One-Sided Device Independence of BB84 Via Monogamy-of-Entanglement Game

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(have they already been successfully attacked, e.g. fair sampling?)

2. Formalize Security ✓

(there is almost universal agreement on how to do this for QKD)

3. Prove security using the laws of quantum mechanics applied to the formalized protocol/assumptions (✓)

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4. Is the protocol feasible?

(using current technology, does the protocol ever output something non-trivial?)

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There does not currently exist a protocol/proof for which both 1. and 4. have a satisfactory answer.

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Solution	Assumption	Feasibility
Ignore them!	fair sampling	key is produced
Randomize!	none	too many errors

Problem is not solved yet!

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Interesting approaches:

- Restrict adversary, e.g. no long-term memory (Pironio et al.)
- Allow some device assumptions: measurement device independent QKD (Lo/Curty/Qi, Braunstein/Pirandola), **one-sided device independent QKD**

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We show that BB84 is one-sided device independent

The Uncertainty Principle

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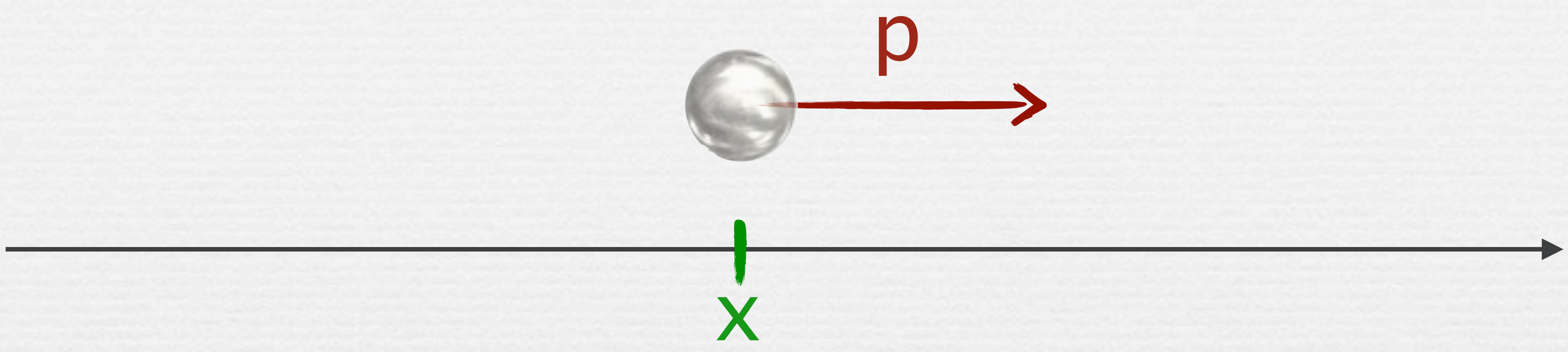

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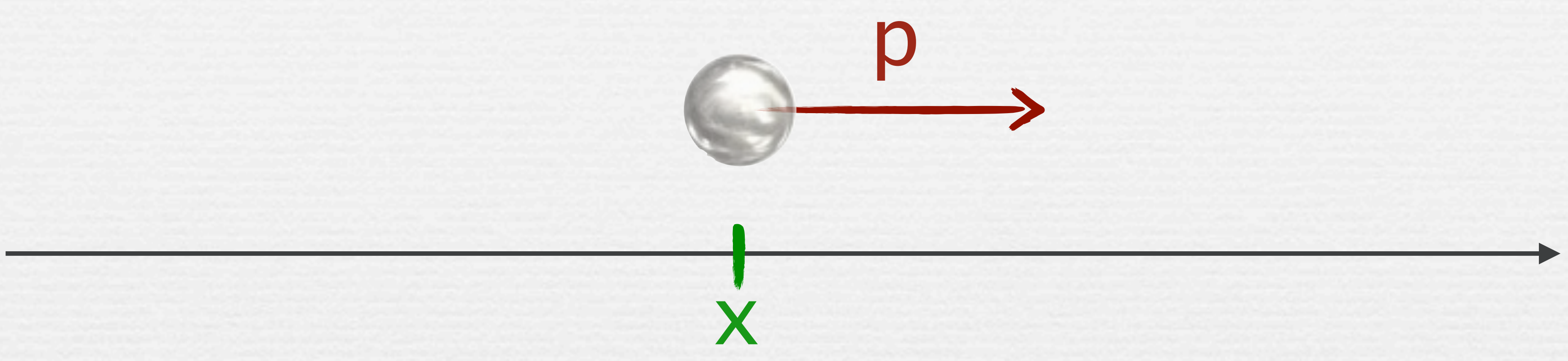

Heisenberg

It is **impossible** that **both** the position x **and** the momentum p are fully determined.

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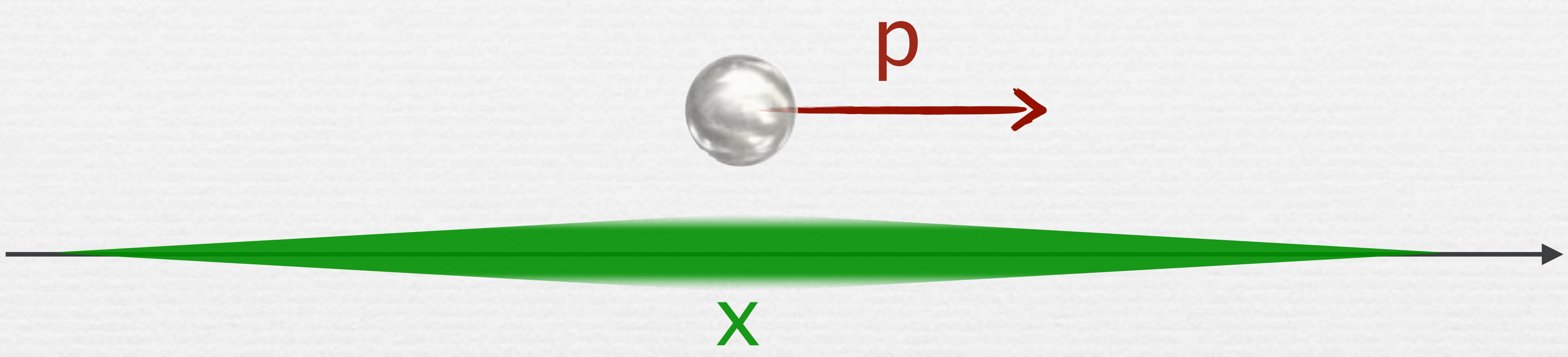

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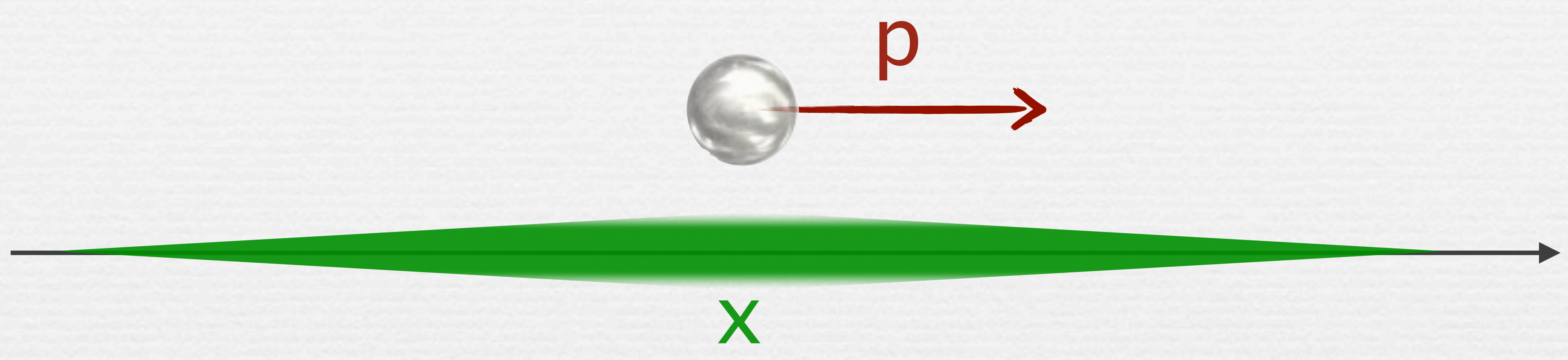

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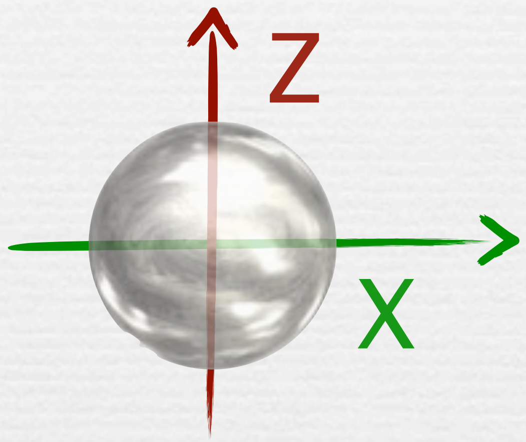
Many different formalizations of this statement have been proposed.

The Uncertainty Principle

Example: Polarization in X and Z direction



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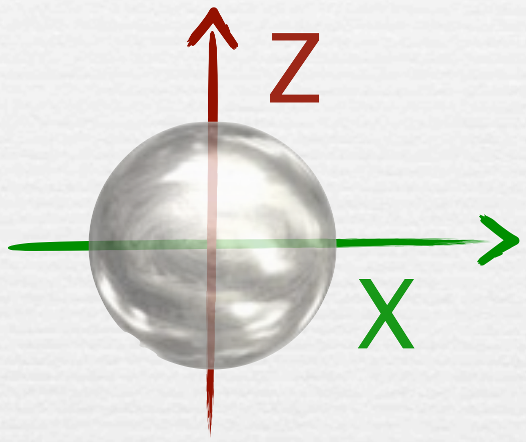

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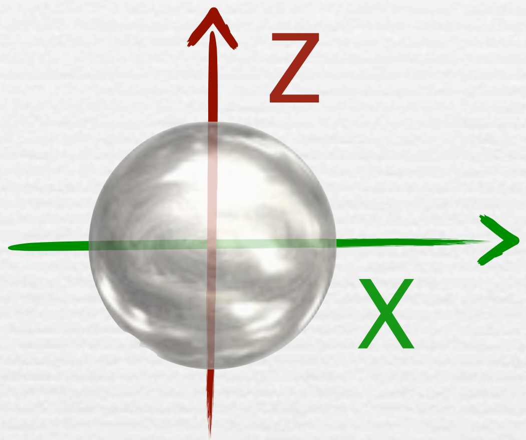

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More formally: $p_{\text{guess}}(X) + p_{\text{guess}}(Z) \leq 1 + \frac{1}{\sqrt{2}}$

Monogamy of Entanglement

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- As given above: is a qualitative statement.
- Exist different quantitative statements.
- Part of our contribution:
  - new way to get a quantitative statement
  - with applications to quantum crypto

A Monogamy (of Entanglement) Game

ALICE
(Game Master)

BOB

CHARLIE

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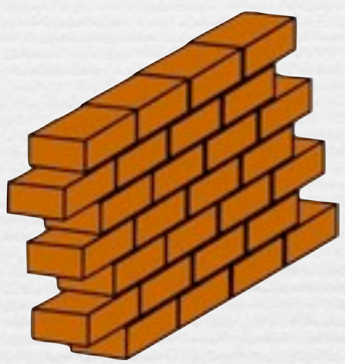

CHARLIE

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Set up:

- $A = A_1 \ldots A_n$: n qubits
- B & C : arbitrary many qubits
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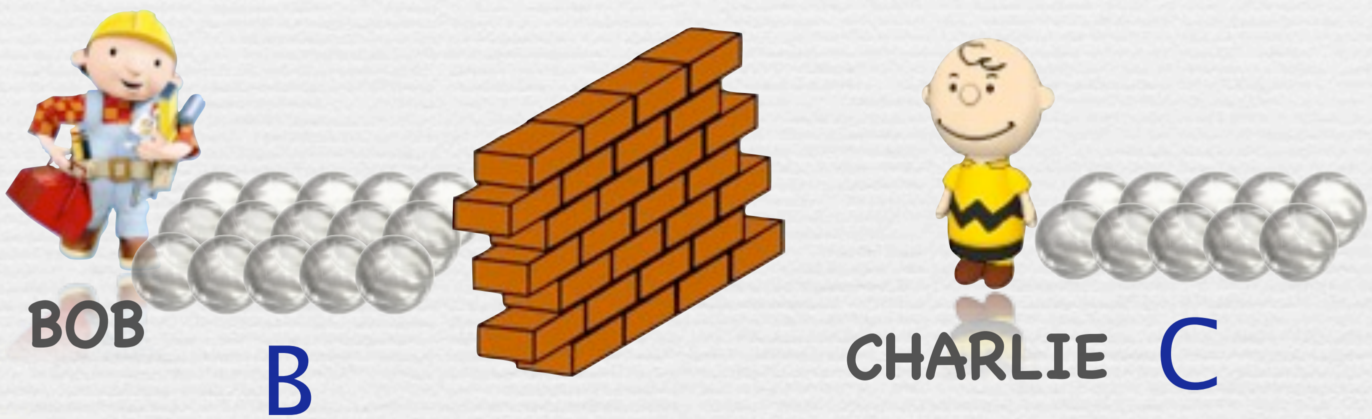

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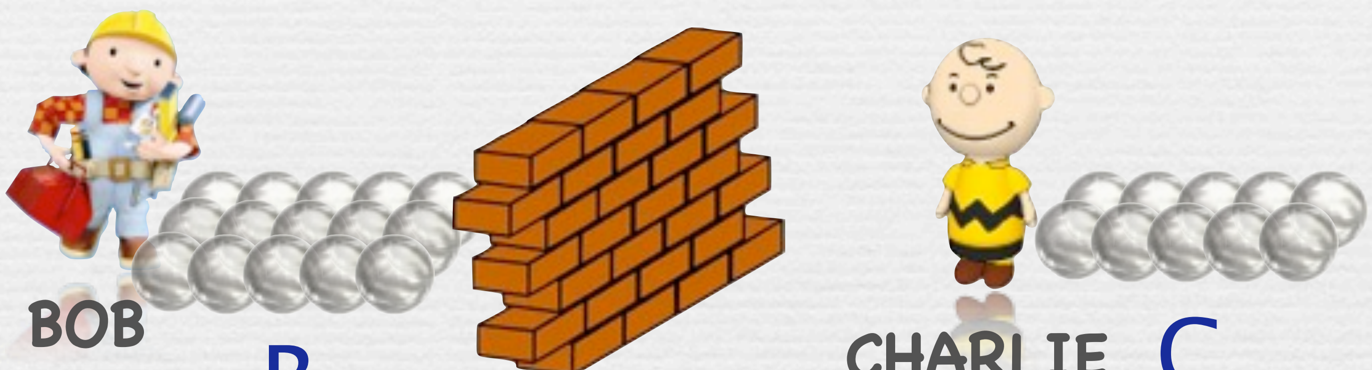

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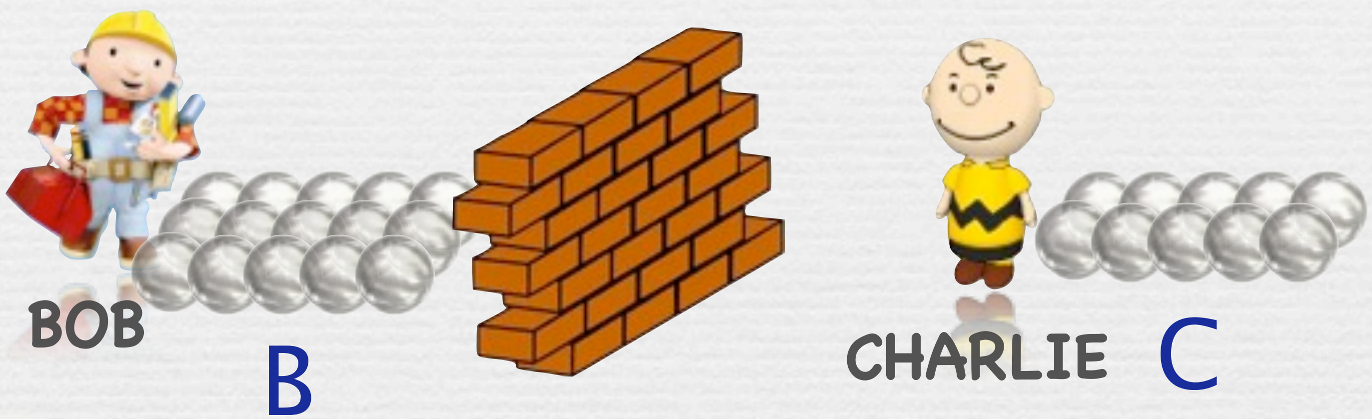

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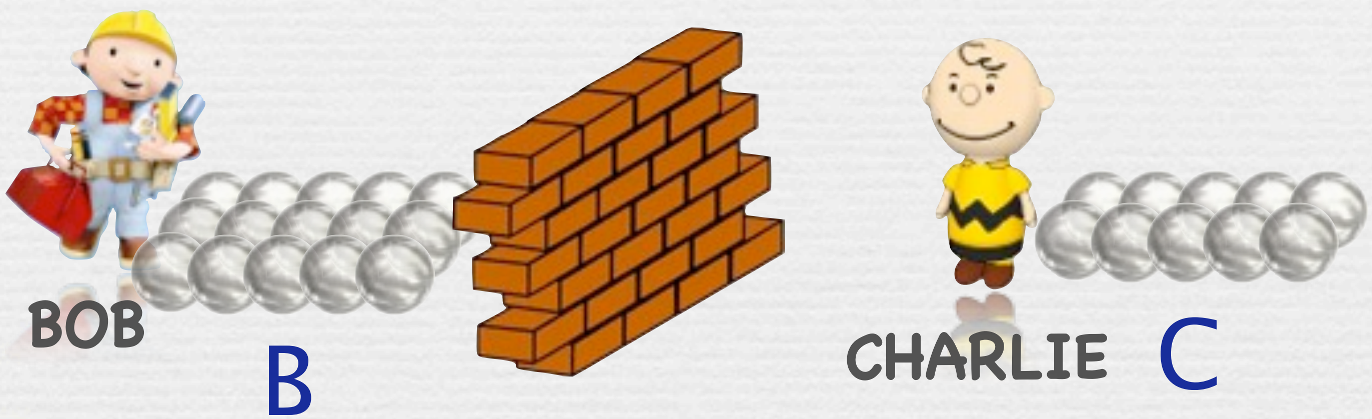

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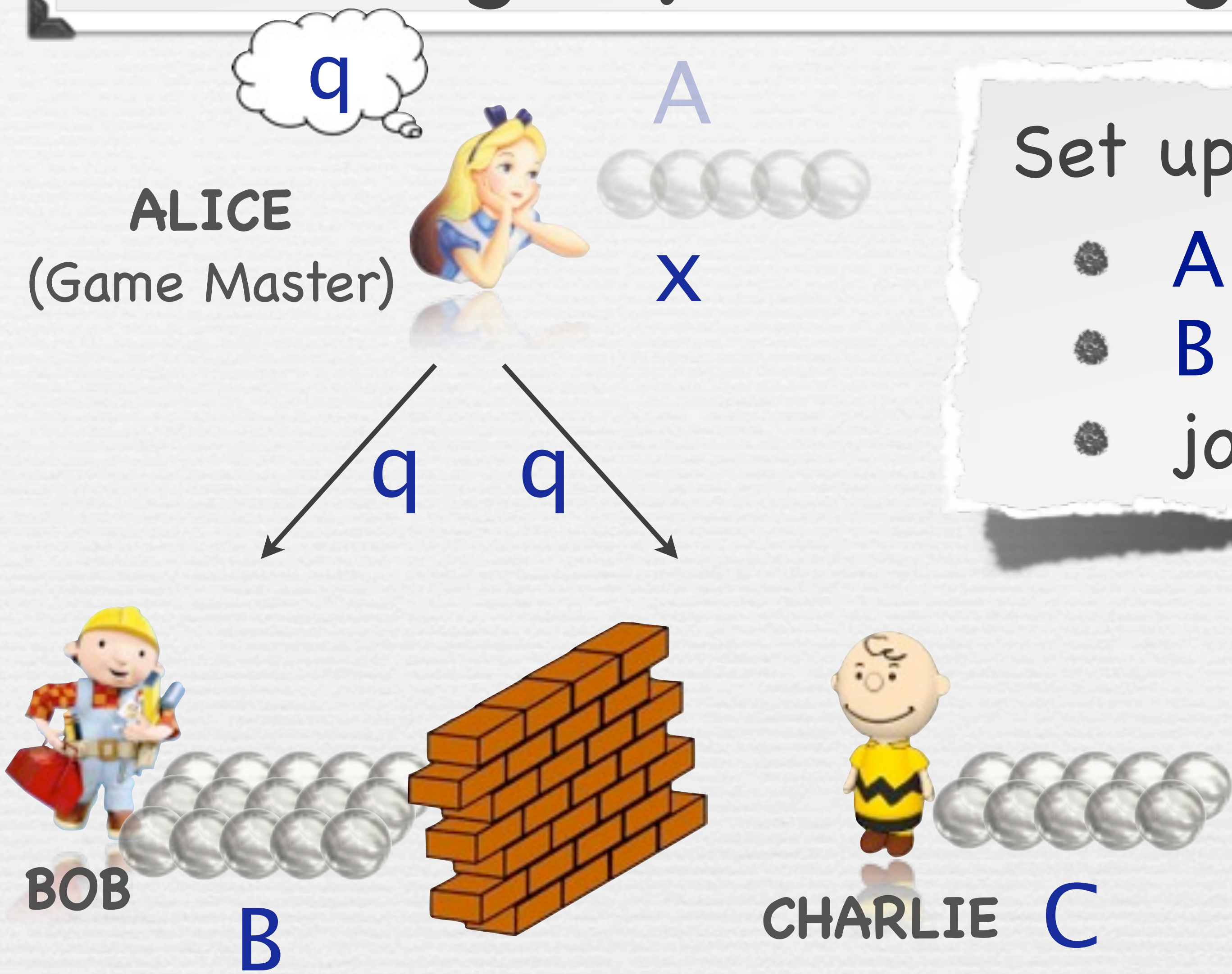

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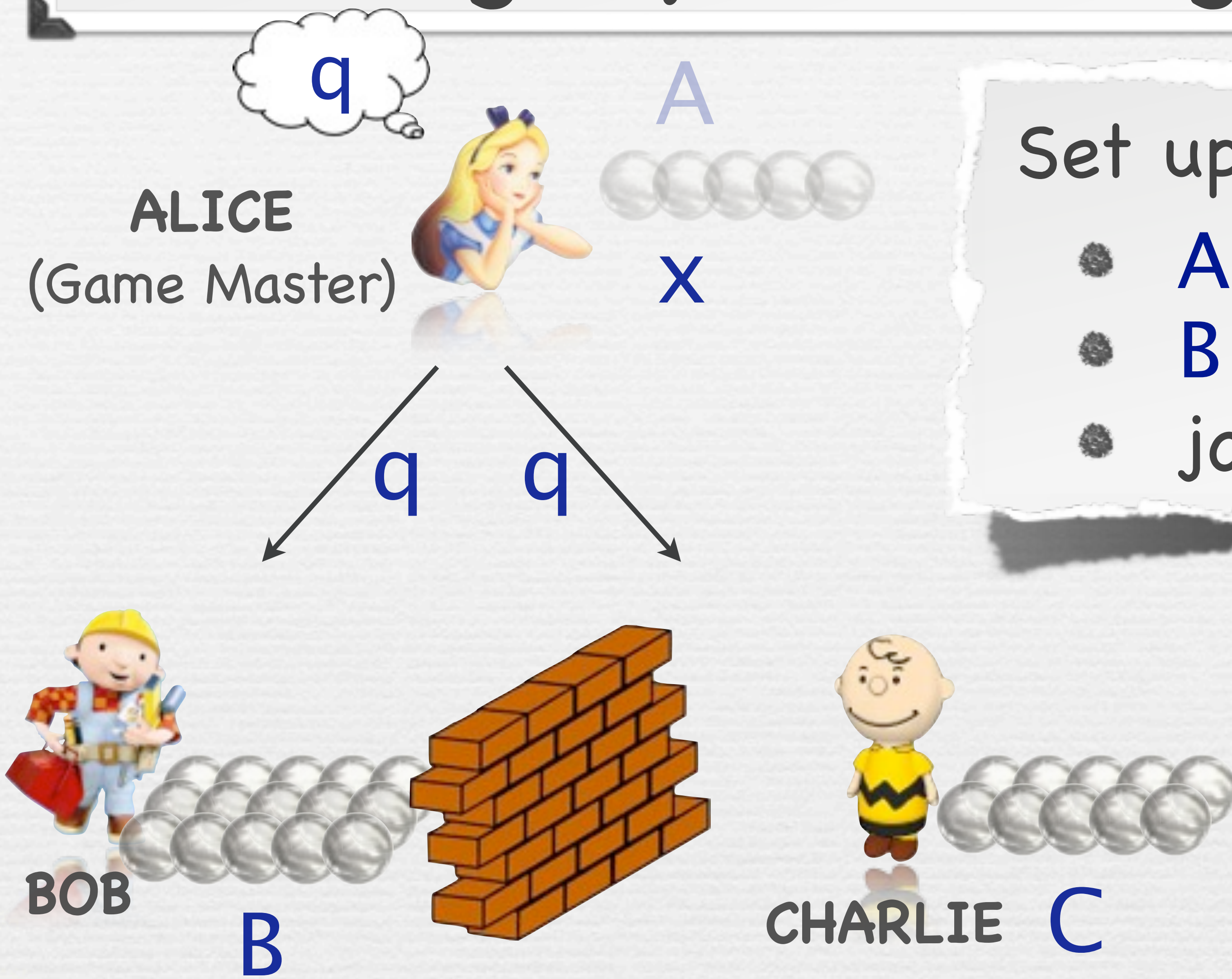

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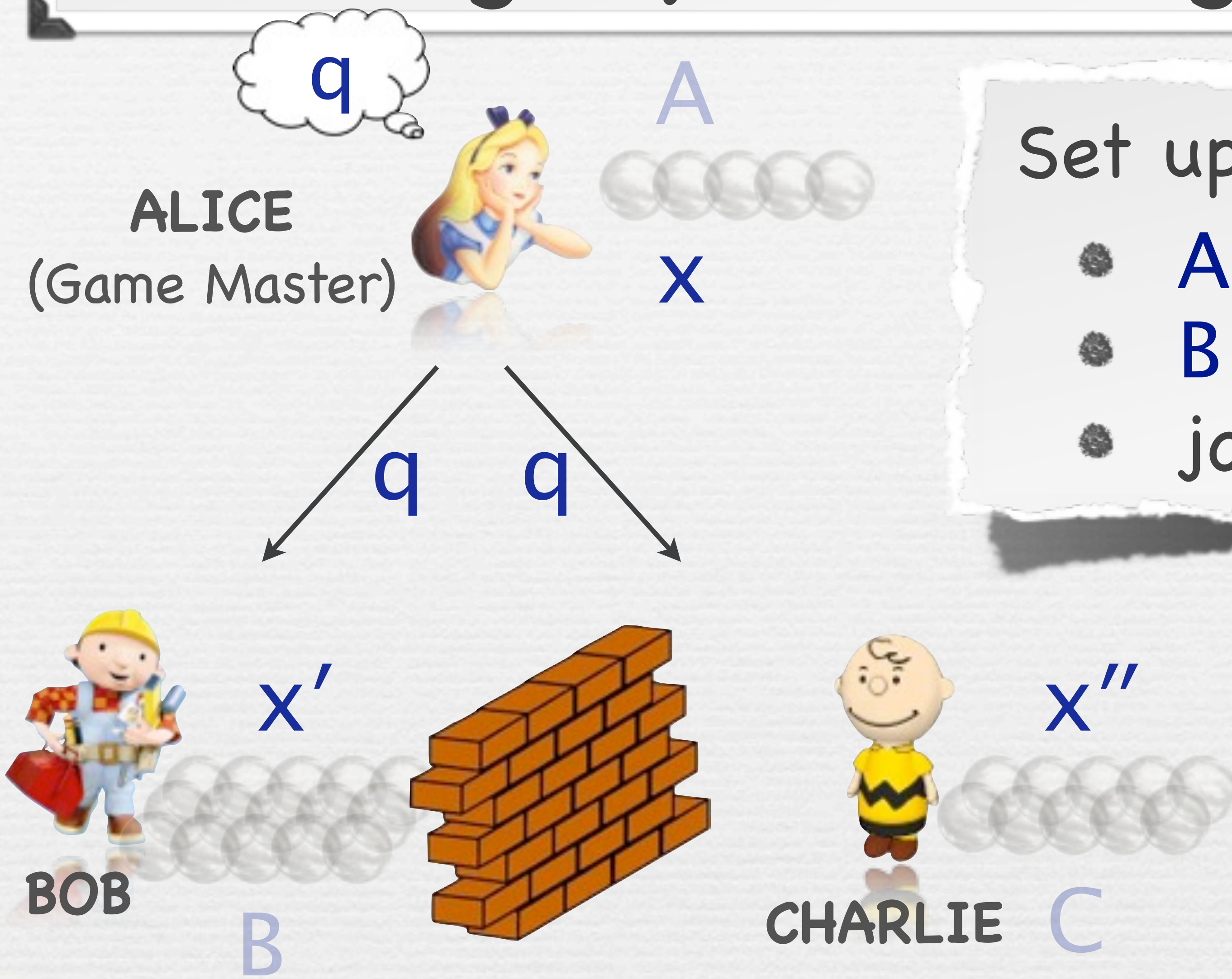

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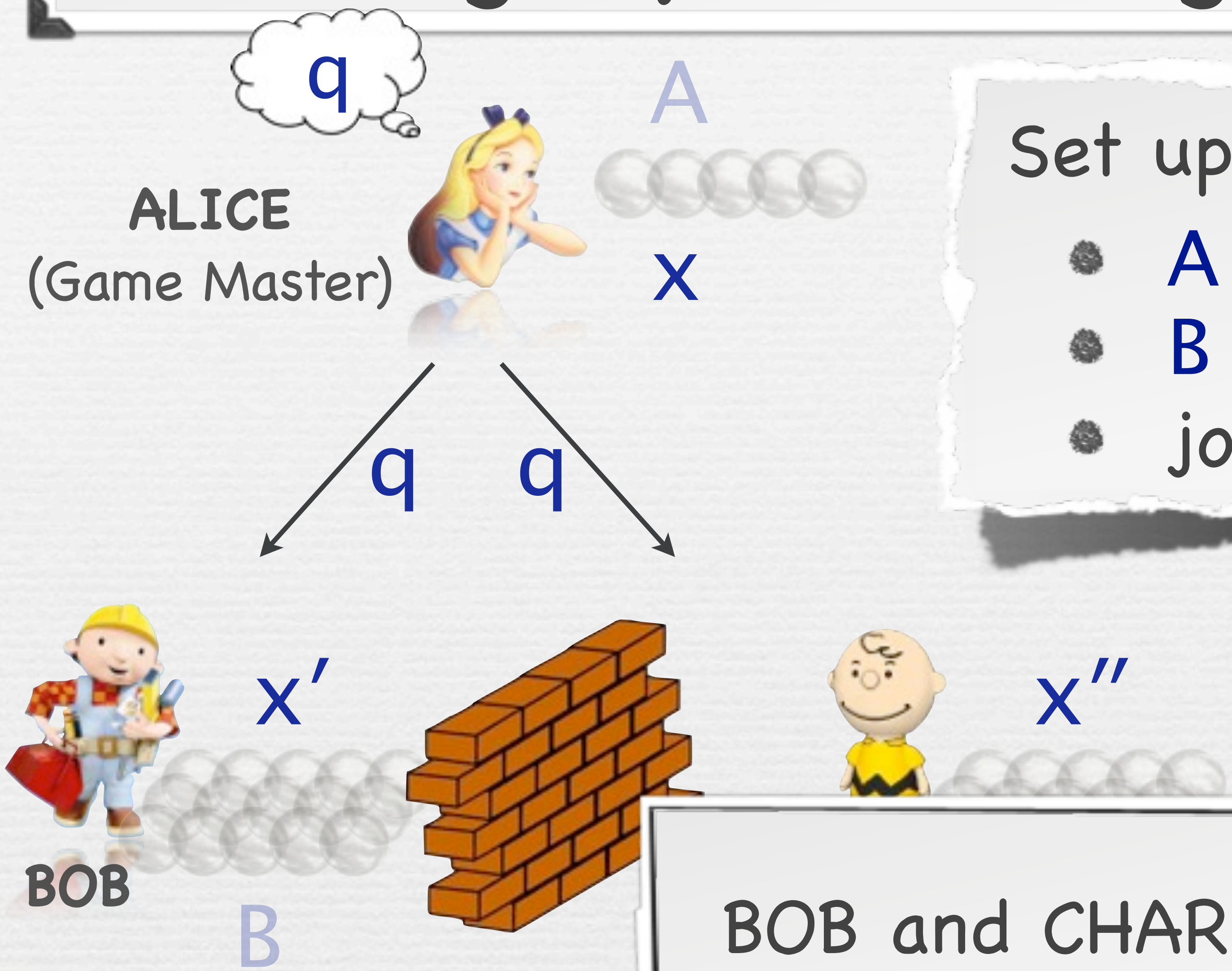

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BOB and CHARLIE:

BOB and CHARLIE jointly **win** if:

both $x' = x$ **and** $x'' = x$.

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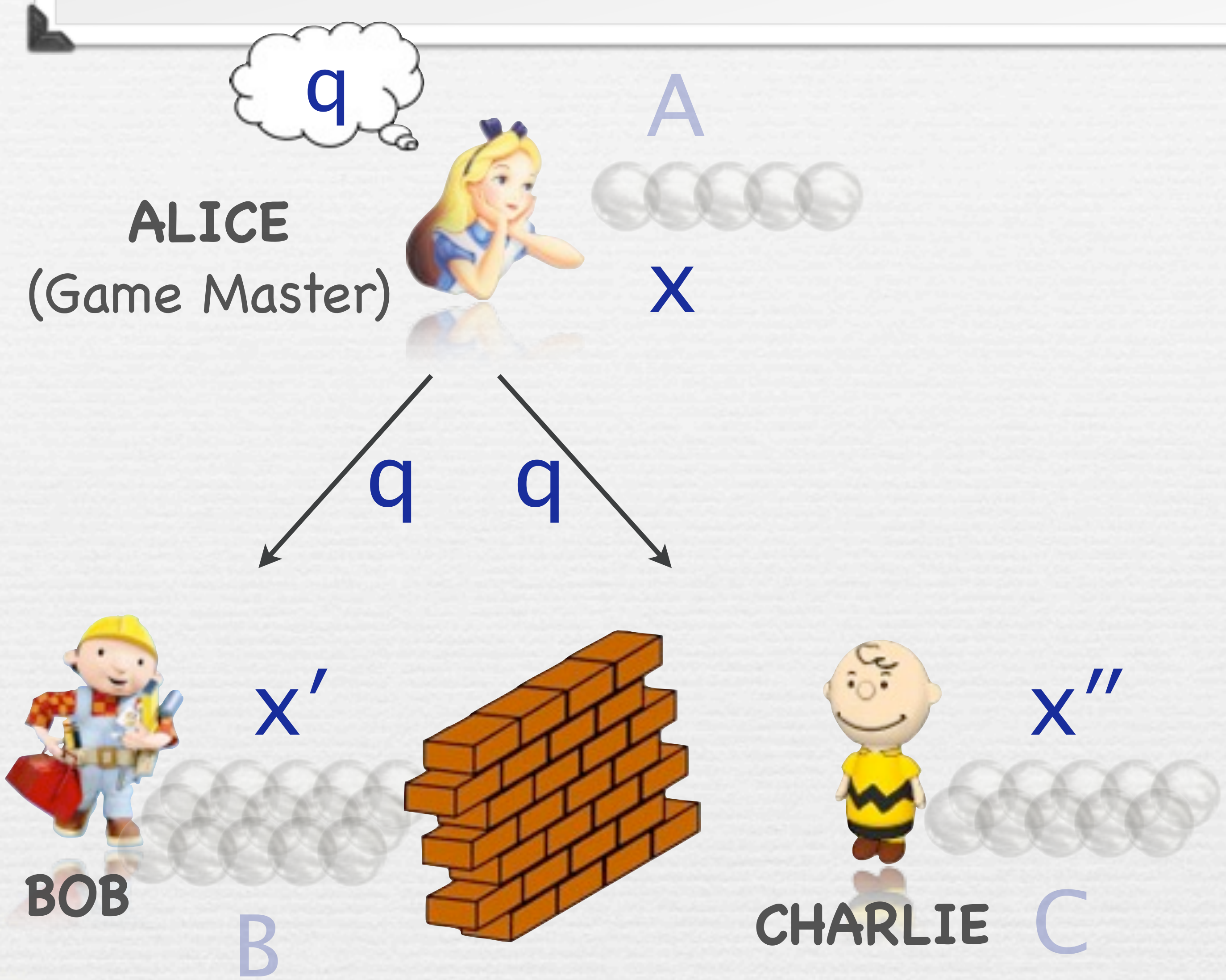

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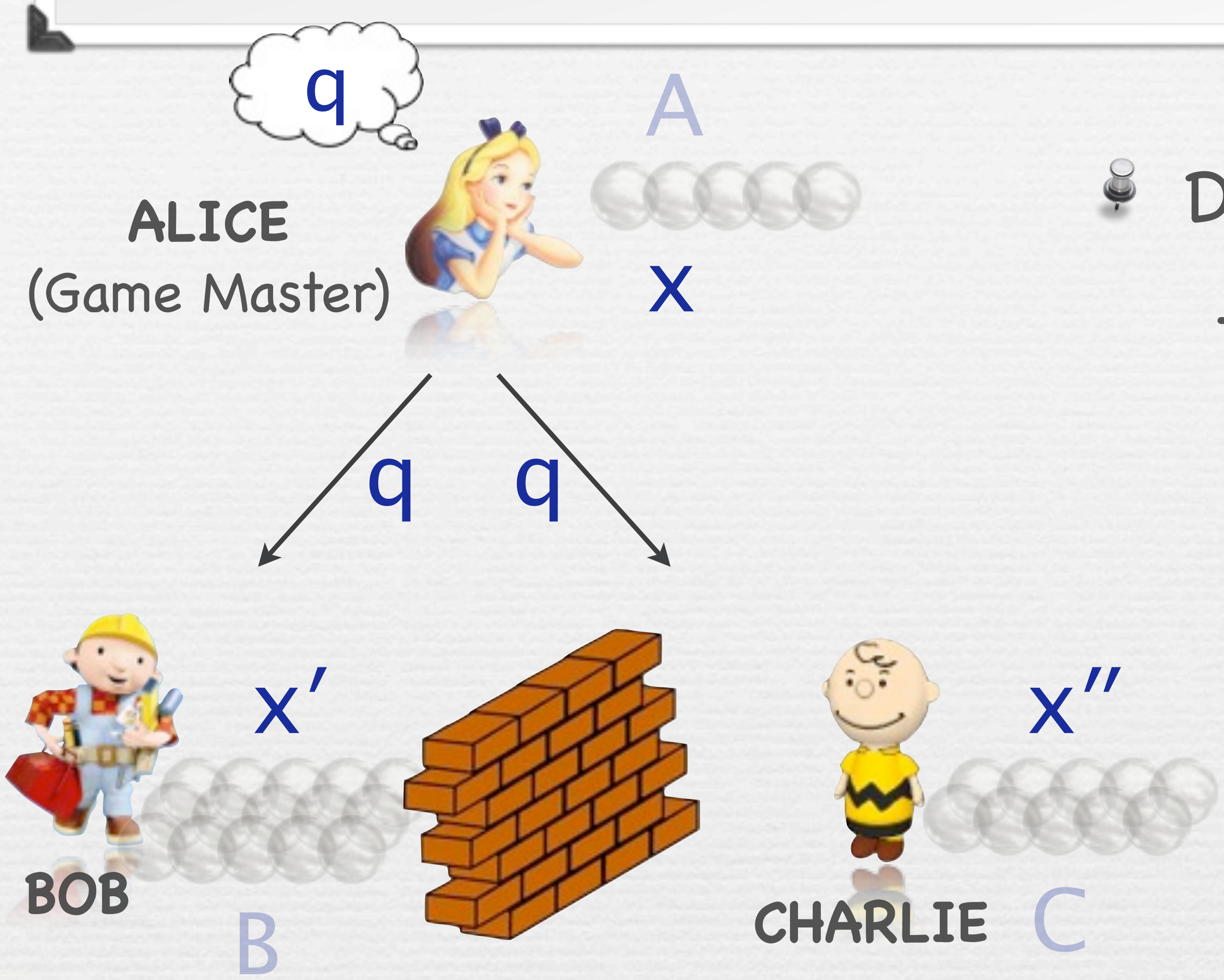

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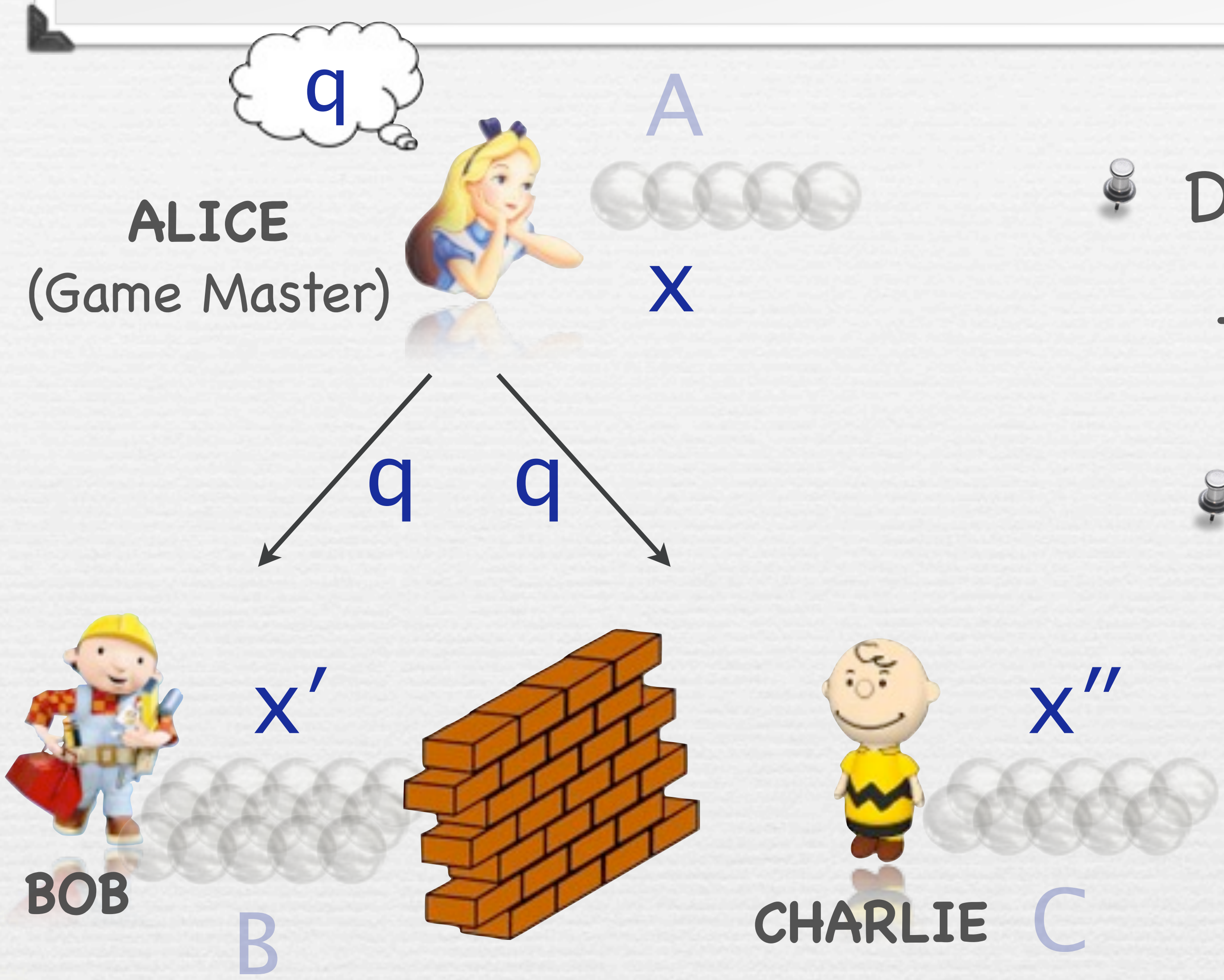

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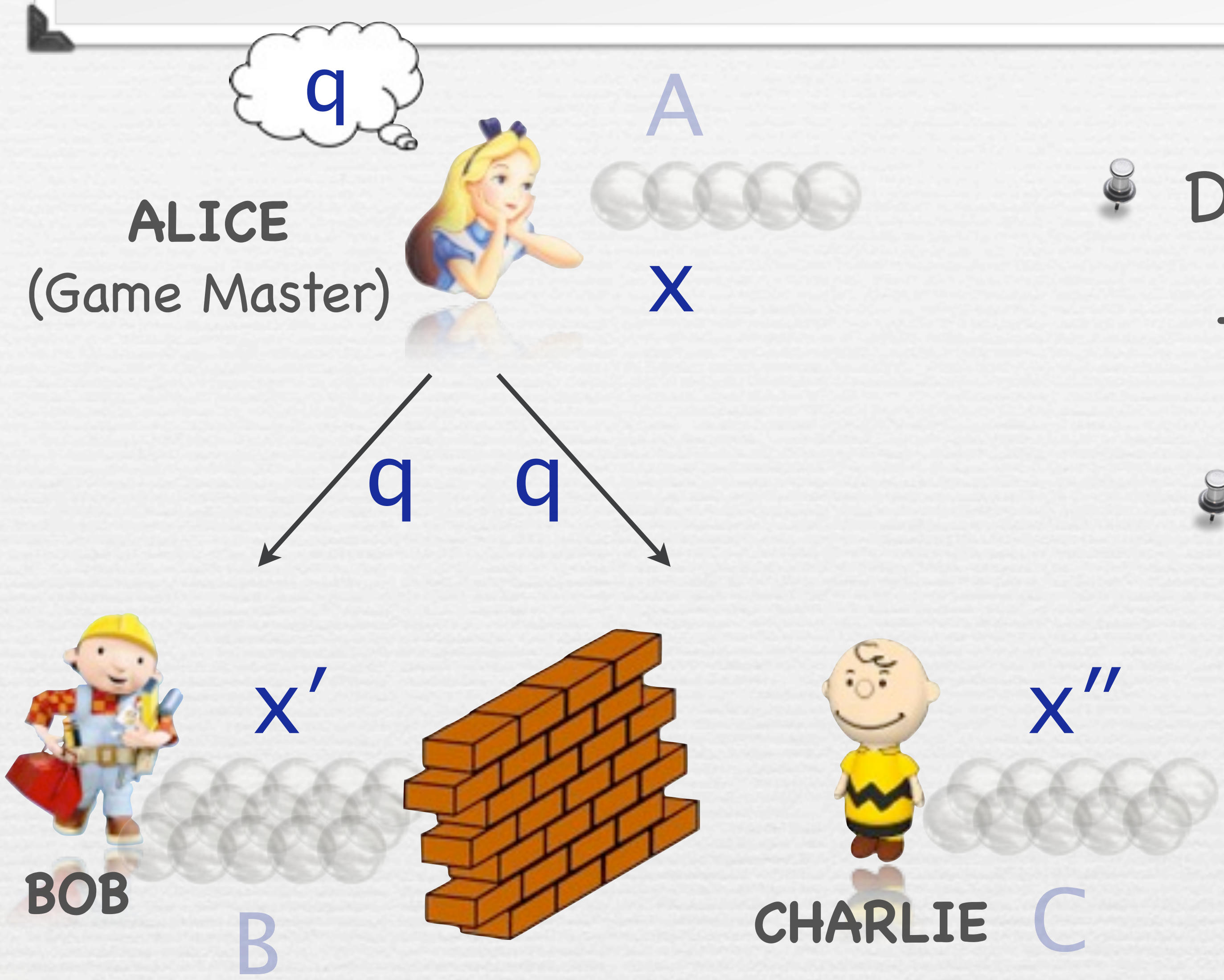

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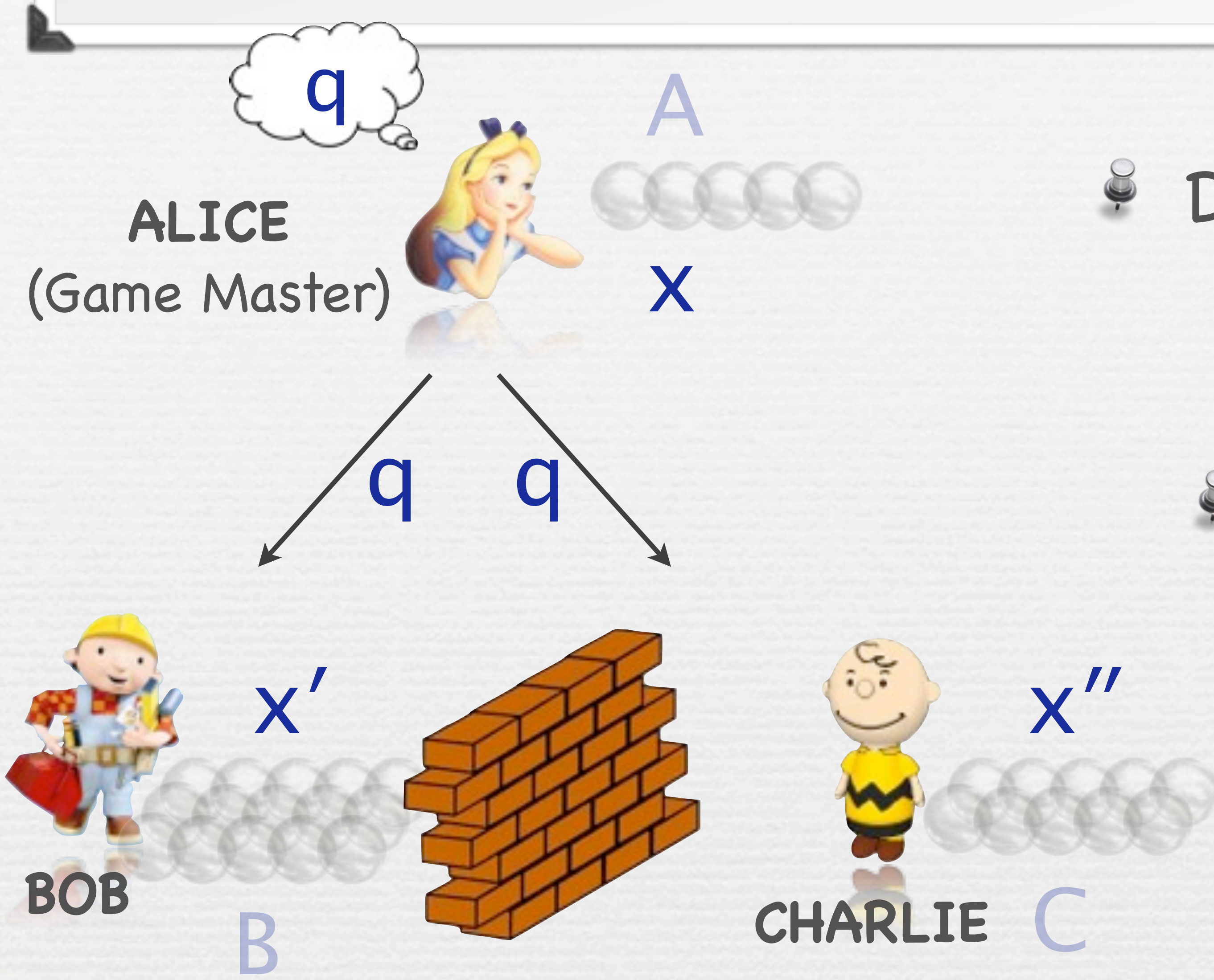

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Thus, we expect:

$$p_{\mathrm{win}}(n) := \max_{\substack{\text{initial states} \\ \& \\ \text{measurements}}} P[X' = X \wedge X'' = X] \approx 0$$

Our Main Technical Result

Formally: $$p_{\text{win}}(n) := \max_{\{P_x^\theta\},\{Q_x^\theta\}} \frac{1}{2^n} \left\| \sum_{\theta,x} H^\theta |x\rangle\langle x| H^\theta \otimes P_x^\theta \otimes Q_x^\theta \right\|$$

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Remarks:

- Bound is **tight** (i.e., $p_{\text{win}}(n)$ = ...)
- Strong parallel repetition: $p_{\text{win}}(n) = p_{\text{win}}(1)^n$
- Is attained **without any entanglement**
  => monogamy completely kills power of entanglement

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Proof:

- very simple
- New **operator-norm inequality**: bounds $||\sum_i O_i||$ for positive operators $O_1,\dots,O_n$ in terms of $||\sqrt{O_i}\sqrt{O_j}||$.

Generalizations

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- **Relaxed winning condition** for Bob and/or Charlie, i.e., $x' \approx x$ and $x'' \approx x$, or $x' \approx x$ and $x'' = x$.

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In the proof:

- We analyze EPR-pair bases version of BB84
- Well known to imply security for standard BB84 QKD

EPR-Pair Based BB84 QKD

ALICE

BOB

CHARLIE

EPR-Pair Based BB84 QKD

ALICE

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EVE

EPR-Pair Based BB84 QKD

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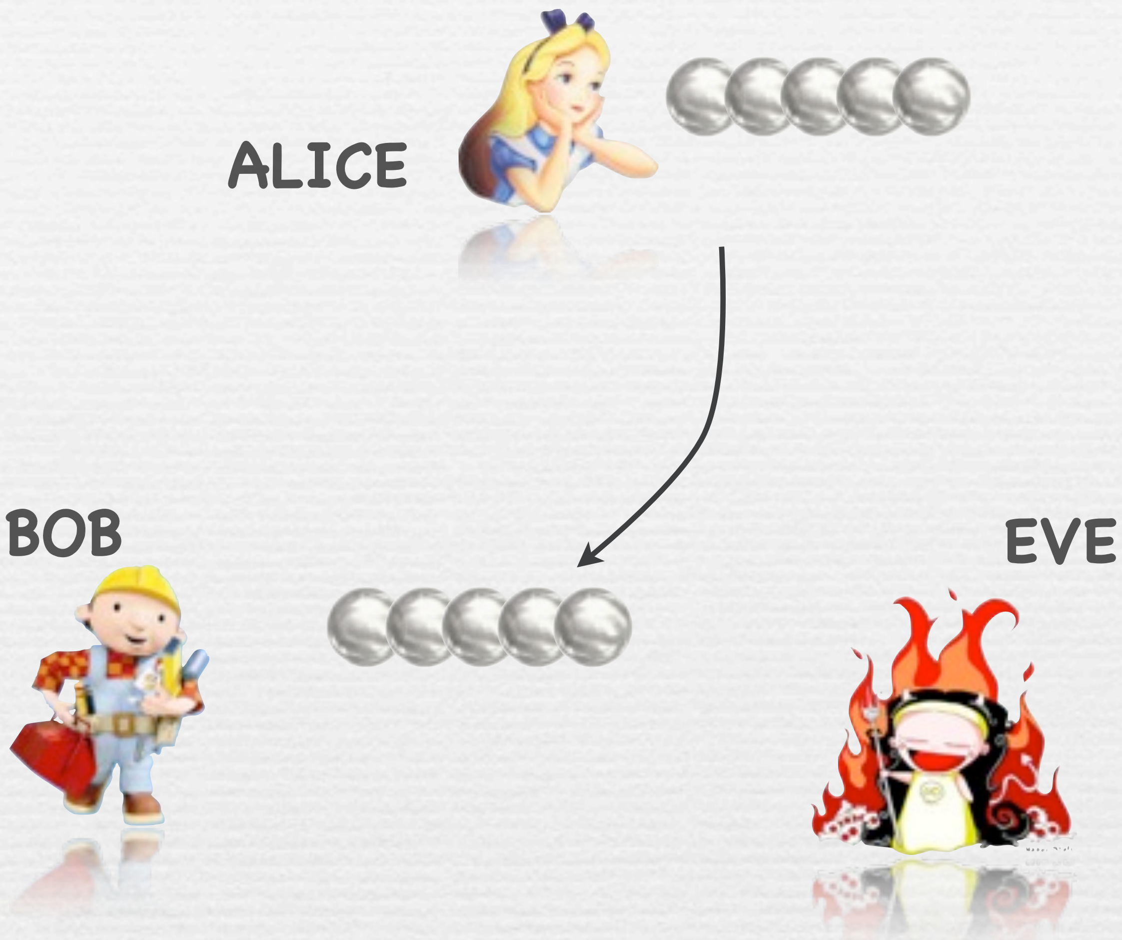
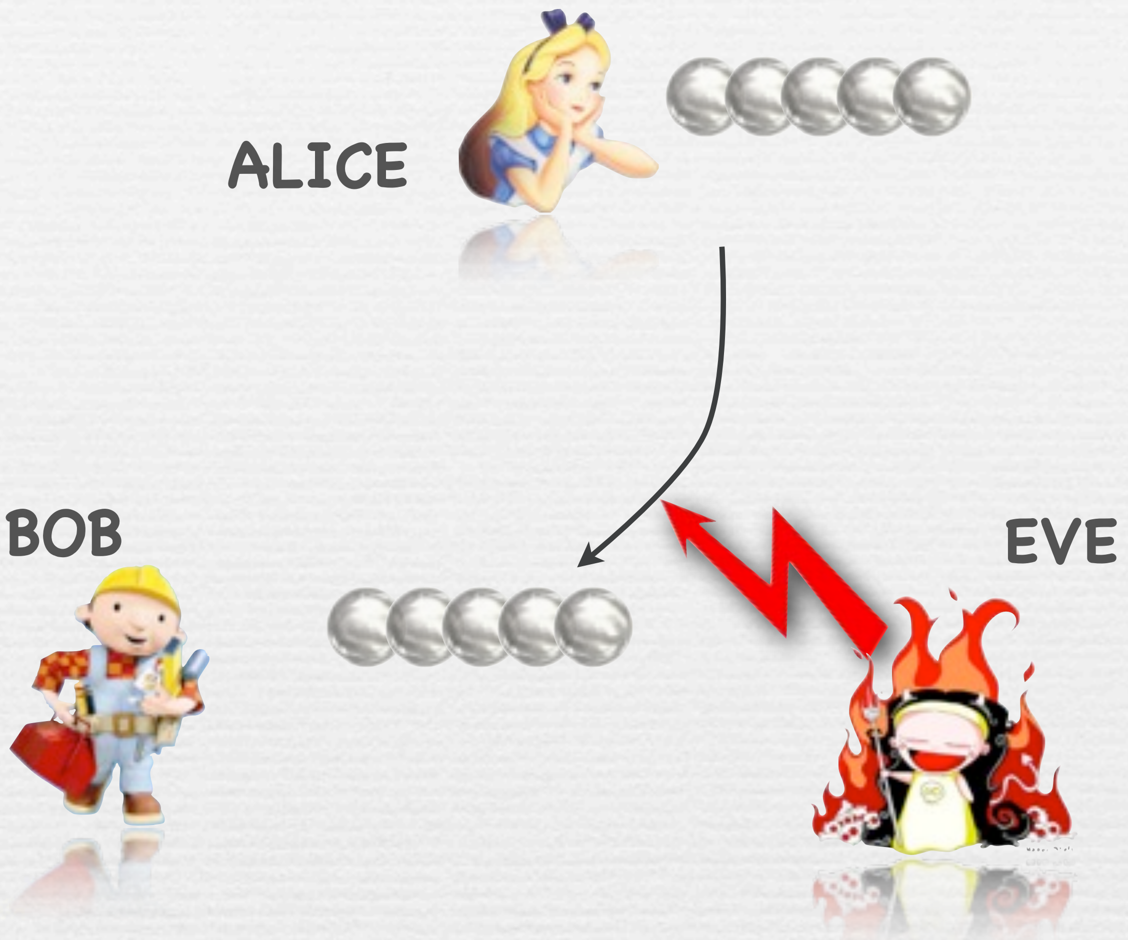


EPR-Pair Based BB84 QKD



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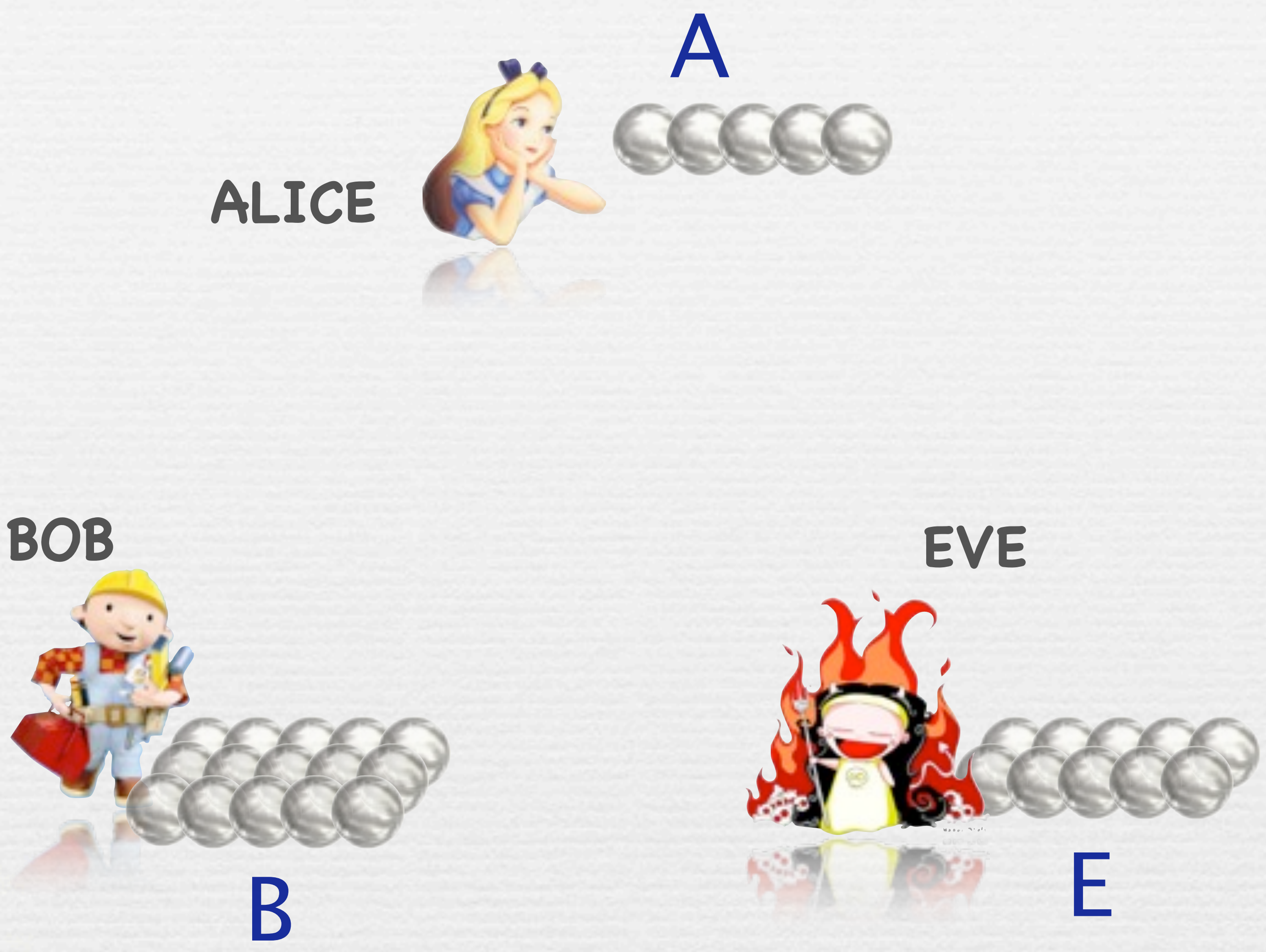

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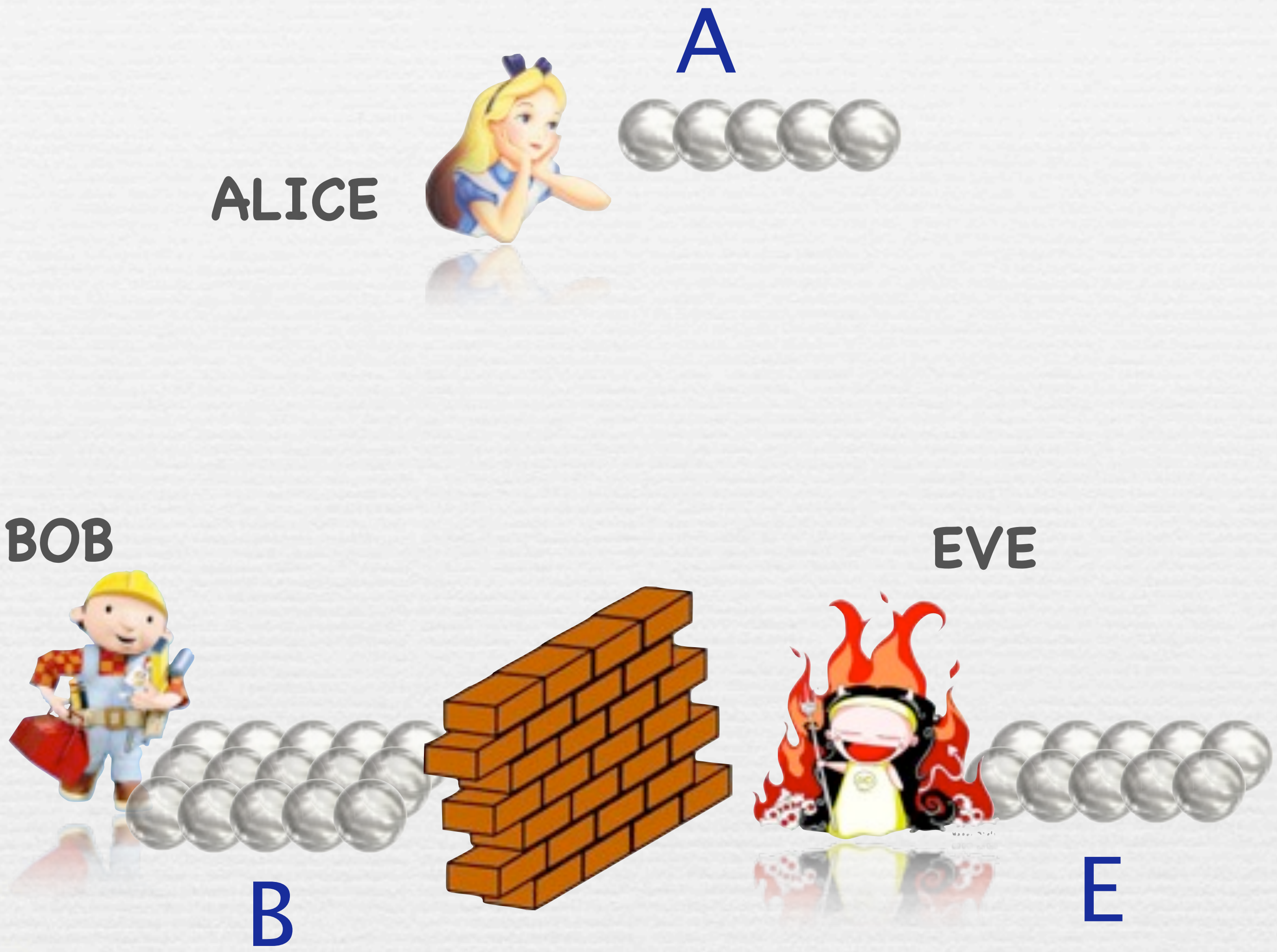

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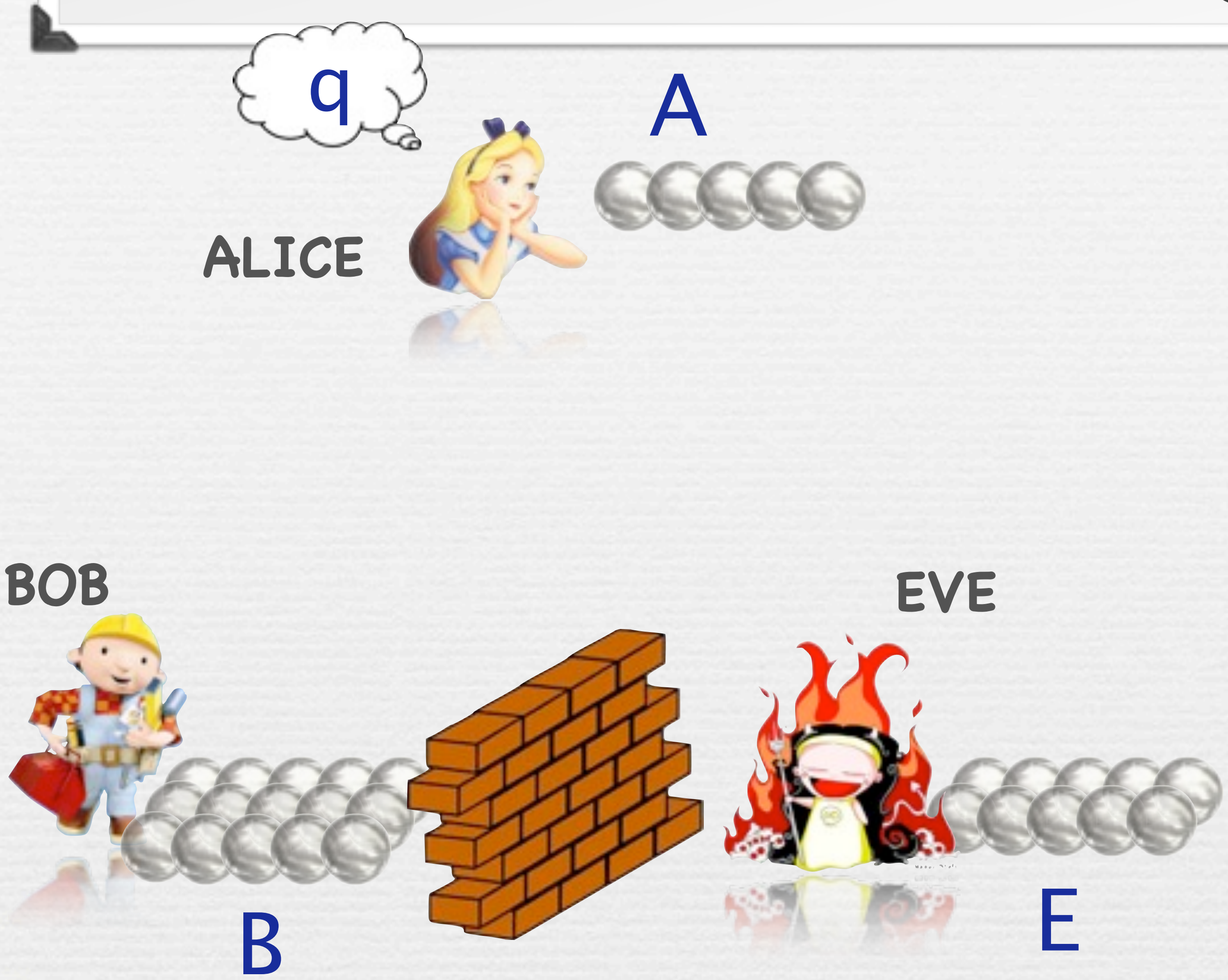

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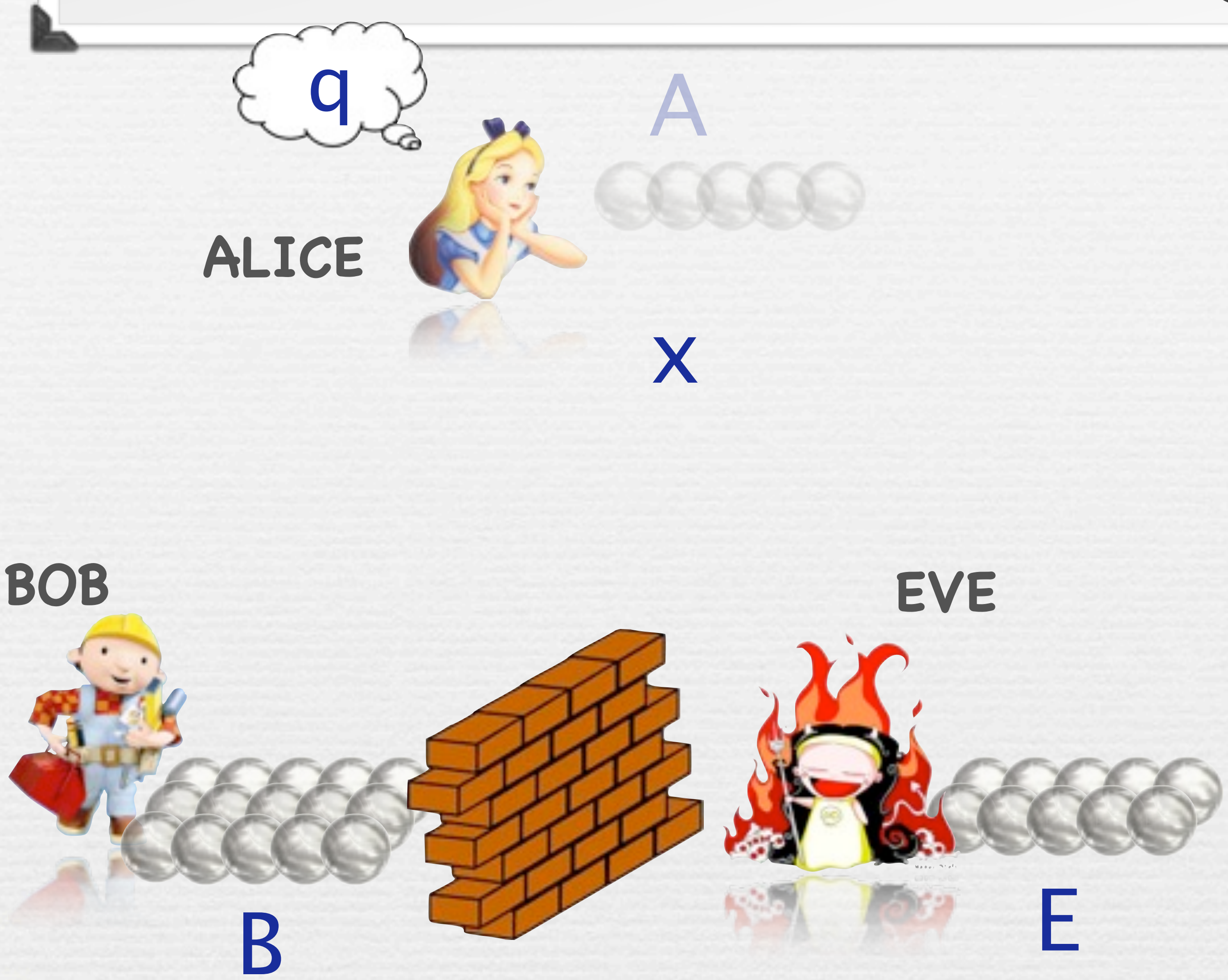

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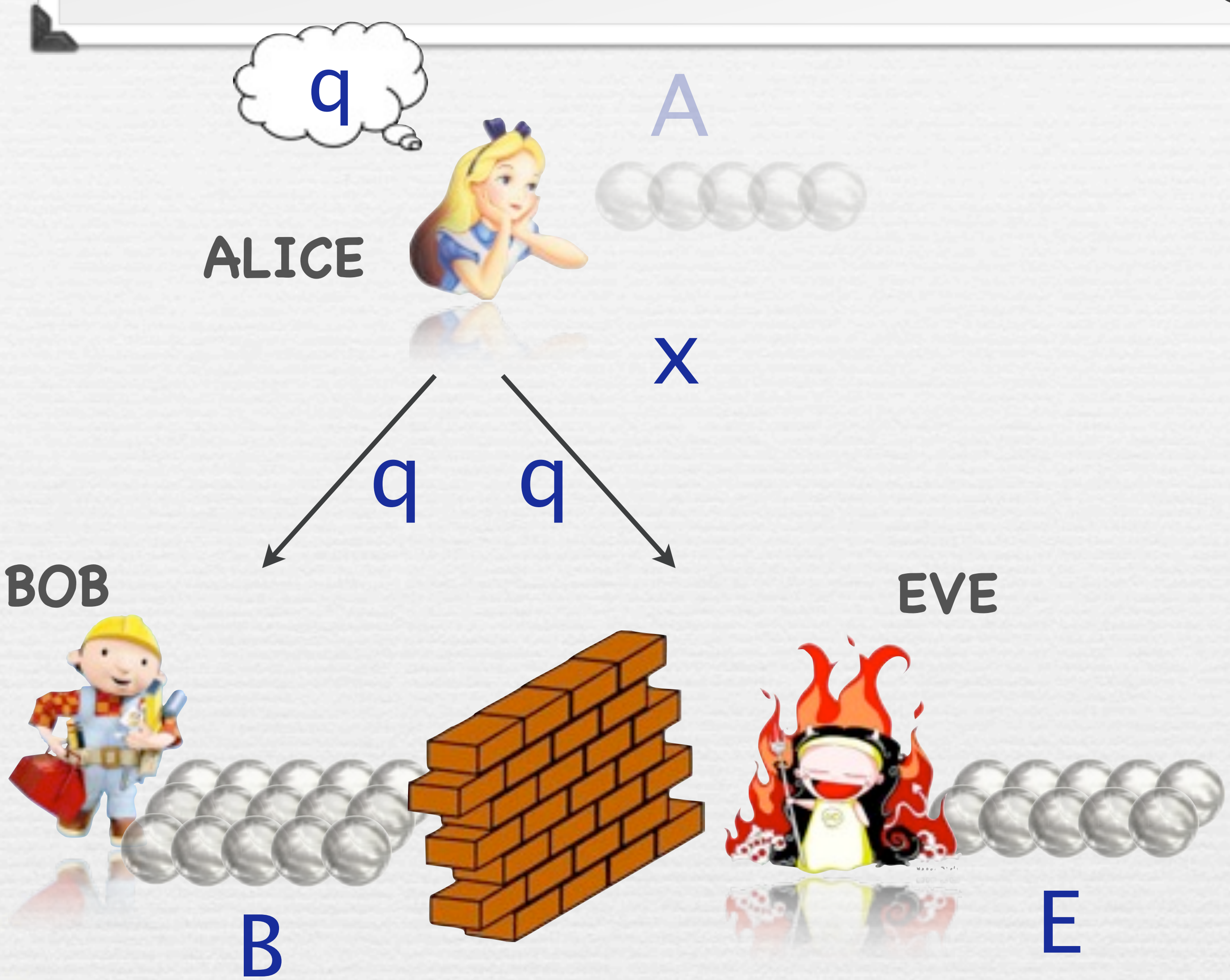

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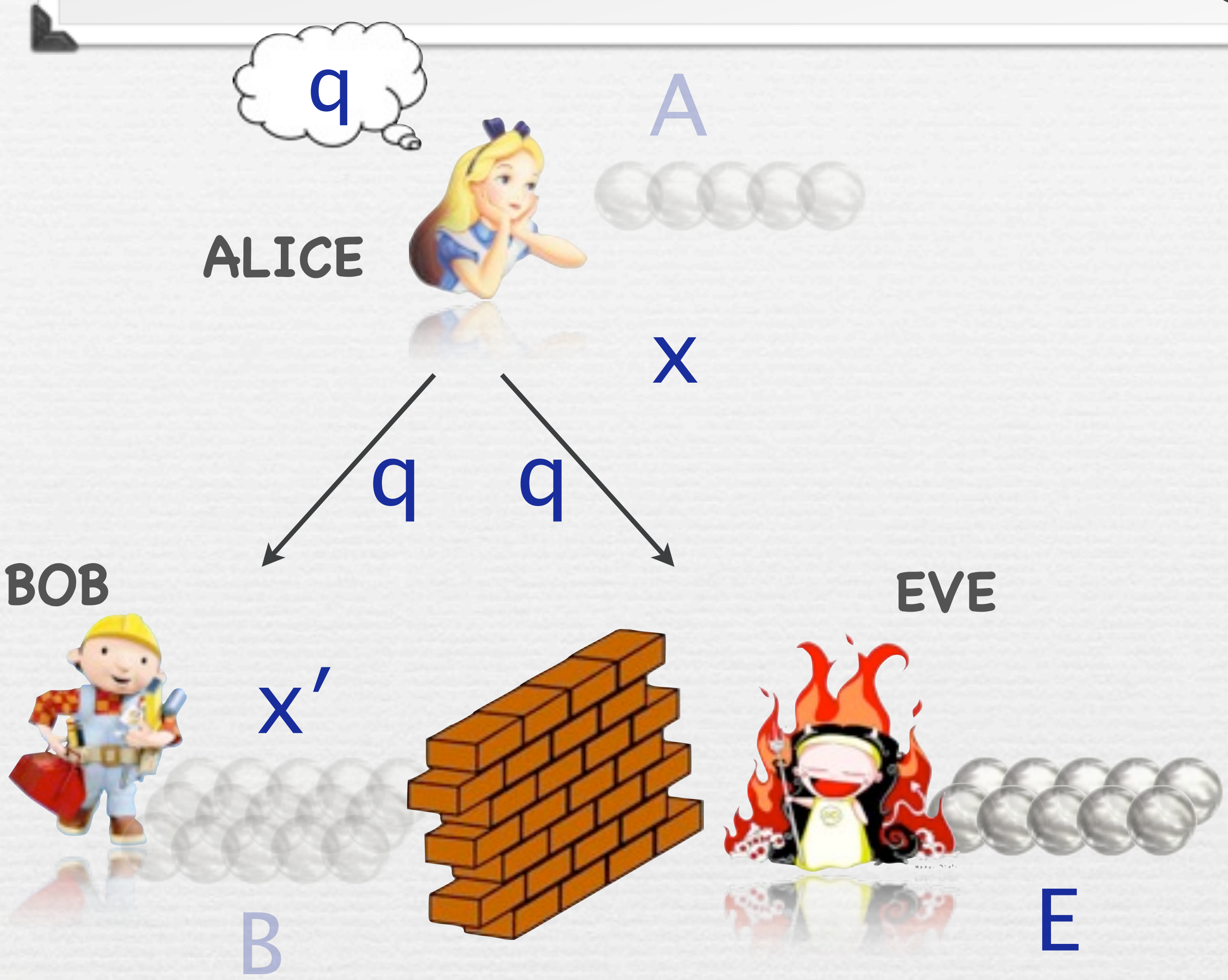

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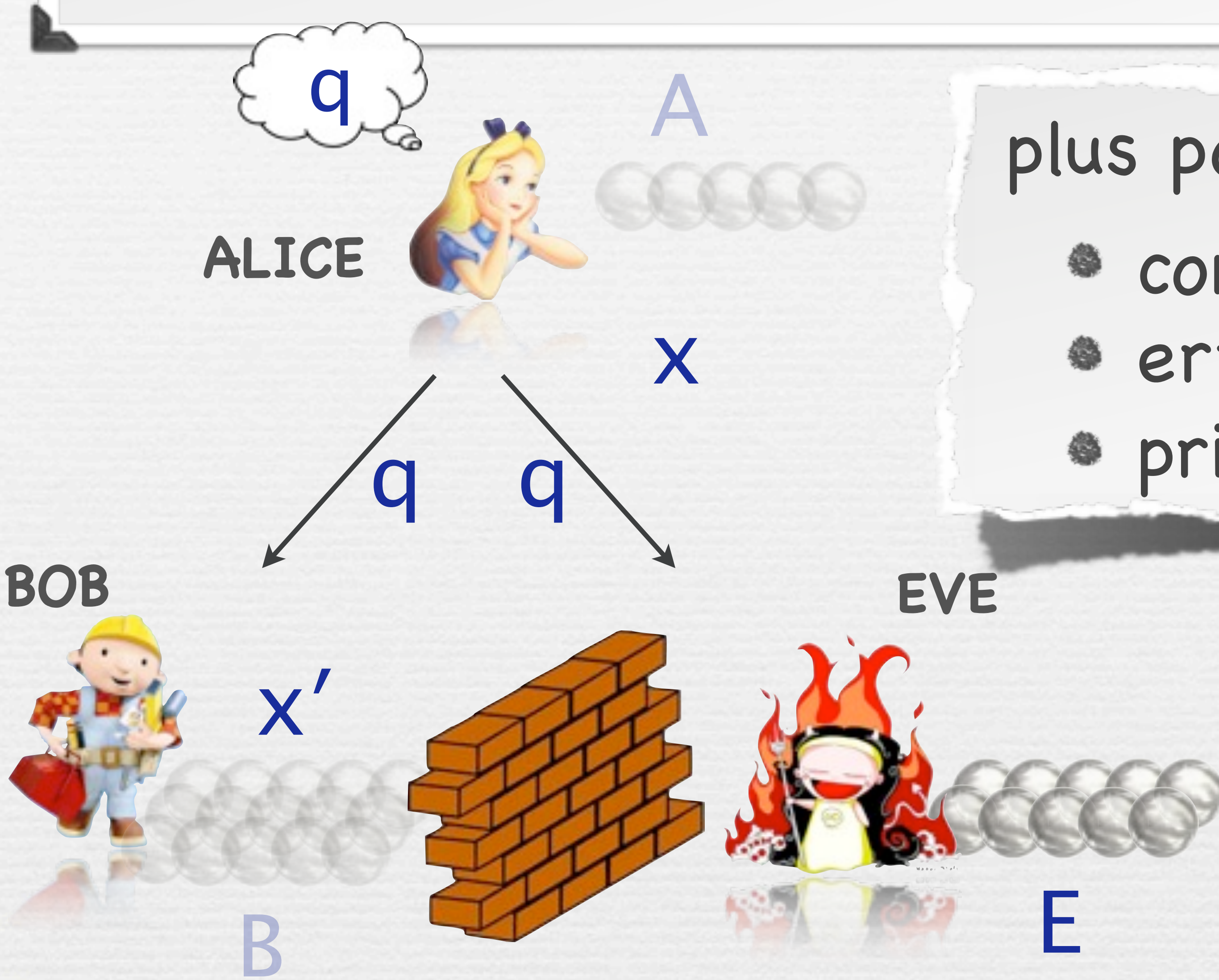

plus post-processing:

- comparing x & x' on random subset
- error correction
- privacy amplification

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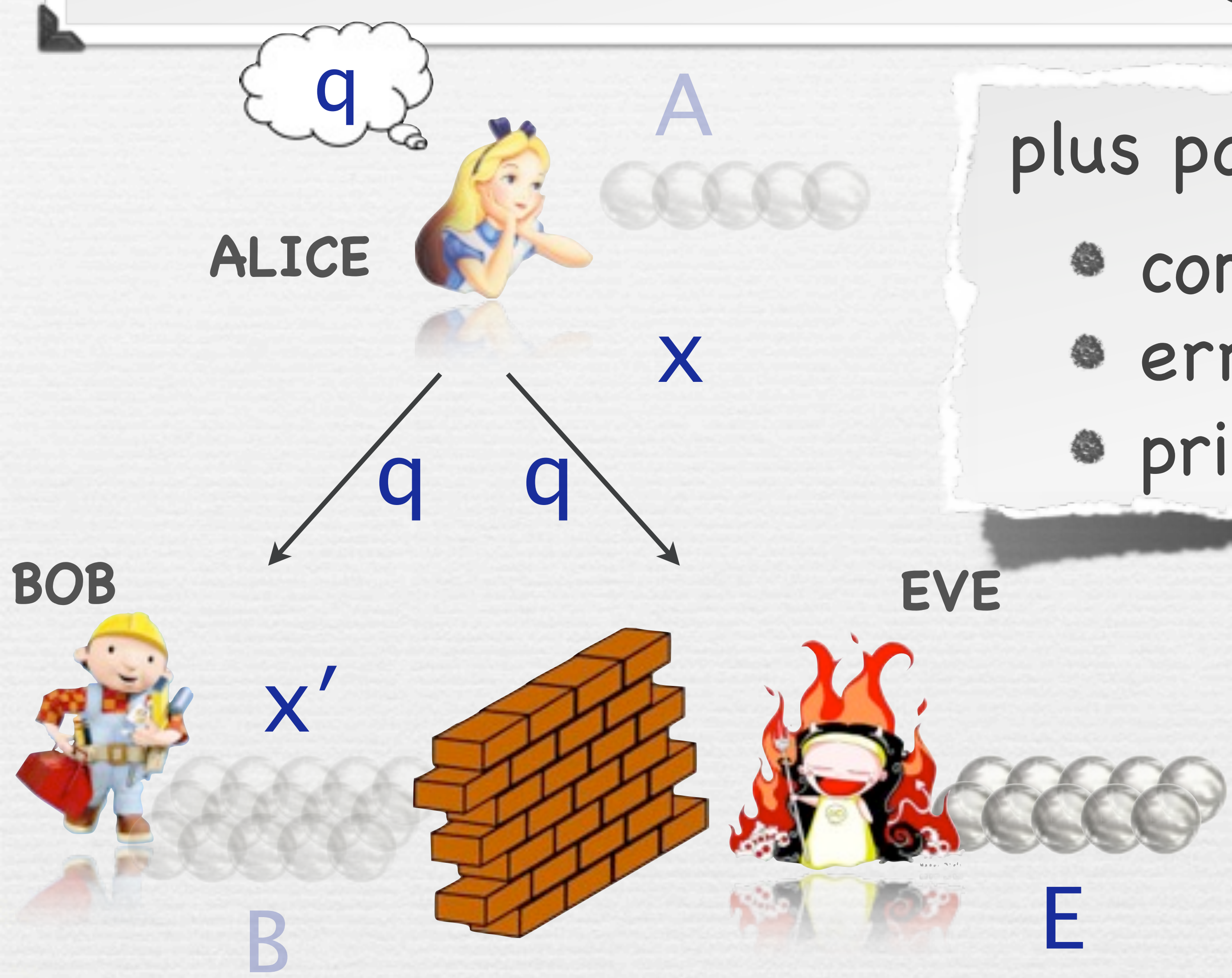

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$H_\infty(X|QE, \text{not abort}) \geq t$

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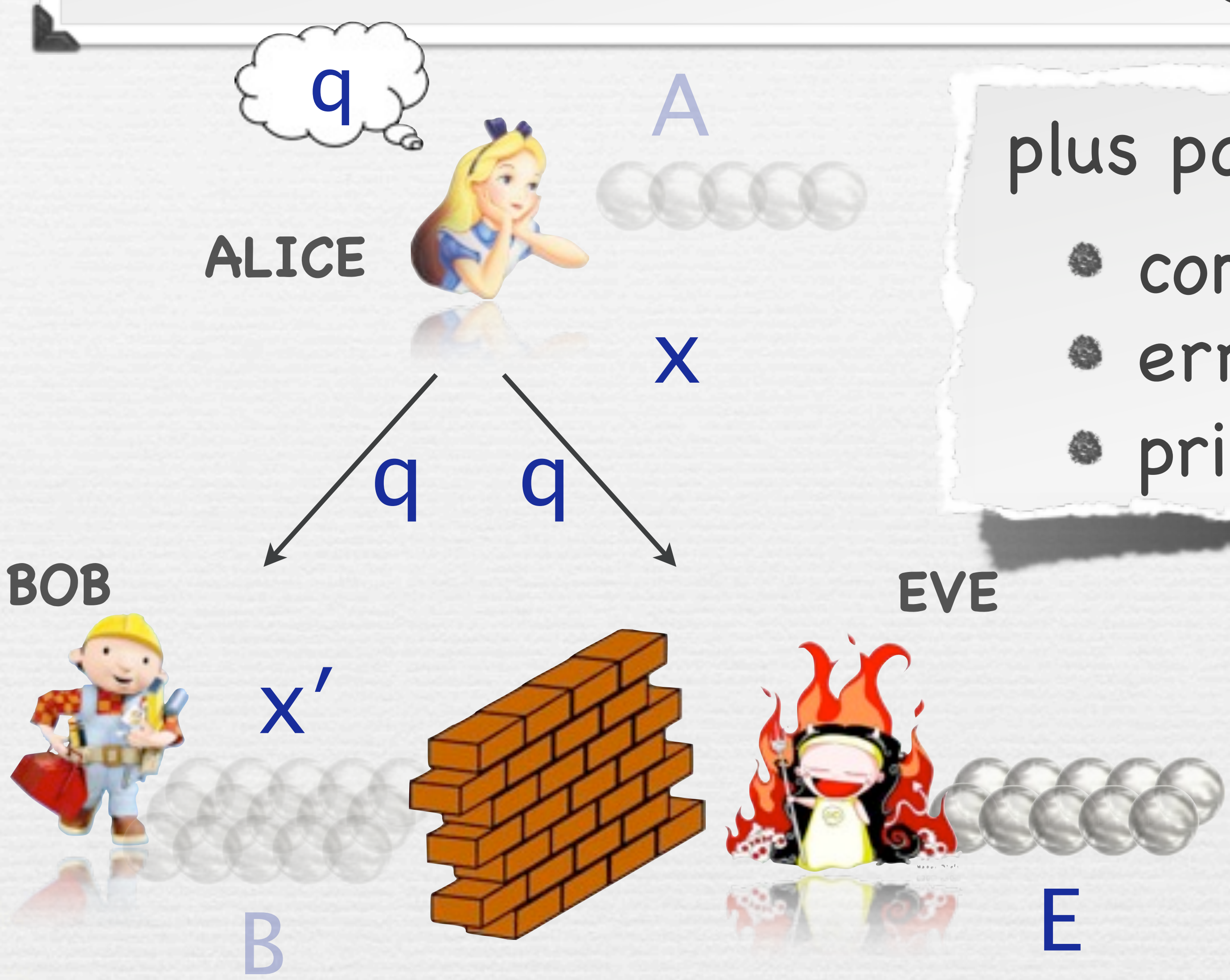

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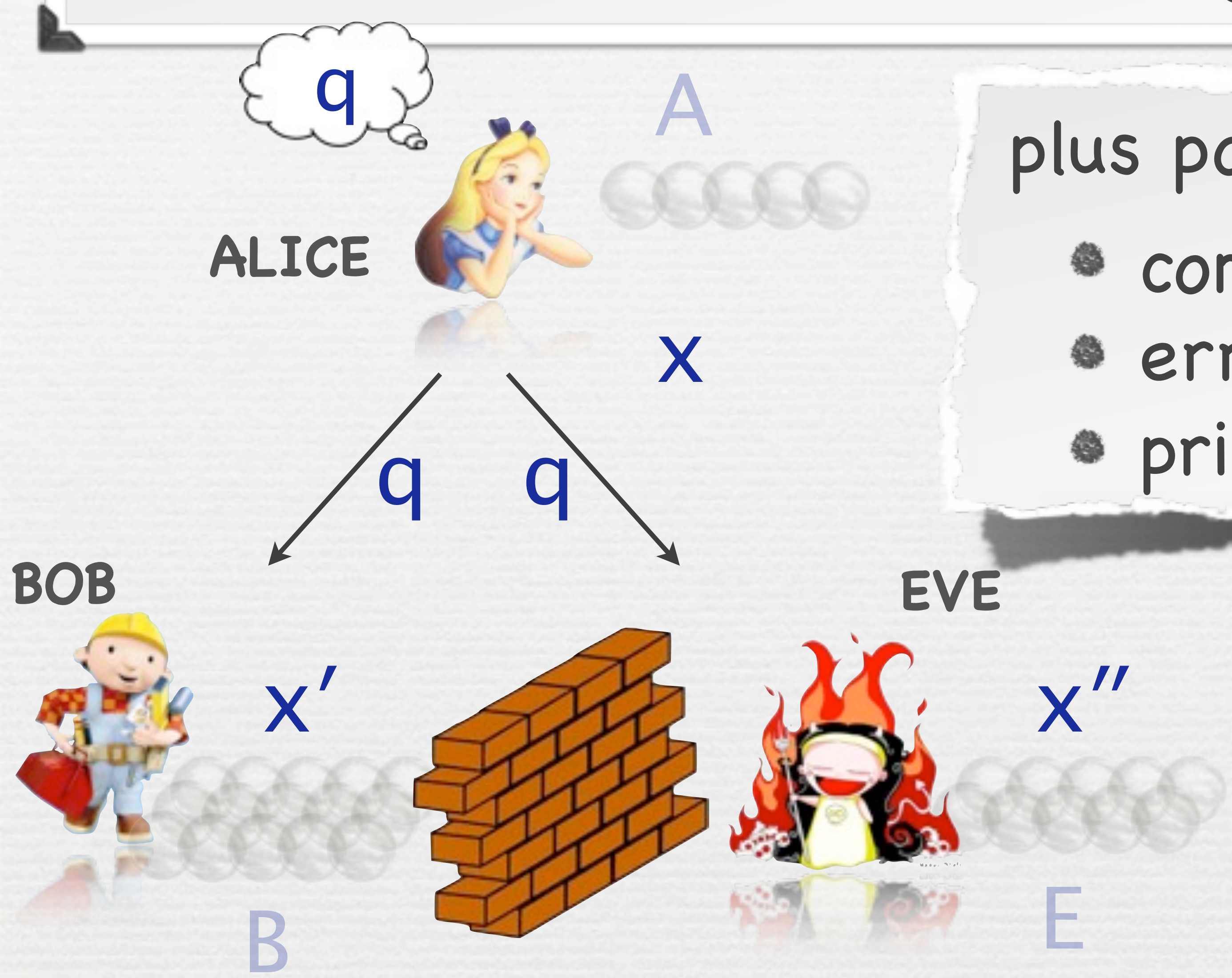

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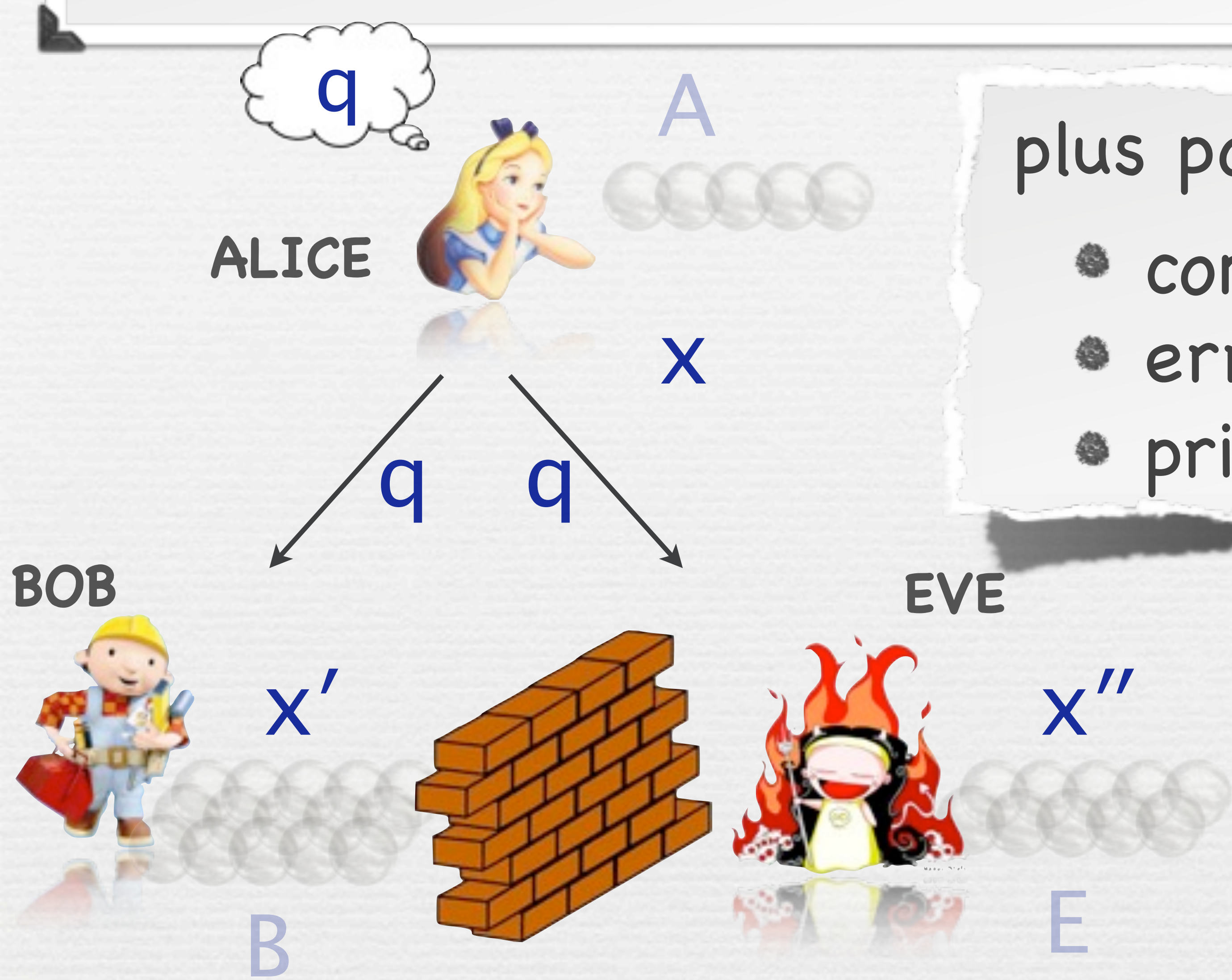

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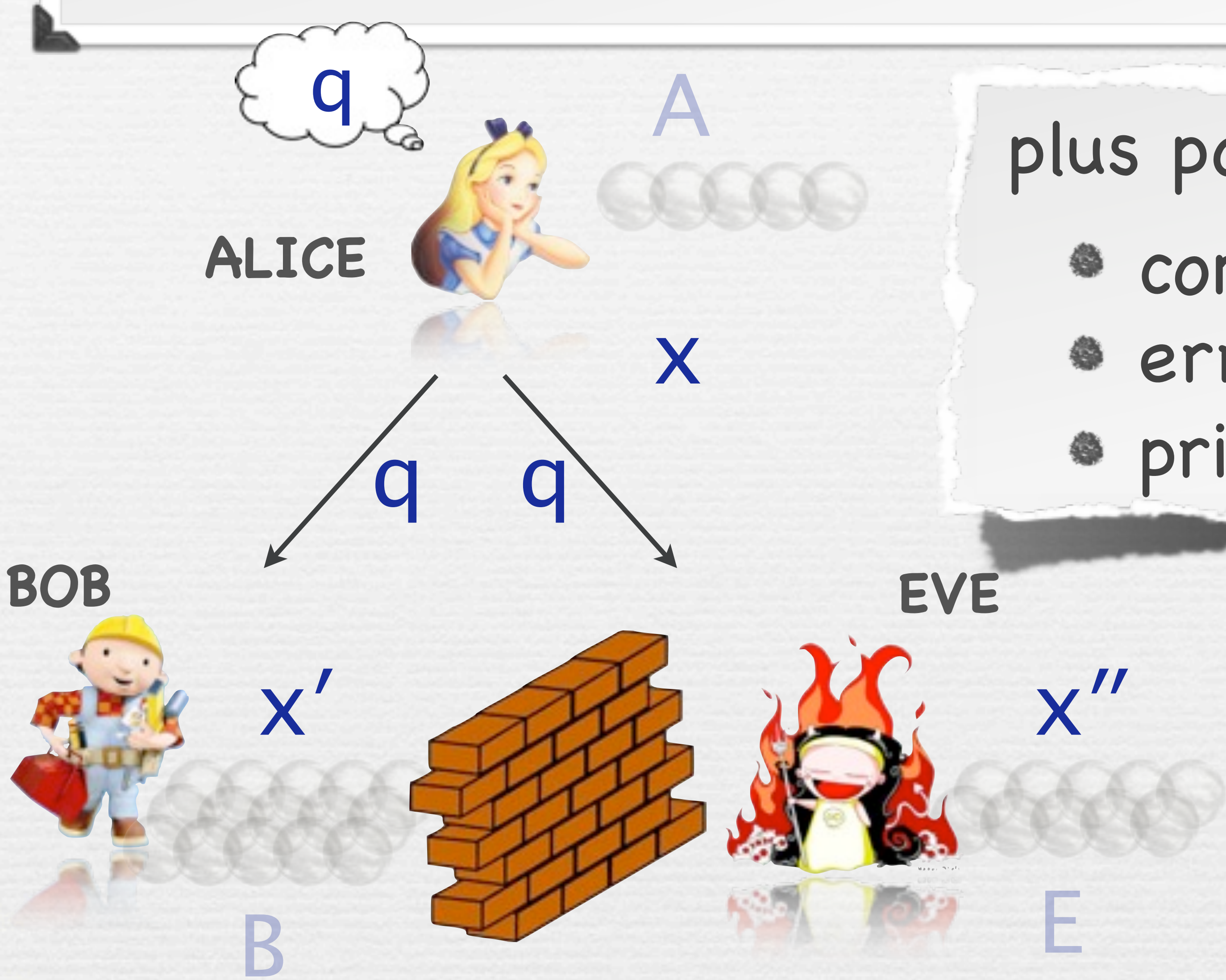

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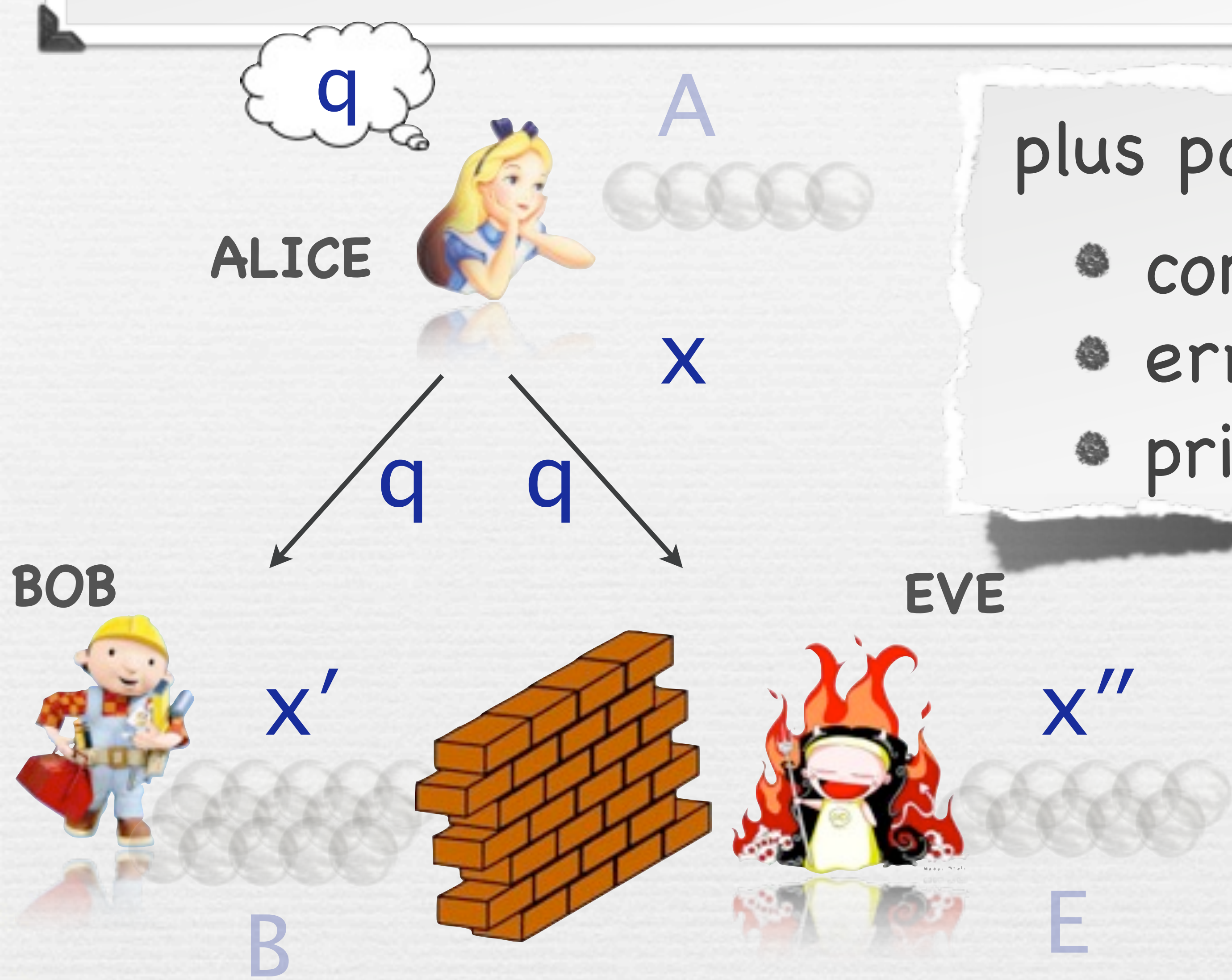
- Monogamy game $\Rightarrow P[X' \approx X \wedge X'' = X] \leq e^n$

  $\Rightarrow P[X' \approx X] \leq e^{n/2}$ (and thus $P[\text{abort}] \approx 1$)

EPR-Pair Based BB84 QKD



plus post-processing:

- comparing x & x' on random subset
- error correction
- privacy amplification

To prove:

$H_\infty(X|QE, \text{not abort}) \geq t$

- For sake of argument: say that Eve measures E
- Monogamy game $\Rightarrow P[X' \approx X \wedge X'' = X] \leq e^n$

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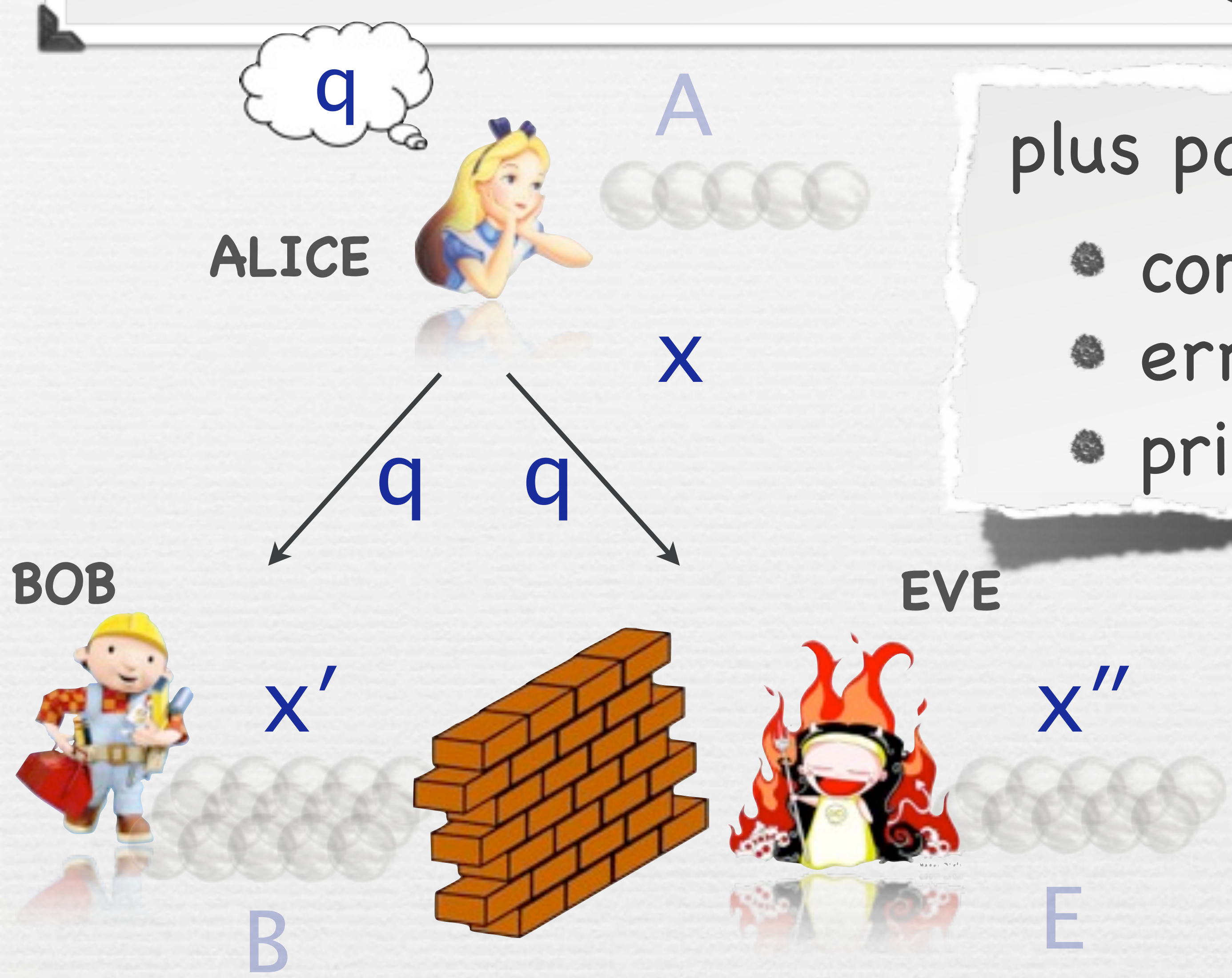

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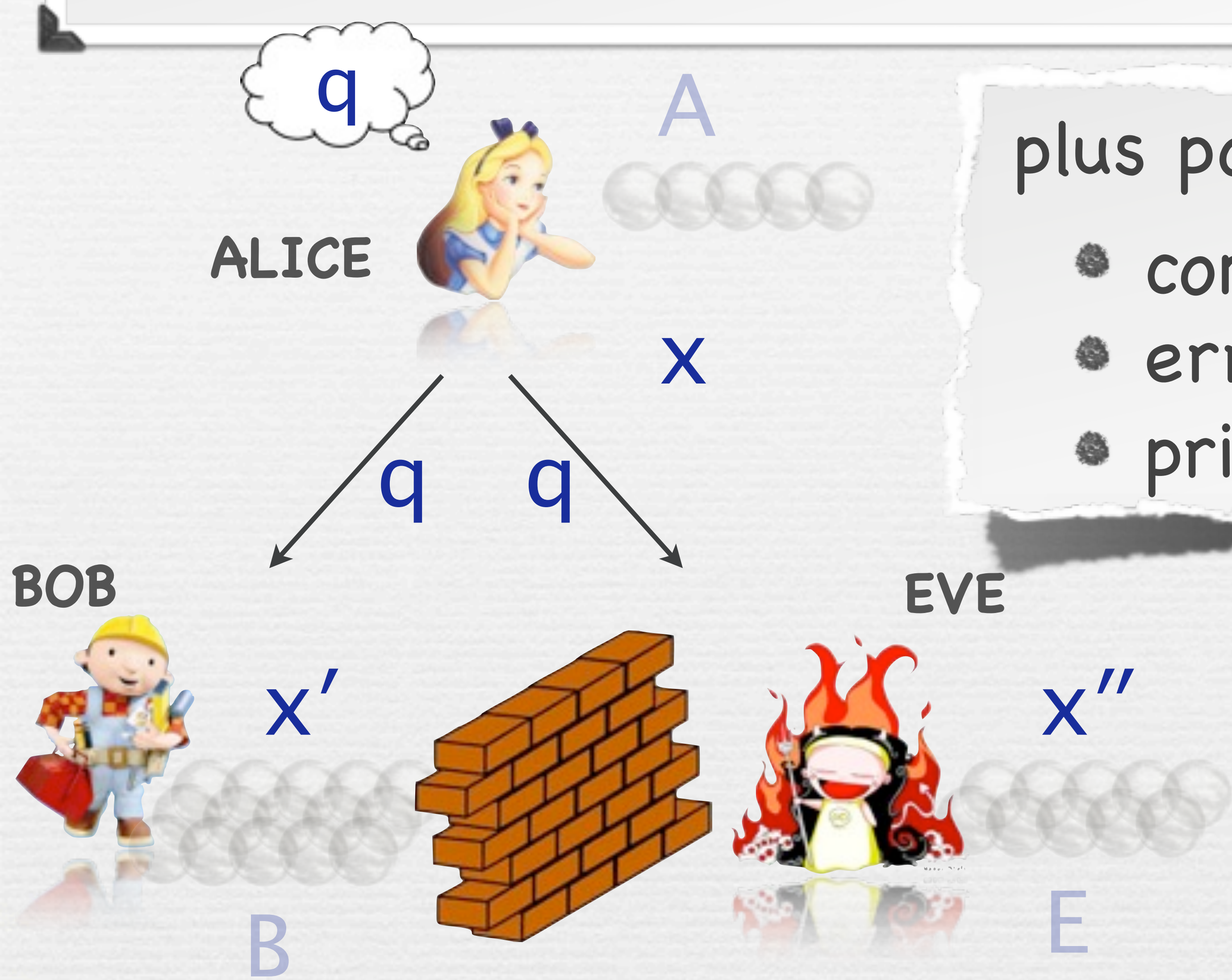

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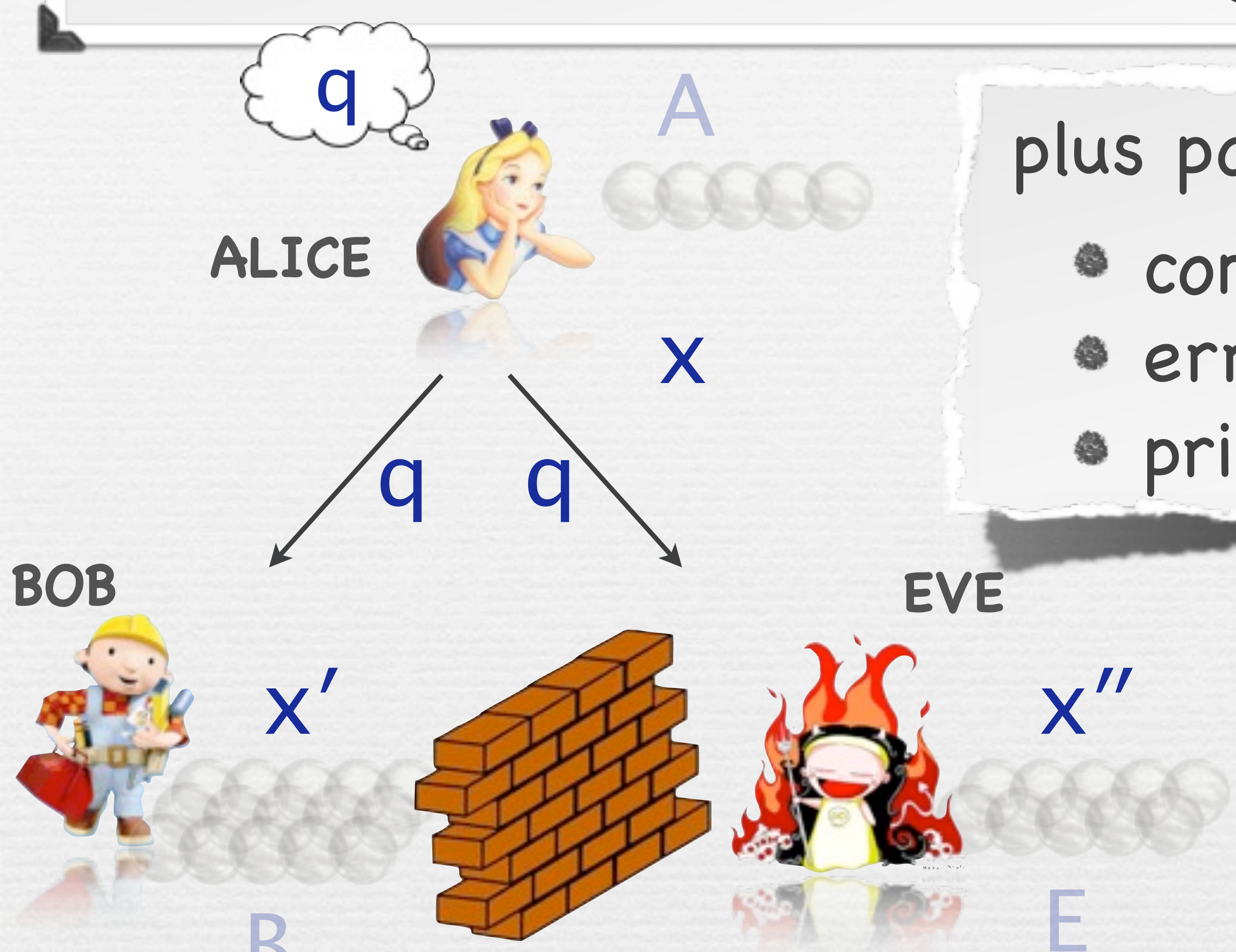

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$\Rightarrow H_\infty(X \mid QE, \overset{\text{not abort}}{\cancel{X' \approx X}}) \geq n/2 \cdot \log(1/e)$ ✓

Comparison with other protocols

	Reichhardt et al. (E91)	Vazirani/ Viddick (E91)	this work (BB84/BBM92)
device assumptions	none	none	trusted Alice (source)
noise tolerance	0%	1.2%	1.5% (11%)
key rate	0%	2.5%	22.8% (100%)
finite key analysis	×	×	✓

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  - winning probability is exponentially small
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- Application I: to BB84 QKD
  - allow a malicious measurement device for Bob
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- Application II: to position-based quantum crypto
  - first 1-round position-verification scheme

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THANK YOU