Photon-Pair States and Violation of CHSH Inequality

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***Abstract*—Entangled states are important for quantum cryptography, being used in several protocols of quantum key distribution. These provide an alternative to single-photon-based protocols. Entangled states are also important in fundamental terms, in the way that the behavior of these states is contradictory with the predicted by local theories. This contradiction is shown by the violation of Bell's inequalities, as the inequality proposed by Clauser, Horne, Shimony and Holt (CHSH). In this work [1] we shown the violation of CHSH inequality using two-particle states. We discussed pure states, an arbitrary state, and the Greenberger-Horne-Zeilinger (GHZ) state. Using the spontaneous four-wave mixing process we generated entangled photon pairs in a highly non-linear fiber, and experimentally verified the violation of CHSH inequality.**

INTRODUCTION

Bell's theorem shows that complete and local theories, i.e., the results of a particle depend only on that particle, lead to different statistical results of quantum theory. Bell's theorem reflects a contradiction between the statistical constrains of correlation measurements imposed by local theories of hidden variable (LTHVs). This theorem also predicts the existence of states composed of pairs of particles which cannot be described by LTHVs, thus violating Bell's inequality. Entangled states can violate Bell's inequality, however not all entangled states violate it. Therefore, we say that quantum entanglement is a necessary but not sufficient condition to verify Bell's theorem.

VIOLATION OF CHSH INEQUALITY

The entangled states arise when more than one particle is in the same Hilbert space, i.e. when it is impossible to factorize its state in individual states. These states are a purely quantum phenomenon, without classical equivalent.

Pure Entangled States

The classic example of entangled-photon-pair states are the pure states or the so-called Bell states. We found that the maximum of violation of Bell's inequality using this state has a Shannon entropy of $S = 2\sqrt{2}$, and it is verified for specific experimental configurations.

General GHZ States

The general GHZ states, which include the pure entangled state, are important in quantum technologies, specially in quantum cryptography. The maximum of Shannon entropy is $S = 2\sqrt{\sin^2(2\phi) + 1}$, which is a sufficient condition to verify Bell's theorem. Then, we can say that general GHZ states violate CHSH inequality when $0 < \phi < \pi/2$, achieving the maximum when $\phi = \pi/4$, which corresponds to the pure entangled state.

General State

Regarding the general state, we found the maximum of Shannon's entropy, considering a state of two particles, starting from the correlation function. The definition of the maximum Shannon's entropy, which is a sufficient condition to verify Bell's theorem, is closely related to the definition of the correlation function. This relationship between these two quantities gives us clues about one possible general solution of Bell's theorem for states of N-particles.

EXPERIMENTAL VALIDATION

In order to verify experimentally the violation of CHSH inequality, we generated polarization-entangled photon pairs in the pure state. Our experimental setup took advantage of the spontaneous four-wave mixing process in a highly non-linear fiber to generate the polarization-entangled photon pairs. Using the experimental configurations determined theoretically for the maximum of Shannon's entropy from the pure entangled state, we obtained an experimental value for Shannon's entropy, $S = 2.47 \pm 0.17$, which is higher than the classical limit of 2. Thus, we verified the violation of CHSH inequality.

CONCLUSION

We addressed comprehensively the fundamental problem of Bell's theorem for systems of two particles. We verified theoretically the violation of CHSH inequality not only for the most important states, such as the pure entangled states and the general GHZ states, but also for an arbitrary state. We also verified experimentally the violation of CHSH inequality of pure entangled states, using an experimental setup which generates polarization-entangled photon pairs directly into a highly non-linear optical fiber, suitable for implementation of quantum cryptographic protocols.

REFERENCES

[1] L. P. Martins, A. J. Almeida, P. S. André, and A. N. Pinto, "Photon-pair states and violation of CHSH inequality," *Microwave and Optical Technology Letters*, vol. 54, no. 11, pp. 2454–2461, 2012.