Model:

Protocol "A sends x to B" is secure!

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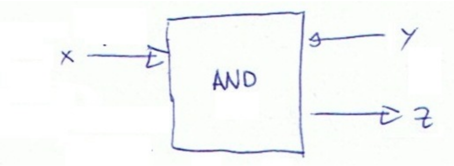

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...since B can always get x by choosing $y = 1$.

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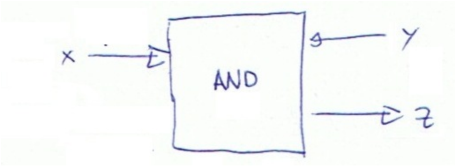

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Semi-Honest Model:

OT required.

Summary MPC

- Real vs. Ideal
- UC (Online) / Sequential (Offline)
- Classical UC $\Rightarrow$ Quantum UC

Further reading:

D. Unruh: "Universally Composable Quantum Multi-Party Computation", arXiv:0910.2912

S. Fehr, C. Schaffner: "Composing Quantum Protocols in a Classical Environment", arXiv:0804.1059

Oblivious Transfer

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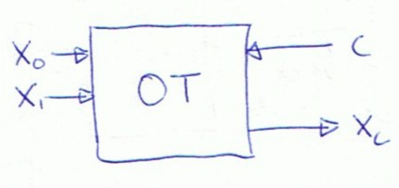

[Wiesner ∼69], [Rabin 83], [Even Lempel Goldreich 85].

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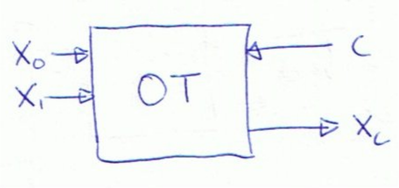

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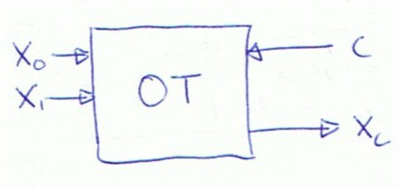

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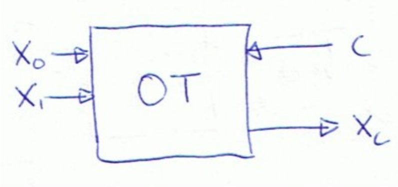

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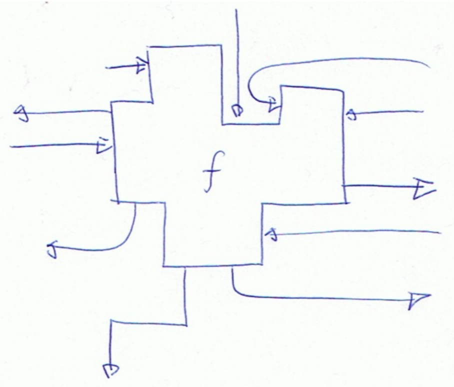
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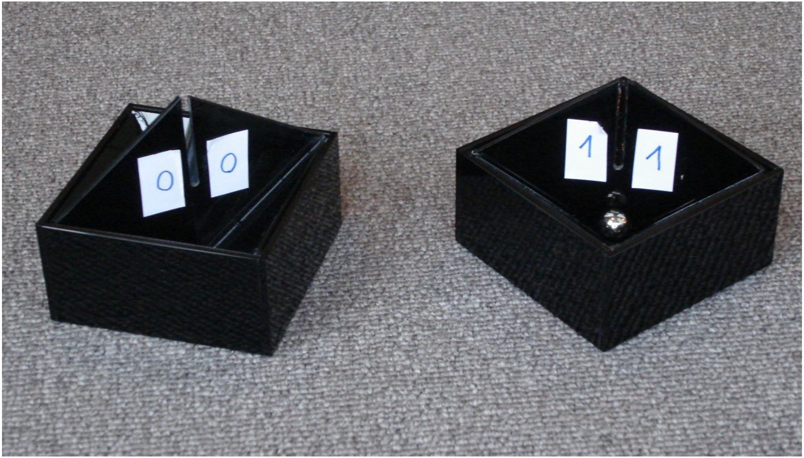
Oblivious Transfer - Model



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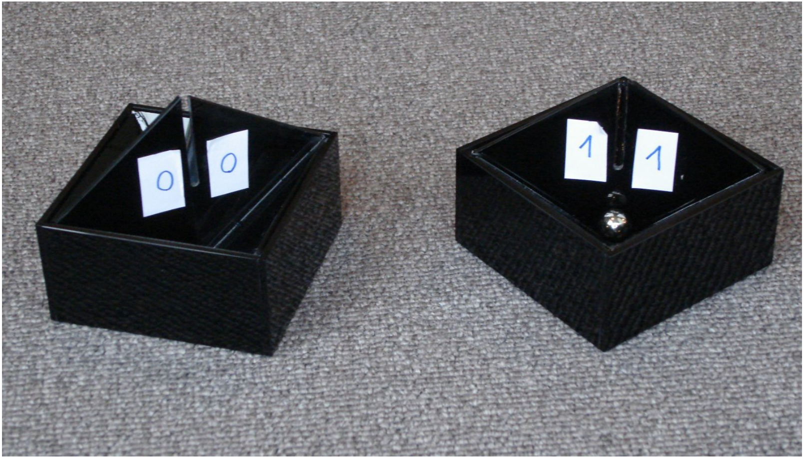

Note: OT does **not** allow input delay!

Oblivious Transfer Impossibility (Classically)

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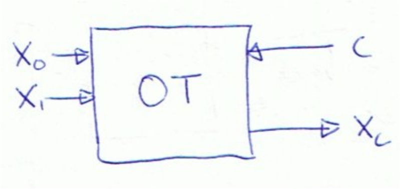
Boils down to:
If Bob doesn't leak his input c, but learns the output x_c, then Alice must send both x_0 and x_1.

Quantum OT?



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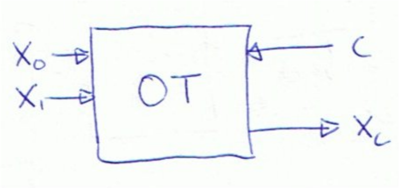
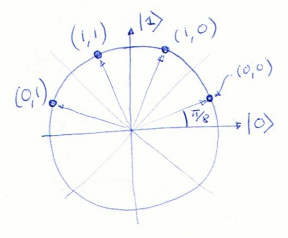


Wiesner: Invented OT to be implemented by a quantum protocol!

Quantum OT?

Encode 2 bits in one qubit:



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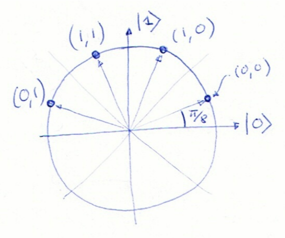

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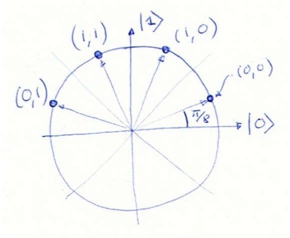

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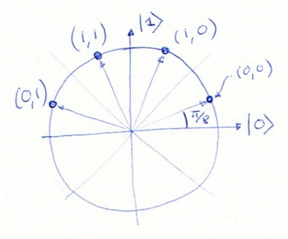

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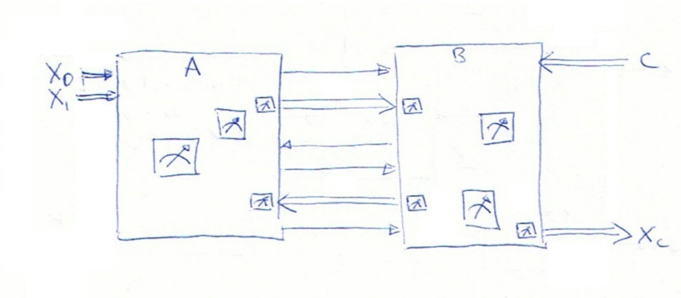
Impossibility of Quantum OT

Impossibility of Quantum OT - Purified Protocol



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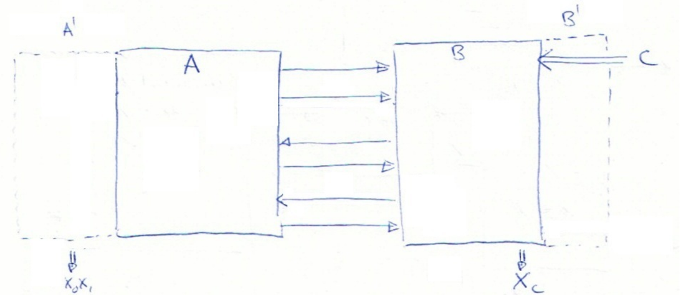

After the protocol execution: pure state $|\rho_c^{AA'BB'}\rangle$.

Equivalence of Purifications

For any $|\rho^{AB}\rangle$, $|\phi^{AB}\rangle$:
If $\rho^A = \phi^A$, then there exists an U^B, such that

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ε: Uhlmann's Theorem.

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Therefore, Bob can change c **after** the protocol is over! **Insecure.**

Stronger: Bob can also get x_0, apply $U^{BB'}$, and get x_1.

Extending OT?

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What if we are given a small number of OTs?
Can we make $n+1$ from n? OTs?

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Given: n OT's. Create $m > n$ OT's.

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Implies that $n+1$ from n OTs is impossible.

Note: Bound is weaker than in the classical setting.

We need Additional Assumptions

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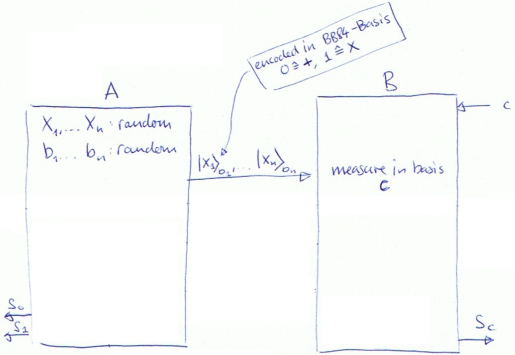
Bounded/Noisy Quantum Storage Model:

Adversary does not have an unlimited, perfect quantum storage.

OT in the Bounded Quantum Storage Model [. . . ,DFRSS07]



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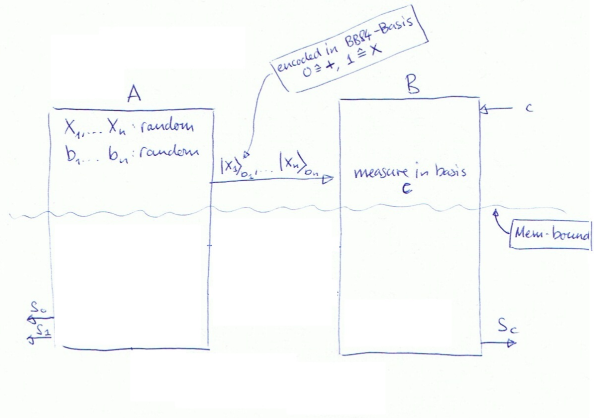

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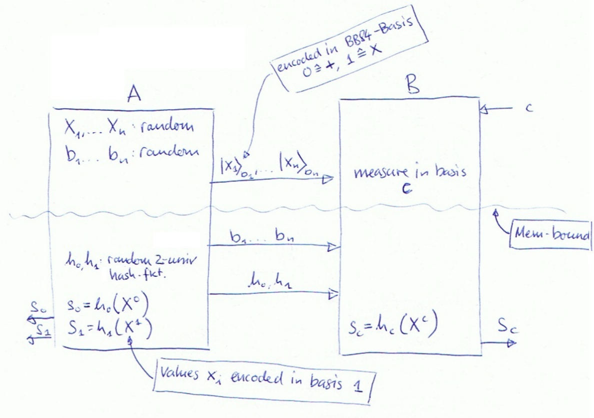

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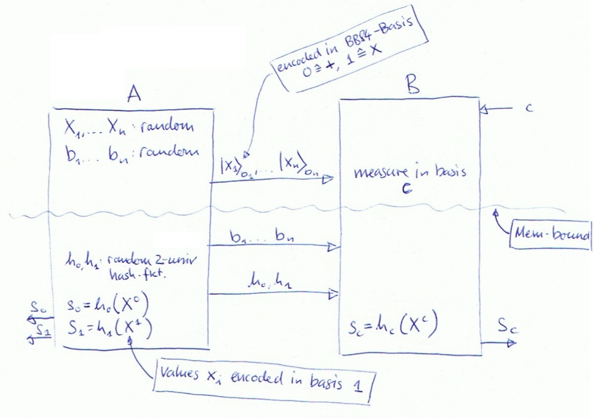

Proof: Uncertainty relation + privacy amplification.

Use OTs from MPC

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Semi-Honest Model

Share Secrets. Evaluate circuit gates, one-by-one.

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Somehow force players to follow protocol.

[Crépeau van de Graaf Tapp 95]: Use bit commitments.

[Ishai Prabhakaran Sahai 08]: Use an MPC-in-the-head.

Summary OT

- OT: Simple + Useful.
- Creating / Extending OT is impossible.
- OT is possible in BQS model.

Further reading:

S. Winkler, J. Wullschleger: "On the Efficiency of Classical and Quantum Secure Function Evaluation", arXiv:1205.5136

I. Damgaard, S. Fehr, R. Renner, L. Salvail, C. Schaffner: "A Tight High-Order Entropic Quantum Uncertainty Relation With Applications", arXiv:quant-ph/0612014

Y. Ishai, M. Prabhakaran, and A. Sahai: "Founding Cryptography on Oblivious Transfer - Efficiently", CRYPTO 08.

Bit Commitment (BC)

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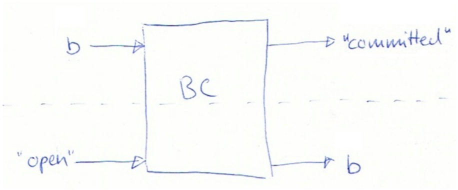
First formally defined in [Bennett Brassard Crépeau 88]

aka: Commitment, Commitment Scheme, Commit-and-Open, Commit-and-Reveal, ...

Bit Commitment



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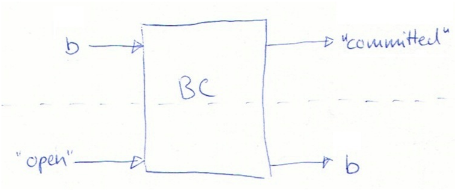

Mostly used to force players to follow the protocol.

BC implementations

Quantum protocol for creating BC

[Mayers 97, Lo Chau 97]: Impossible. Basically the same proof as for OT.

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Basic Idea:

- Use a protocol very similar to the BQSM-protocol from before.
- Bob commits to **all** his measurement basis and outcome.
- Cut-And-Choose: Alice asks Bob to open a small subset and checks.

Summary BC

- Quantum BC is impossible.
- OT $\rightarrow$ BC.
- Quantum: BC $\rightarrow$ OT.

Further reading:

C. Crépeau, J. van de Graaf, A. Tapp: "Committed Oblivious Transfer and Private Multi-Party Computation", www.cs.mcgill.ca/∼crepeau/PS/CGT95.ps

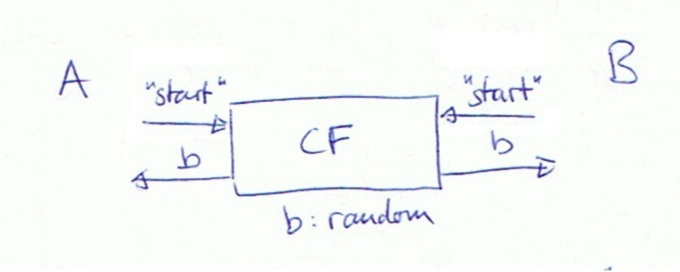
Niek J. Bouman, Serge Fehr: "Sampling in a Quantum Population, and Applications", arXiv:0907.4246

Coin Flip

Coin Flip

Introduced by [Blum 81]

Coin Flip



Relativistic Coin Flip

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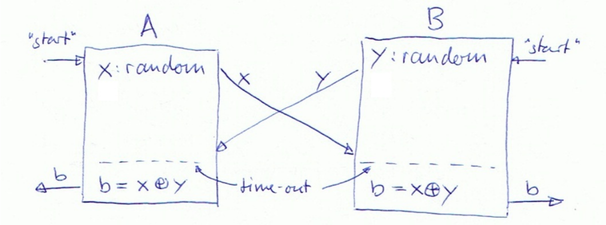

Coin Flip from BC

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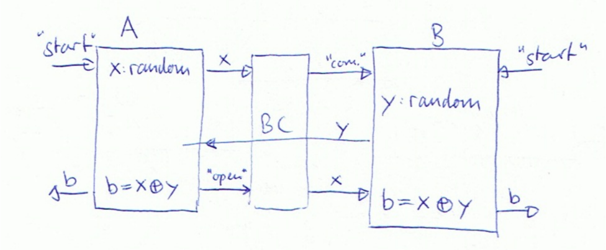

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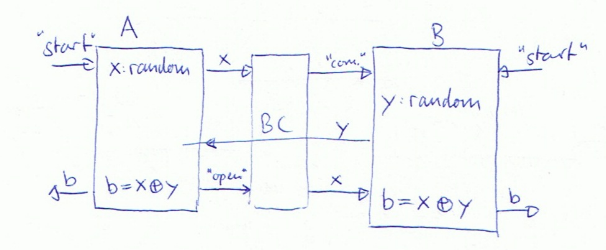
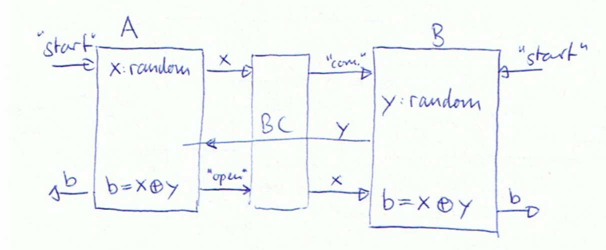


Secure?

Coin Flip from BC

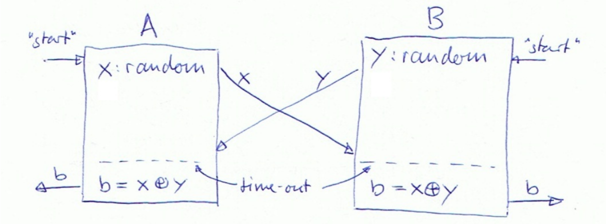


Secure?

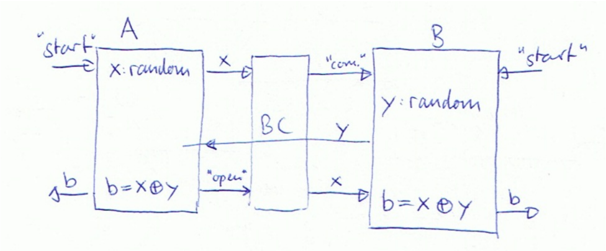
Alice can refuse to open!

Coin Flip from BC

But we can also abort here!



Coin Flip from BC - Problem



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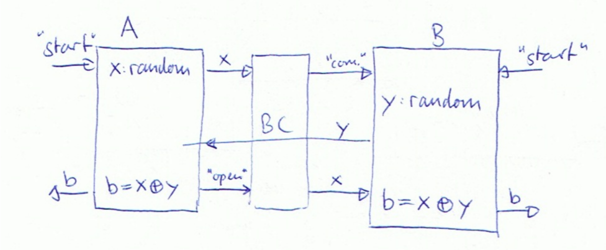

Unfair, because Alice can **SELECTIVELY** abort. E.g., for $y = 0$.

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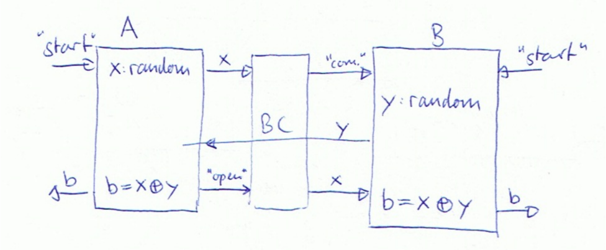

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But should we care! We then know that she is cheating!

Forest-Crossing Problem

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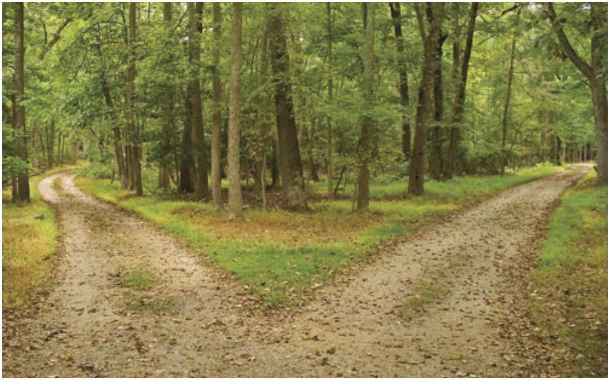

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[Moran Naor Segev 09]

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Most Fkt. with 2 outputs have this problem.

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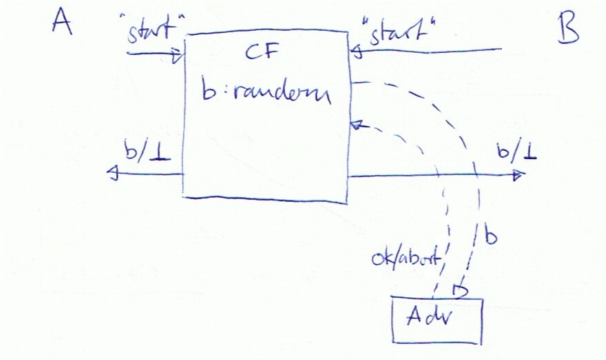

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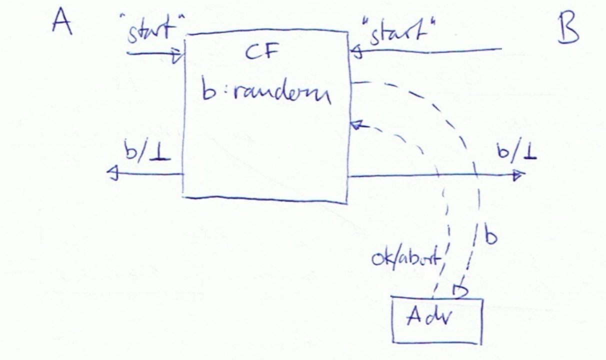

Equivalent to "Strong Coin Flip".

Coin Flip Variants

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Note: WCF cannot be unfair.

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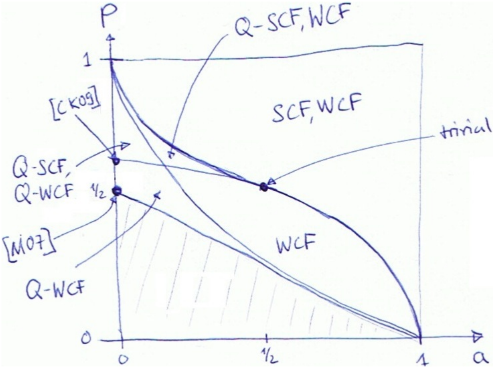
Long line of research: [Aharanov Ta-Shma Vazirani Yao 00, Ambainis 01, Spekkens Rudolph 01, Kitaev 02, Spekkens Rudolph 02, Mochon 04, Hofheinz Müller-Quade Unruh 06, Mochon 07, Nguyen Frison Huy Massar 08, Chailloux Kerenidis 09, Hänggi W. 11]

WCF and SCF Bounds.

a: abort probability, p: max. probability of a value.



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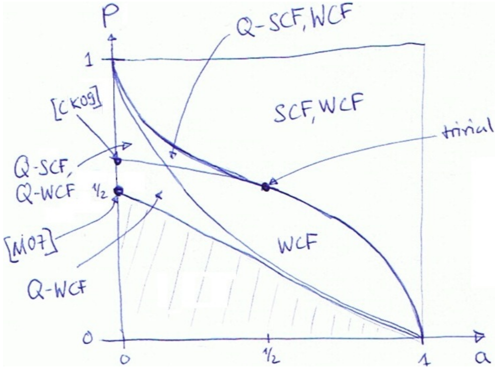

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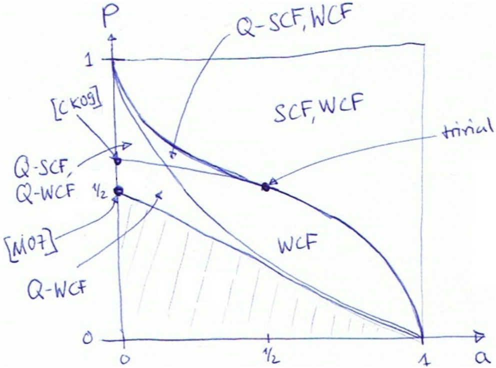

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Fair CF???

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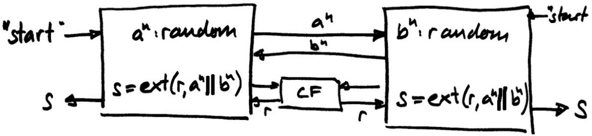

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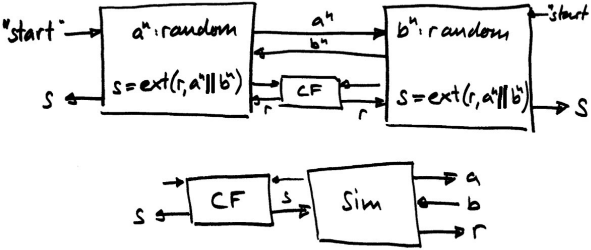

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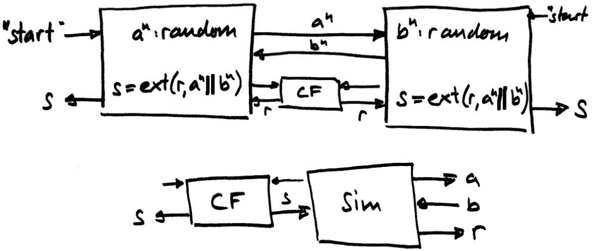

It is unlikely that Sim can find a r with:

$$s = ext(r, a^n \| b^n) \; .$$

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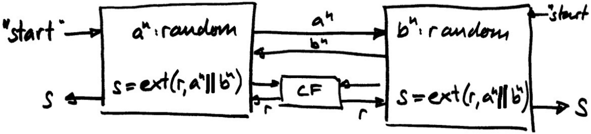

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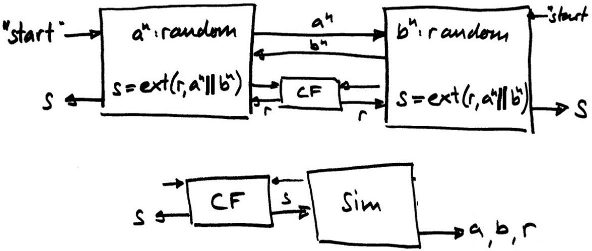

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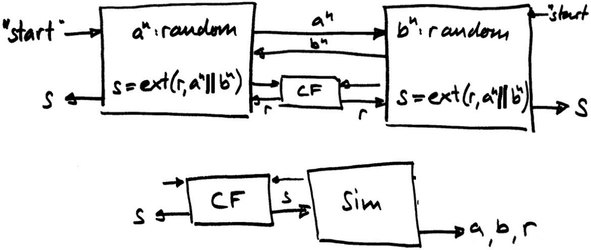

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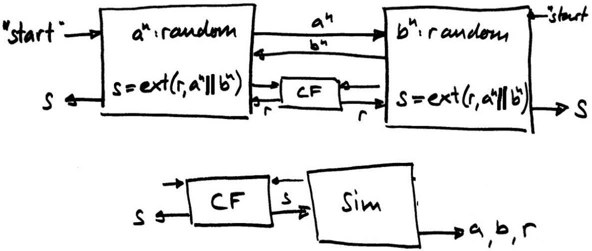

Works also against quantum adversary. UC?

Summary Coin Flip

- Three types: fair CF, (unfair) SCF, WCF.
- BC $\rightarrow$ SCF.
- Quantum WCF possible, others not.
- Optimal quantum SCF achieved by classical protocol using WCF.

Further reading:

R. Cleve: "Limits on the security of coin flips when half the processors are faulty", STOC 86

C. Mochon: "Quantum weak coin flipping with arbitrarily small bias", arXiv:0711.4114

D. Hofheinz, J. Müller-Quade, D. Unruh: "On the (Im-)Possibility of Extending Coin Toss", on eprint.iacr.org/2006/177

Last Slide

Some interesting open problems:

- Efficiency bounds for WCF.
- [Cleve 86] in quantum setting.
- Improve OT impossibility bounds.
- Q/C bounds for fair (non-aborting) coin flip.
- Improve OT protocols: many bit-OT instead of one string-OT.

Thanks.