Tamper-Resistant Cryptographic Hardware in the Isolated Qubits Model

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How to build tamper-resistant cryptographic devices?

<u>End goal:</u>

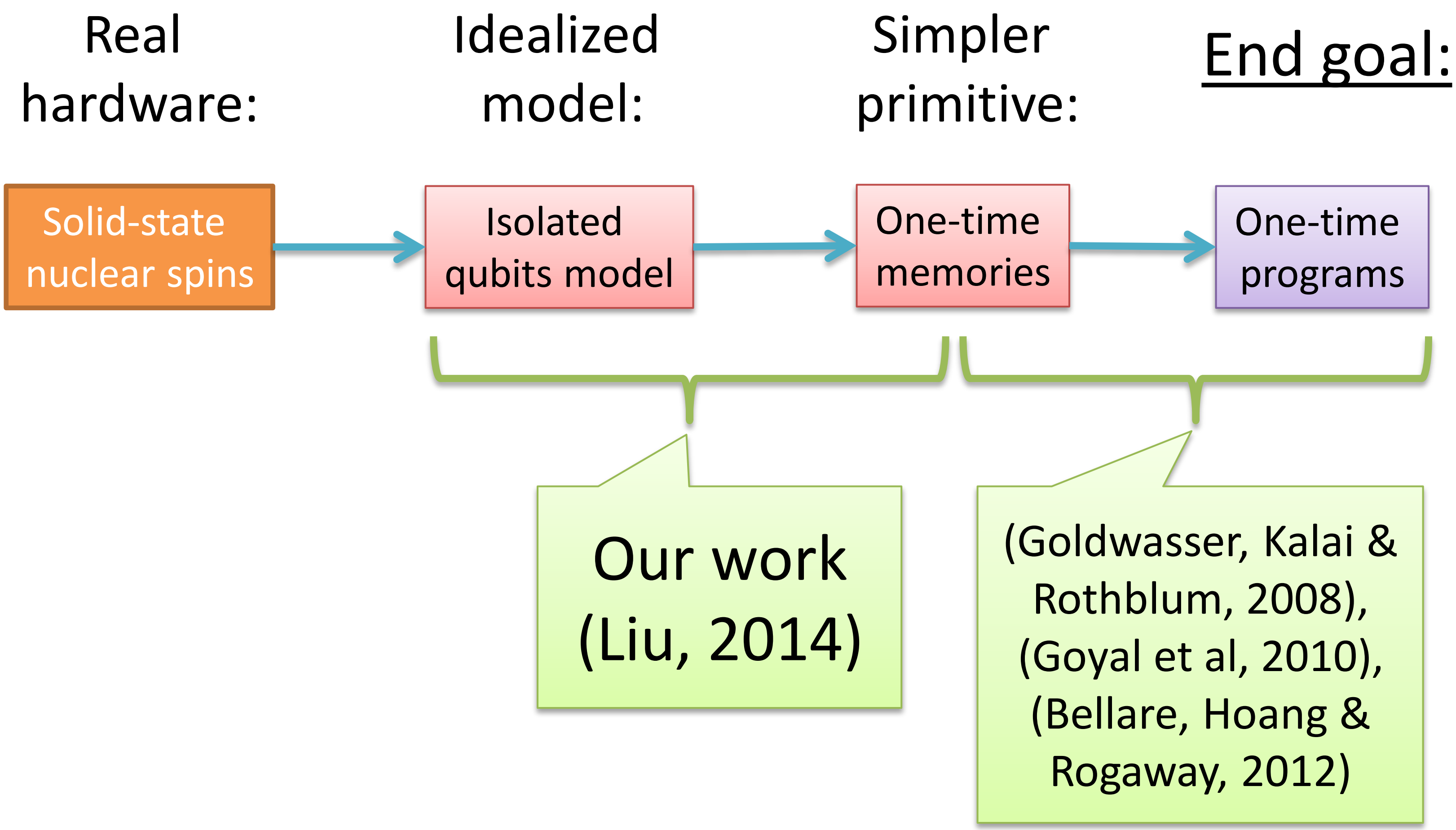
One-time programs

Program can be run <u>only once</u>, on an input supplied by the user

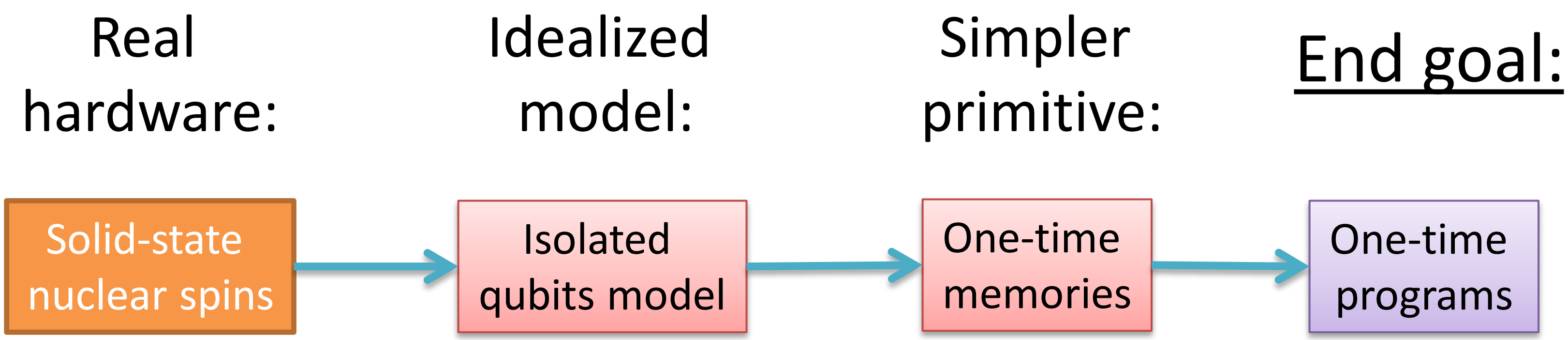
Intermediate results of the computation are <u>hidden</u>

Related to program obfuscation and copy-protection

How to build tamper-resistant cryptographic devices?



How to build tamper-resistant cryptographic devices?



- Why is quantum information useful?
  - Quantum states cannot be cloned, measurement disturbs the state, etc.
  - But it's more subtle than that… quantum bit-commitment, oblivious transfer are <u>not</u> possible (Mayers; Lo and Chau)

Our results (1/2)

- One-time memories based on "conjugate coding"
  - Old idea due to Wiesner, <u>not</u> secure against quantum adversaries
  - We show how to instantiate it, so that it <u>is</u> secure against a natural sub-class of quantum adversaries
    - "Isolated qubits model"
  - Construction has several desirable properties:
    - "Single-shot security"
    - Security against general LOCC adversaries
    - Efficiently implementable
    - But it leaks information…

Y.-K. Liu, CRYPTO 2014

Our results (2/2)

- How to stop leakage: privacy amplification in the isolated qubits model
  - Usual solution: use an extractor, w/ random seed
  - Trouble: OTM's are non-interactive
    - No way to generate a random seed that is unknown to the adversary
  - Instead, use a deterministic extractor
    - Can be secure because adversary is restricted to LOCC

Y.-K. Liu, in preparation

Isolated qubits are fun

- For theorists:
  - Another model, where many interesting cryptographic tasks are possible!
    - Known constructions seem very far from optimal!
    - Based on simple probabilistic constructions, crude bounds
- For experimentalists:
  - Another family of interesting quantum devices that can be realized
    - Very different from quantum repeaters
    - Want long coherence times, good single qubit operations, no entanglement swapping

This talk

- Overview
  - One-time memories, why they are useful
  - Isolated qubits model
- How to construct OTM’s in the isolated qubits model
  - Leaky OTM’s
  - Privacy amplification

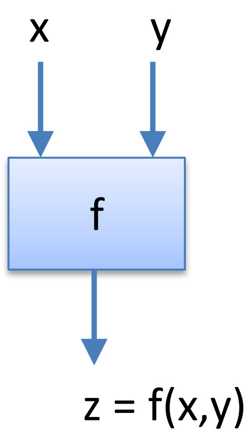
One-time memories

- One-time memory contains two messages s,t
  - Adversary can choose to read s or t, but not both
  - “Non-interactive oblivious transfer”

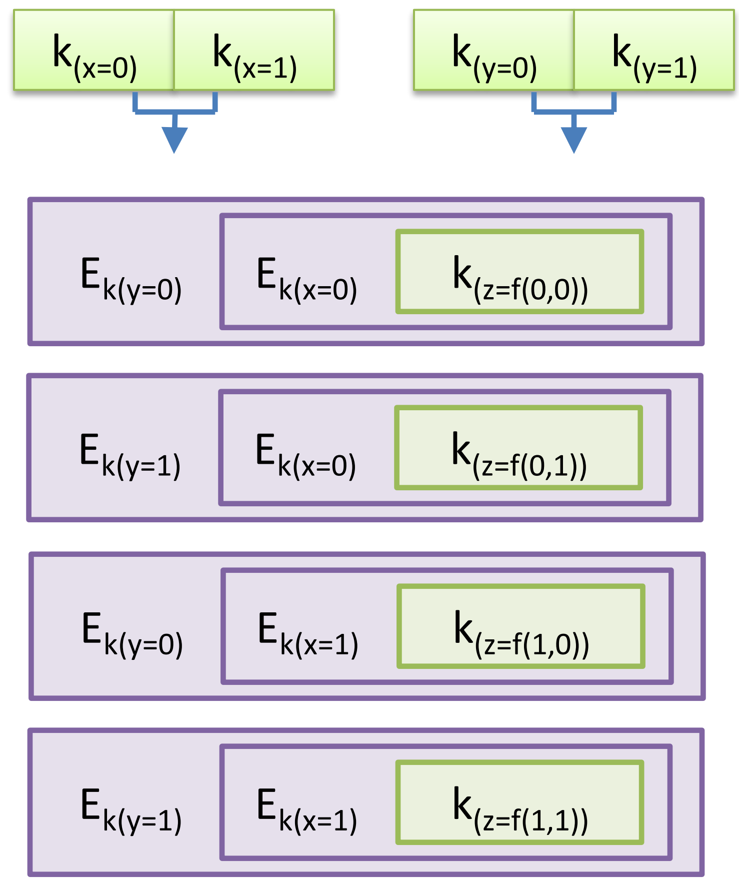
One-time programs from one-time memories

- Use Yao's garbled circuits (Goldwasser et al, 2008)

One gate from the original circuit:



One-time program:



One-time memories

E_k( ) means encryption with key k

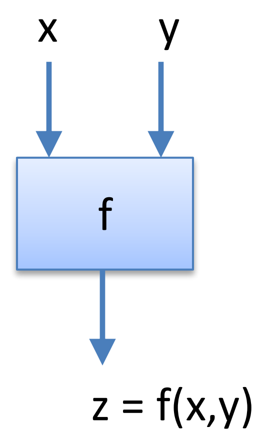
Also note: adaptive security (Bellare et al, 2012), quantum one-time programs (Broadbent et al, 2012)

One-time programs from one-time memories

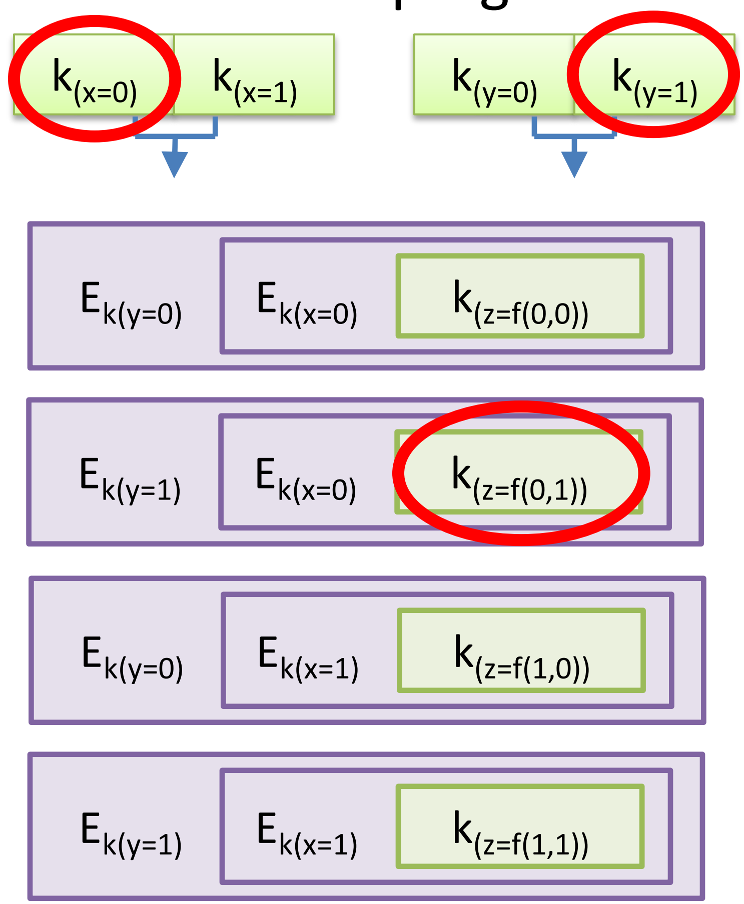
- Use Yao's garbled circuits (Goldwasser et al, 2008)

Choose inputs
x=0, y=1

One gate from the original circuit:



One-time program:



One-time memories

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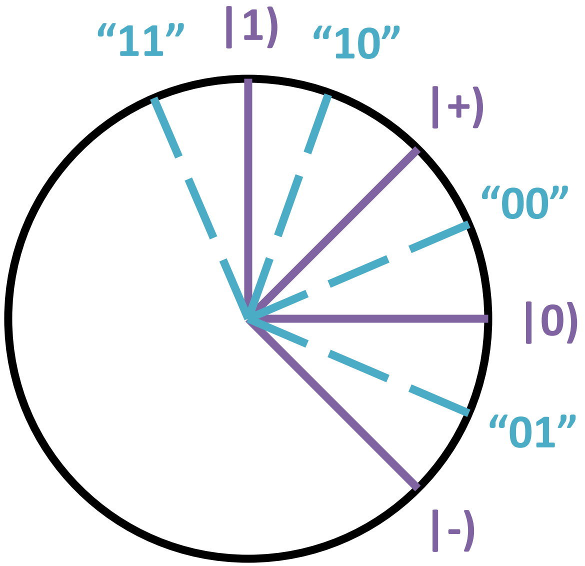
Also note: adaptive security (Bellare et al, 2012), quantum one-time programs (Broadbent et al, 2012)

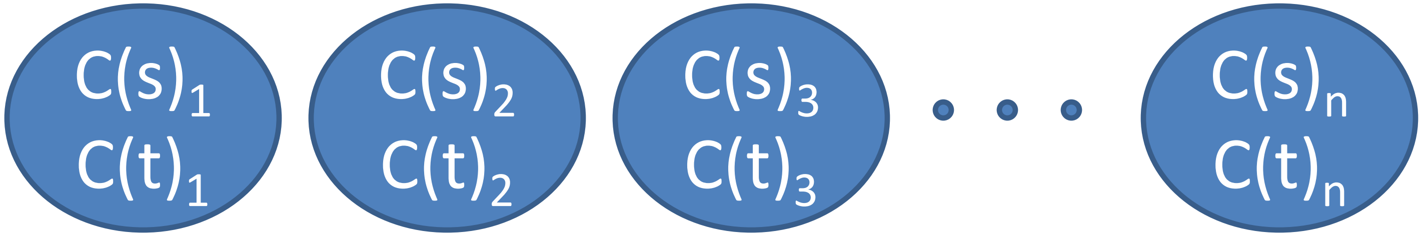
One-time memories using qubits?

- Conjugate coding (Wiesner, ~1970)
  - Encode two classical bits (x,y) into one qubit
  - Measure in standard basis: learn x, w/ prob ≈ 0.85
  - Measure in Hadamard basis: learn y, w/ prob ≈ 0.85



One-time memories using qubits?

- Conjugate coding (Wiesner, ~1970)
  - Take two strings (s,t), apply a classical error-correcting code C, then encode using n qubits

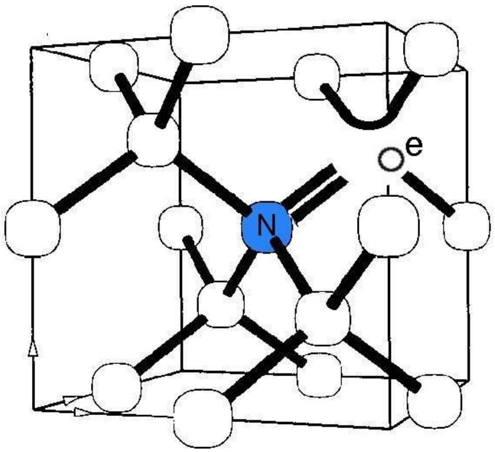


- Bad news: can recover both messages, using many-qubit entangling operations
  - Run the classical decoding algorithm in superposition
  - Recover s without collapsing the superposition
  - Then repeat the procedure to recover t

Isolated qubits model

- We propose a new class of quantum devices: isolated qubits
  - Single qubit operations are allowed
  - Cannot perform operations that entangle multiple qubits
  - LOCC = “local operations and classical communication”

- Modeled on nuclear spins in solid-state materials
  - Easier to build than quantum computers
  - Can still be secure in a world with quantum computers



Credit: http://en.wikipedia.org/wiki/Nitrogen-vacancy_center

Related work

- "Nonlocality without entanglement" [Bennett et al, 1999]
  - There exist quantum operations that are "one-way" with respect to parties who are restricted to LOCC
- Quantum bit-commitment secure against k-local adversaries [Salvail, 1998]
  - Relies on interactive privacy amplification, won't work here
- Quantum bounded storage model [Damgaard et al, 2005]
- Quantum tokens [Pastawski et al, 2012]
- Password-based identification [Bouman et al, 2012]

This talk

- Overview
  - One-time memories, why they are useful
  - Isolated qubits model

- How to construct OTM's in the isolated qubits model
  - Leaky OTM's
  - Privacy amplification

How to construct OTM's

- Step 1: Leaky string-OTM's
  - Conjugate coding
  - Device stores two strings, leaks at most a constant fraction of the information

- Step 2: Deterministic privacy amplification
  - "Almost-perfect" single-bit OTM
  - Device stores two bits, leaks an exponentially small amount of information

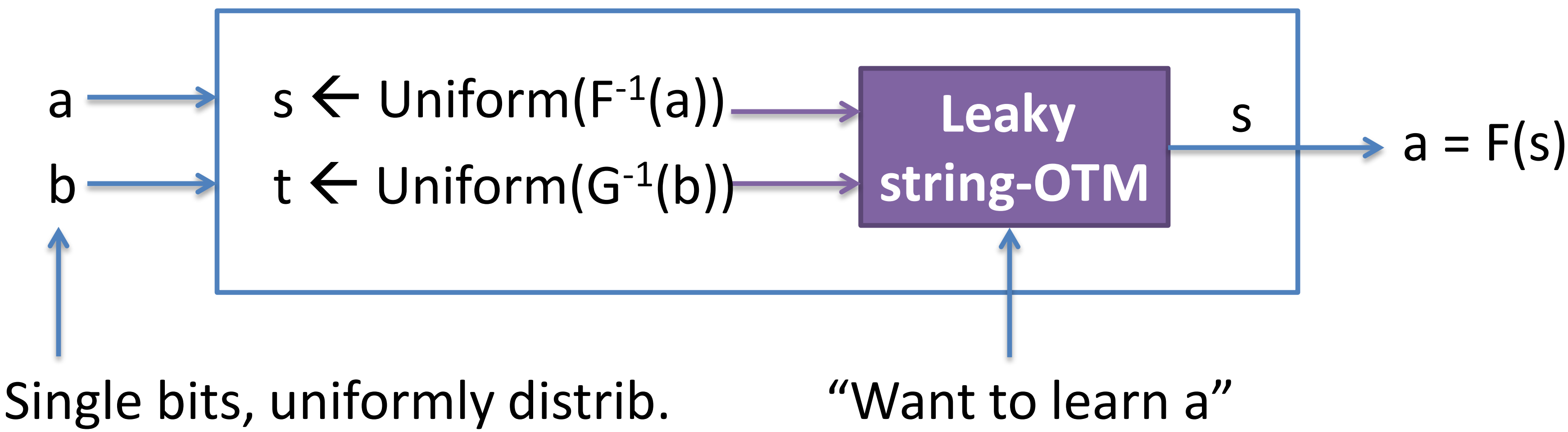
Assume we have a leaky string-OTM

- Device stores two messages S and T, each ℓ bits long
  - Assume they are uniformly distributed
  - Ideal security goal: adversary can learn either S or T, but not both

- A weaker ("leaky") notion of security:
- For any LOCC adversary, $H^{\varepsilon}_{\infty}(S,T|Z) \geq (0.5 - \delta)\,\ell$
  - Z is the adversary's output

Step 2: Deterministic privacy amplification

- Given a leaky string-OTM, construct an "almost-perfect" bit-OTM
  - Choose two (r-wise independent) random functions $F, G: \{0,1\}^{\ell} \rightarrow \{0,1\}$
    - Set ℓ, r to be polynomial in the security parameter k
    - Fix F and G permanently, as part of the construction



“Almost perfect” security

- With high probability over the choice of F and G, the following holds:
- For every LOCC adversary, there exists a binary random variable C, such that:
- $H^{\varepsilon}_{\infty}(A|C=0, Z) \geq 1 - 2^{-\Omega(k)}$
- $H^{\varepsilon}_{\infty}(B|C=1, Z) \geq 1 - 2^{-\Omega(k)}$
  - where Z is the adversary’s output, and $\varepsilon \leq 2^{-\Omega(k)}$
  - Note: adversary’s strategy may depend on F and G!
  - Random variable C comes from “entropy splitting” [Damgard et al]

Proof of security

- First, prove security wrt a single fixed meas. outcome
  - For any fixed measurement outcome M,
    with high probability over the random functions F and G,
    the scheme is secure
- Proof
  - Leaky string-OTM: $H^{\varepsilon}_{\infty}(S,T|M) \geq \Omega(k)$
  - Entropy splitting: ∃ random variable C, $H^{\varepsilon}_{\infty}(S|C=0, M) \geq \Omega(k)$
  - Want to bound: $\mathrm{bias}(A|C=0, M) = E_A((-1)^A|C=0, M)$
    $= \Sigma_s (-1)^{F(s)} \Pr(S=s|C=0, M)$
  - This is a sum of r-wise independent random variables $(-1)^{F(s)}$
  - Use Hoeffding-like large-deviation bound
  - Note $\Sigma_s \Pr(S=s|C=0, M)^2 = 2\text{^}[-H_2(S|C=0, M)] \leq 2^{-\Omega(k)}$

Proof of security

- Covering argument
  - Construct an ε-net for the set of all tensor product measurement outcomes
  - This has cardinality ≤ $2^{\text{poly}(k)}$ (singly, not doubly exponential, because adversary is restricted to LOCC measurements)

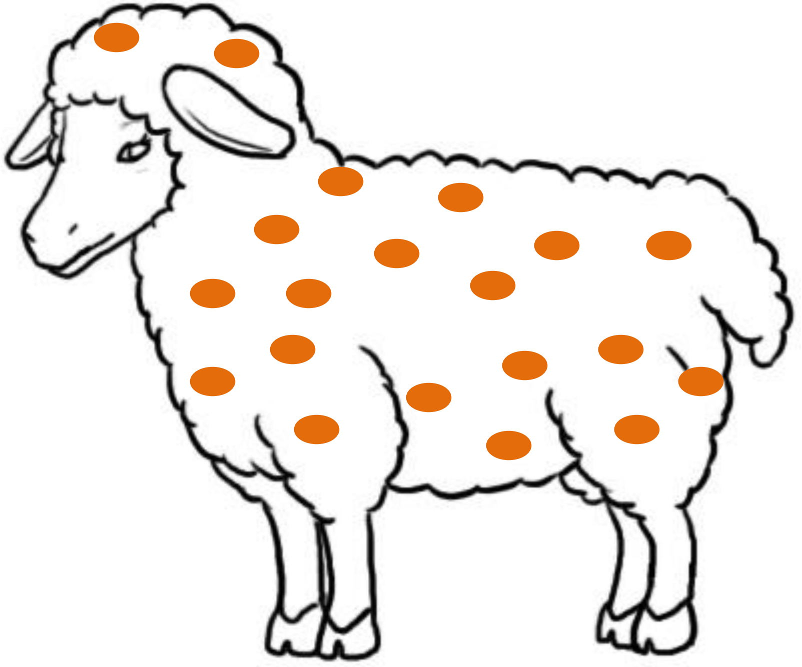
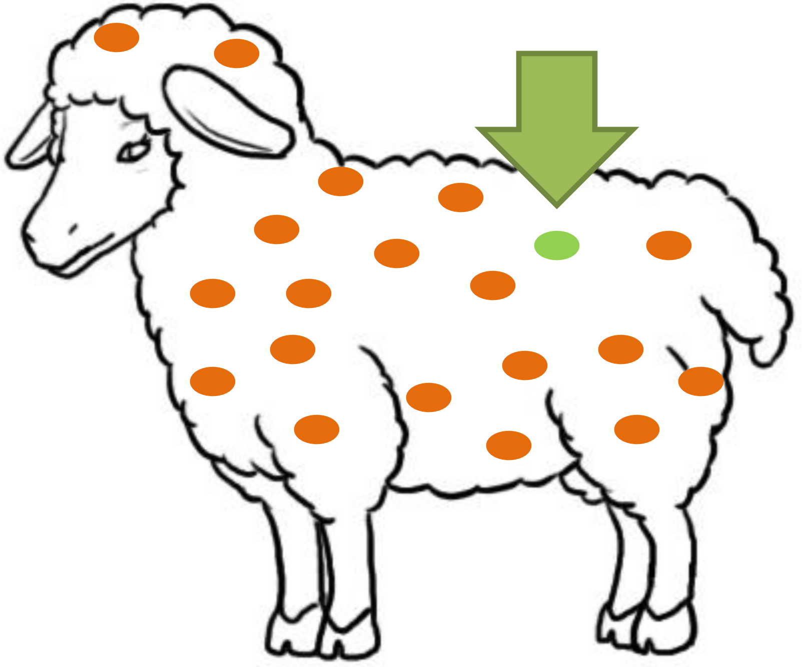


Image credit: http://www.wikihow.com/Draw-Sheep

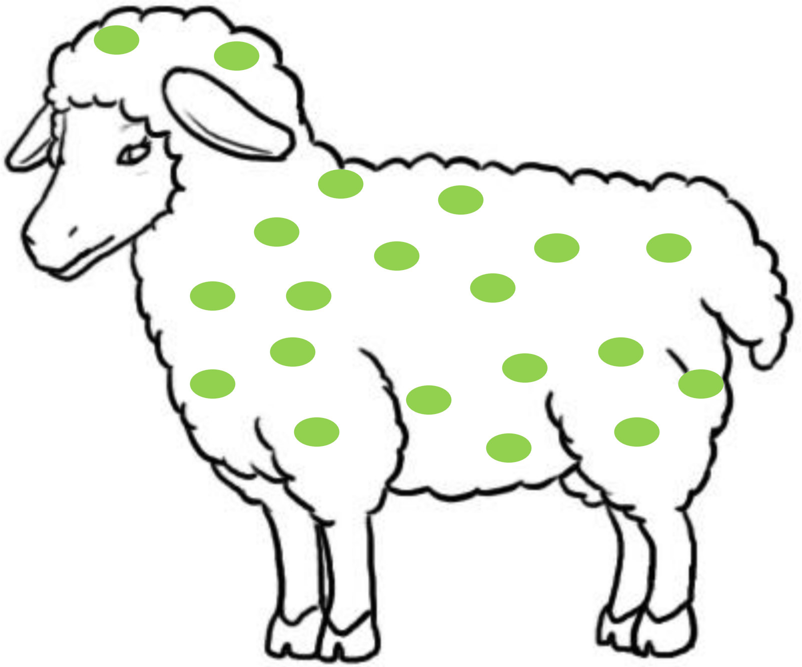
Proof of security

- Covering argument
  - Construct an ε-net for the set of all tensor product measurement outcomes
  - This has cardinality $\leq 2^{\mathrm{poly}(k)}$
- Prove security at one point in the ε-net
  - For any fixed measurement outcome M, with high probability over the random functions F and G, the scheme is secure
  - Failure probability is $\leq 2^{-\mathrm{poly}(k)}$



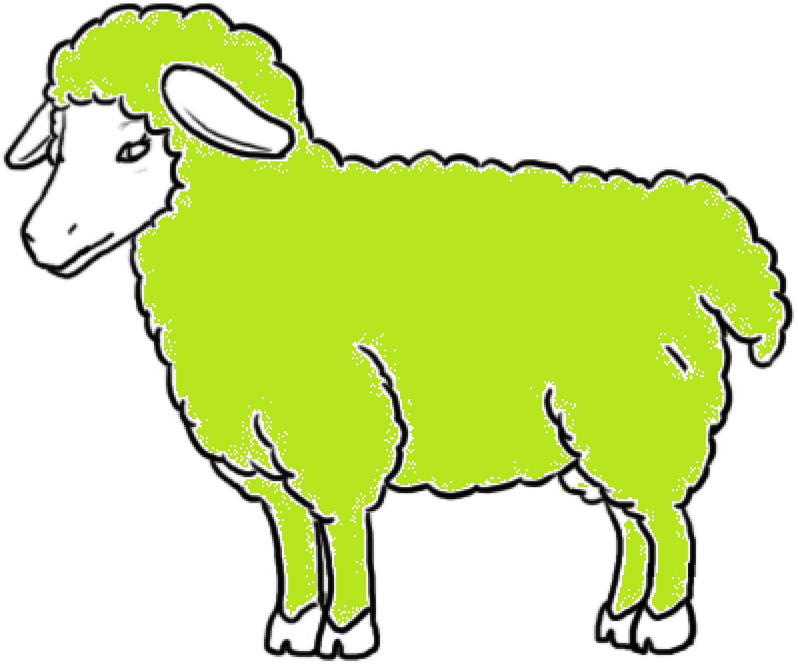
Proof of security

- Then use the union bound over all M in the ε-net
  - With high probability over F and G,
    for all M in the ε-net (simultaneously),
    the scheme is secure

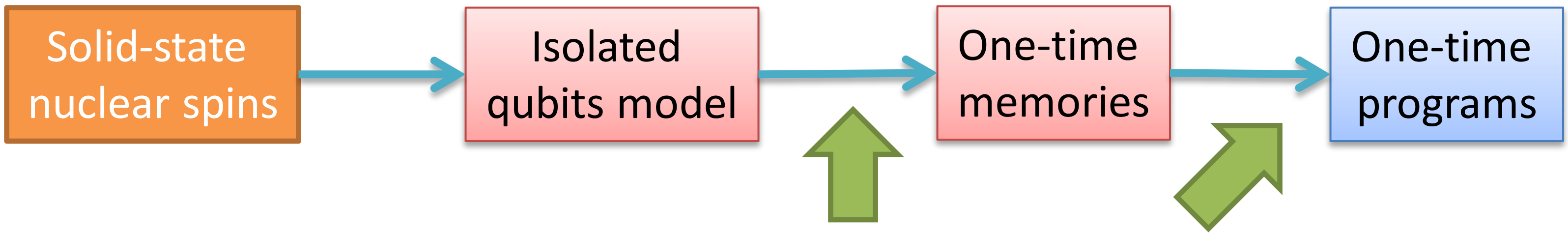


Proof of security

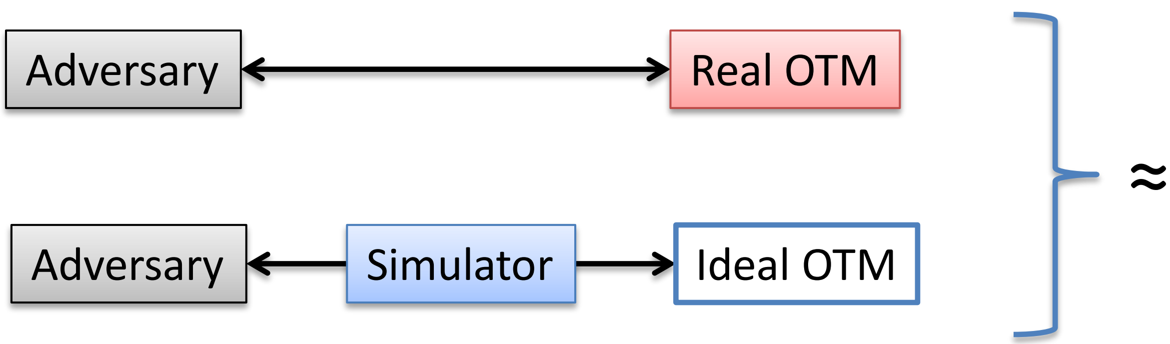
- Then use the union bound over all M in the ε-net
  - With high probability over F and G,
    for all M in the ε-net (simultaneously),
    the scheme is secure
- "Continuity argument"
  - Security does not change much
    when we perturb M
  - So for all tensor product M
    (simultaneously),
    the scheme is secure



Outlook



- Deterministic privacy amplification helps us to control information leakage
  - This helps to construct one-time programs…
  - Can our OTM's achieve composable security?



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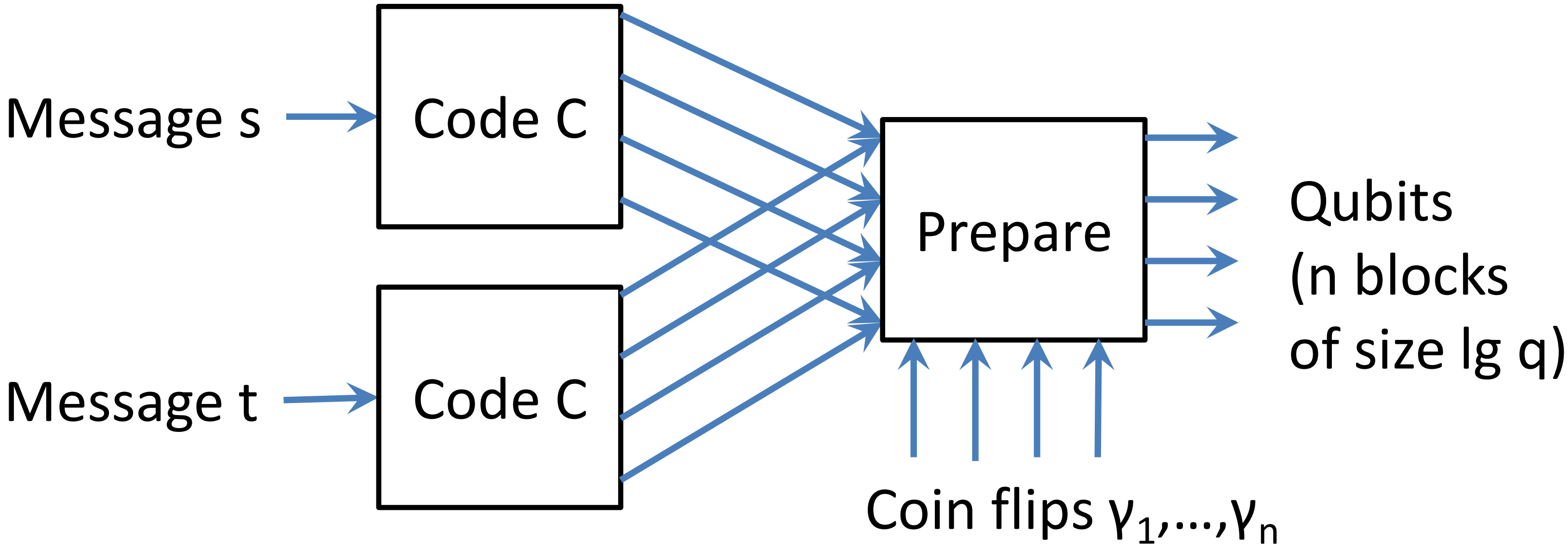
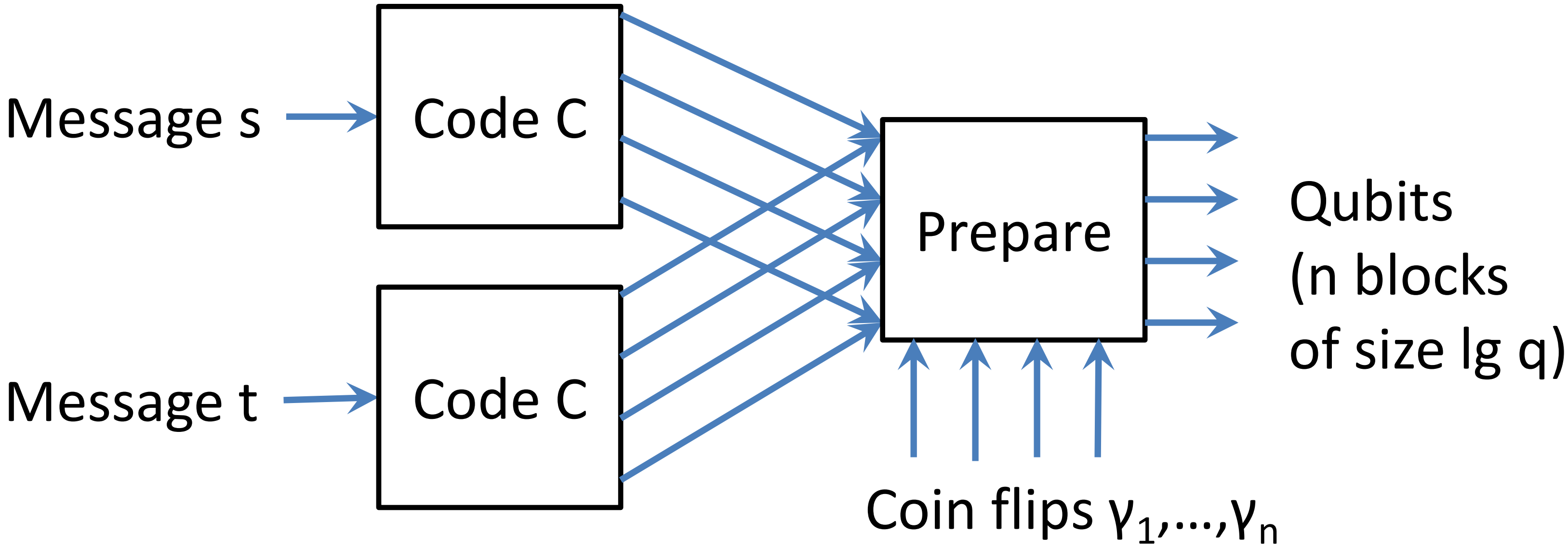
Step 1: Leaky string-OTM's



- To prepare the i'th block of qubits:
- If $\gamma_i = 0$, use the i'th block of C(s) and the |0⟩, |1⟩ basis
- If $\gamma_i = 1$, use the i'th block of C(t) and the |+⟩, |–⟩ basis

Step 1: Leaky string-OTM's

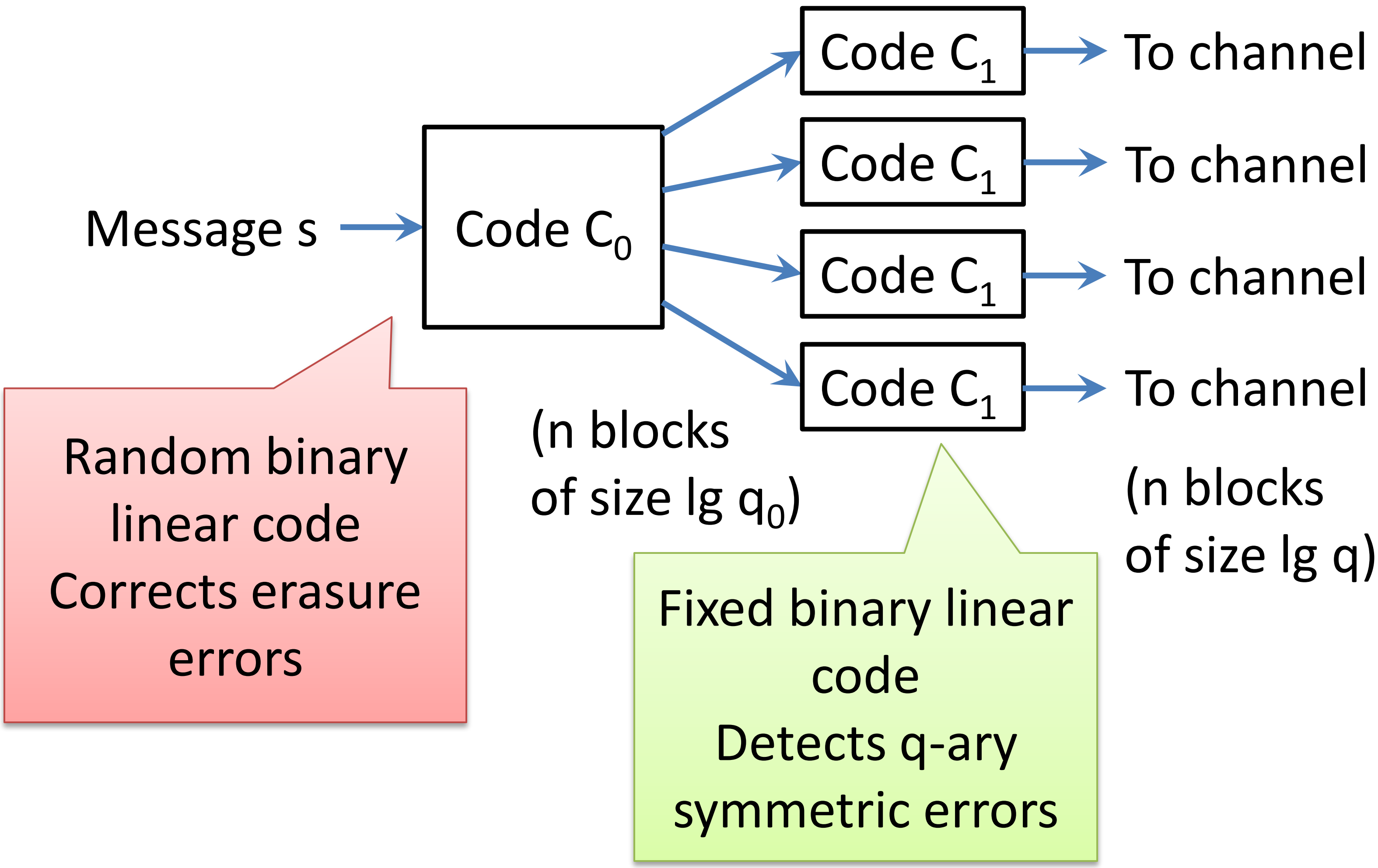


- To read s: measure qubits in standard basis
- To read t: measure qubits in Hadamard basis
- This is equivalent to receiving C(s) or C(t) through a q-ary symmetric channel

Choosing the code C

- To ensure security:
  - C should approach the capacity of the q-ary symmetric channel
  - C should be "unstructured"
    - One way to formalize this: let C be linear over GF(2)
    - Generator matrix has full rank
    - Suppose message S is uniformly distributed
    - Then codeword C(S) will have a large subset of bits that are uniformly distributed
  - Also, C should be efficiently decodeable

Good codes for the q-ary symmetric channel



Good codes for the q-ary symmetric channel

- For large q (growing with n), this approaches the capacity of the q-ary symmetric channel
- Efficient decoding: solving linear systems of equations over GF(2)

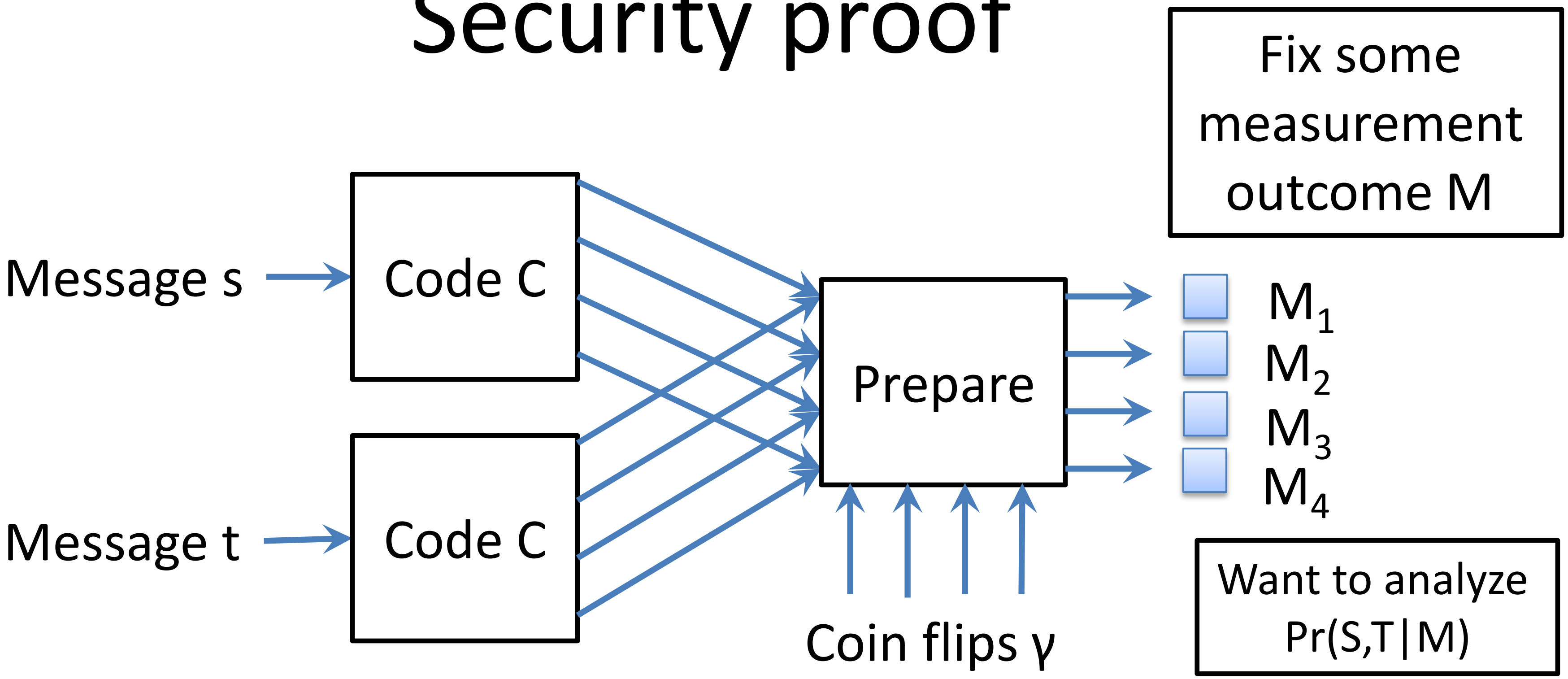
- Other constructions:
  – Interleaved Reed-Solomon codes, interleaved AG codes [Bleichenbacher et al; Shokrollahi; Brown et al]

Security proof

- Prove security against separable adversaries
  - Every POVM element is a tensor product of 1-qubit operators
  - Separable adversaries include LOCC as a special case
  - LOCC can be complicated: e.g., adaptive sequences of weak measurements

Given a separable adversary A

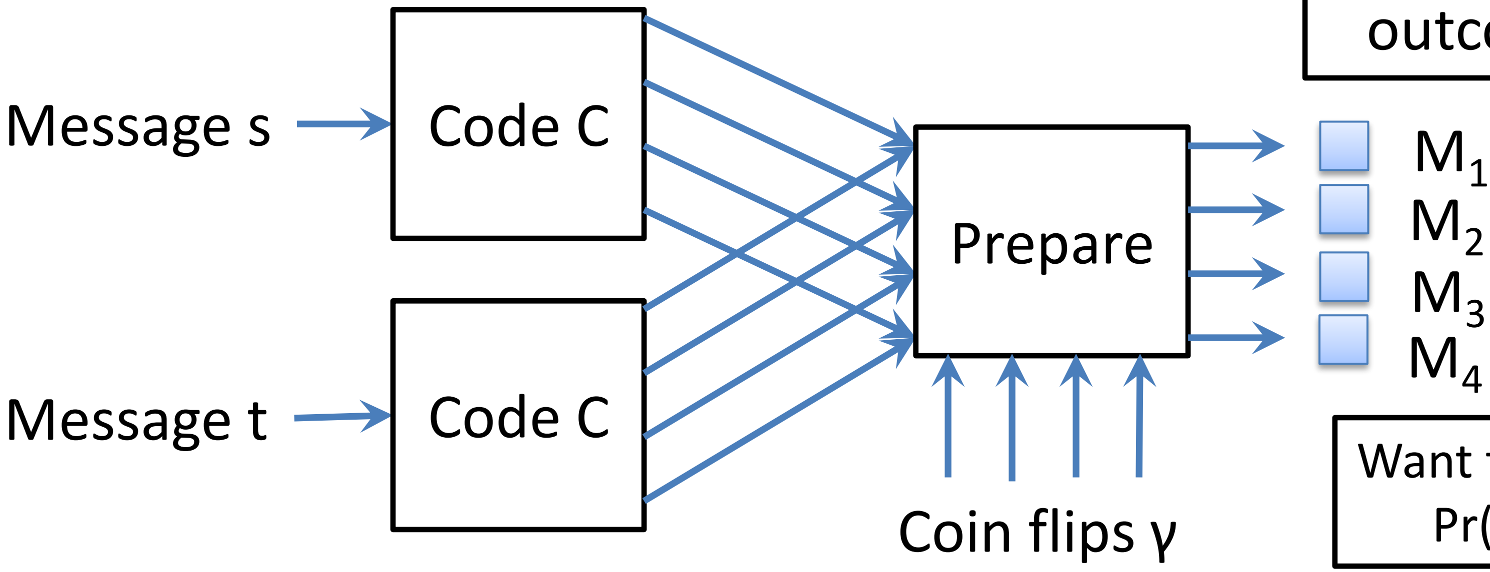
Security proof



Given a separable adversary A

Security proof

Fix some measurement outcome M

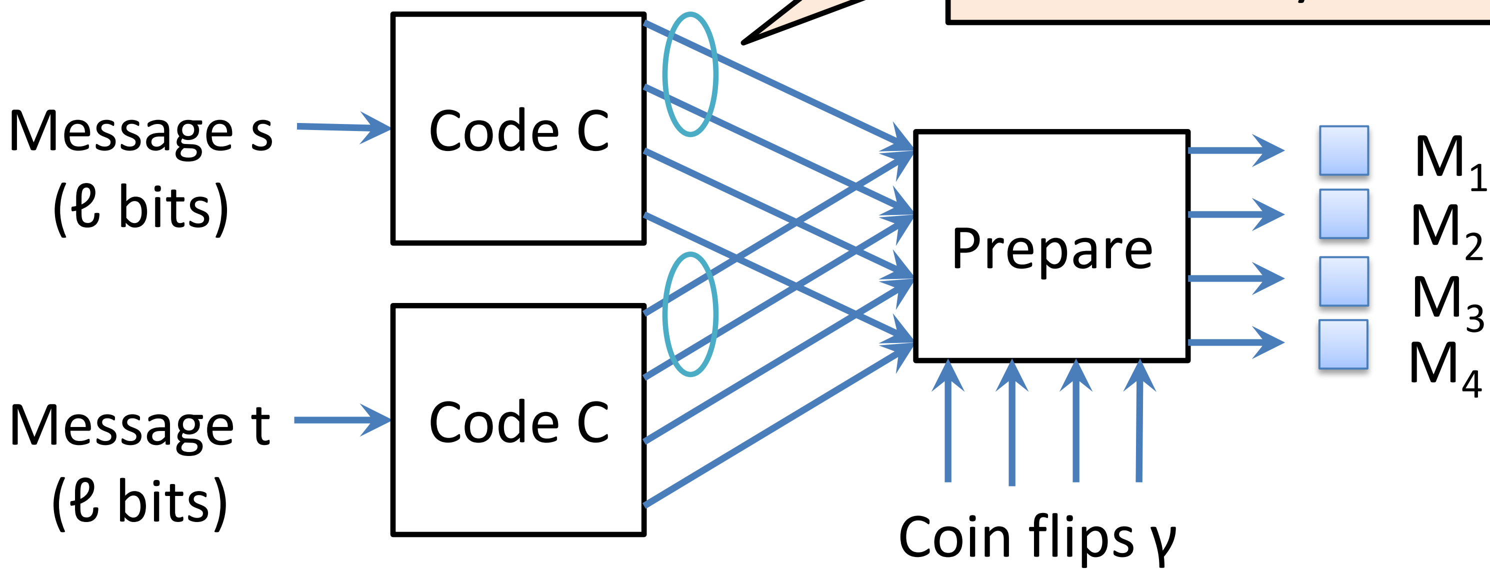


Want to analyze Pr(S,T|M)

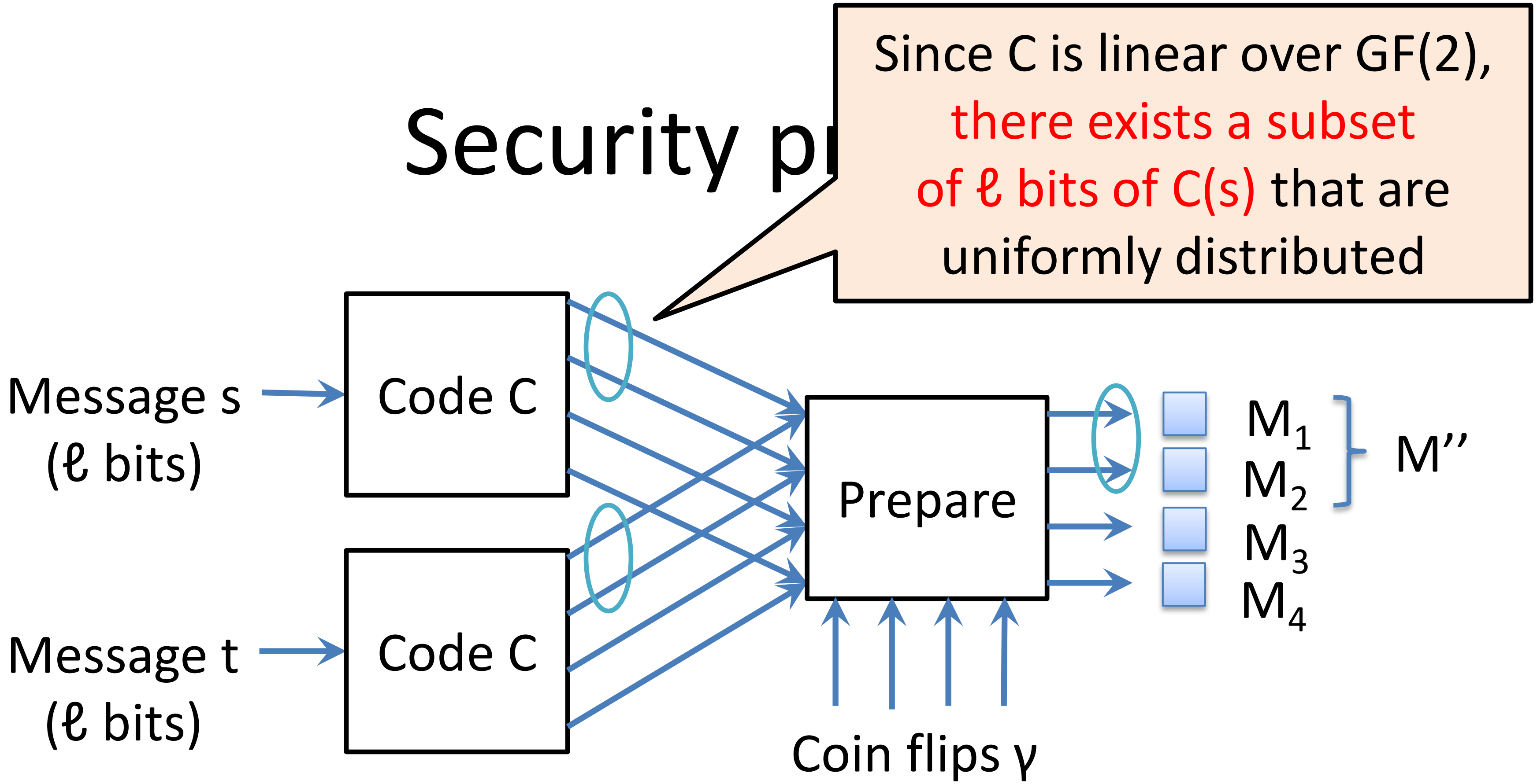
- Consider a fictitious adversary A' that measures each qubit once, such that M_1,M_2,M_3,… are possible outcomes
- Call this event M'
- Then Pr(S,T|M) = Pr(S,T|M')

Security pr

Since C is linear over GF(2), there exists a subset of ℓ bits of C(s) that are uniformly distributed

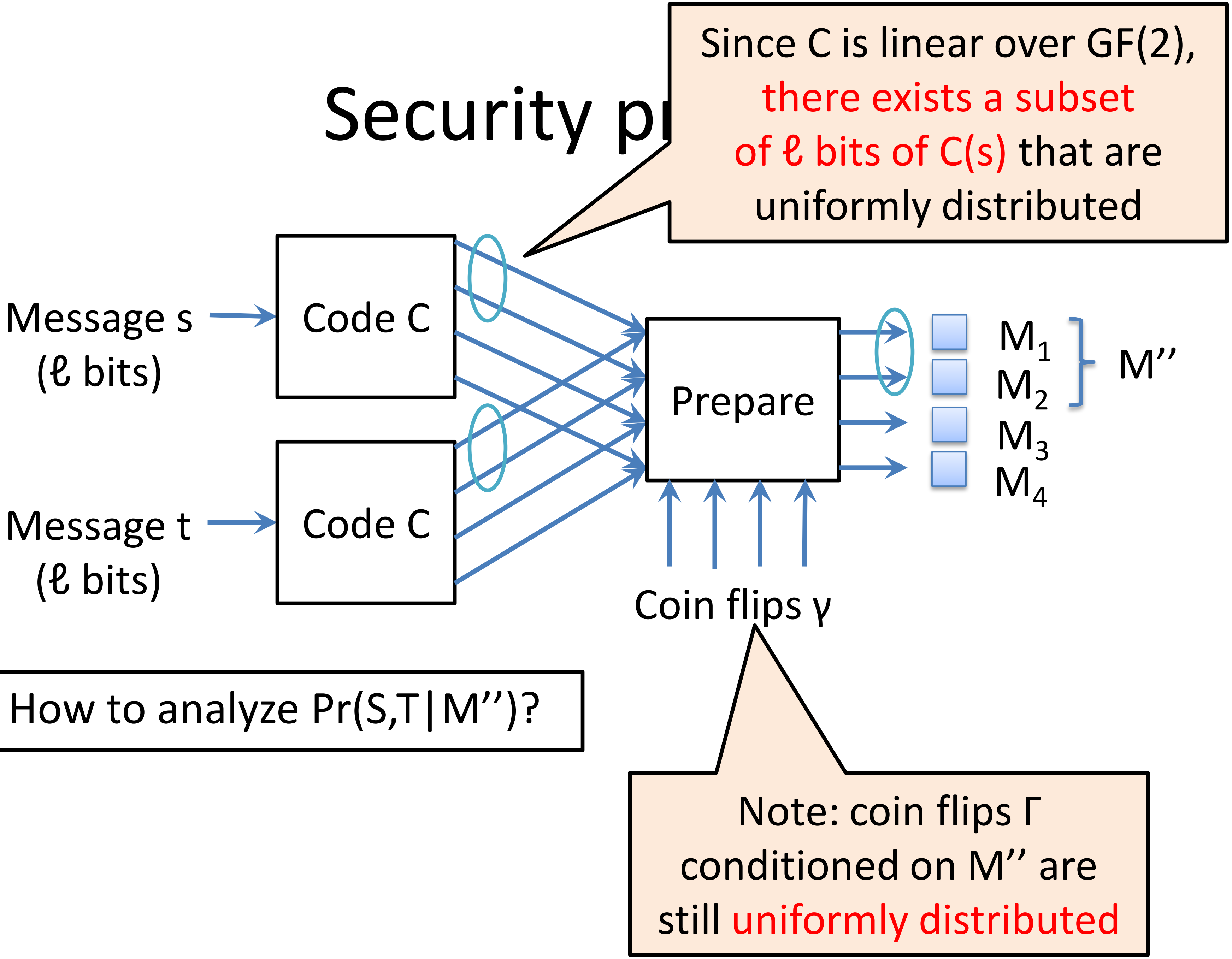


Security pr



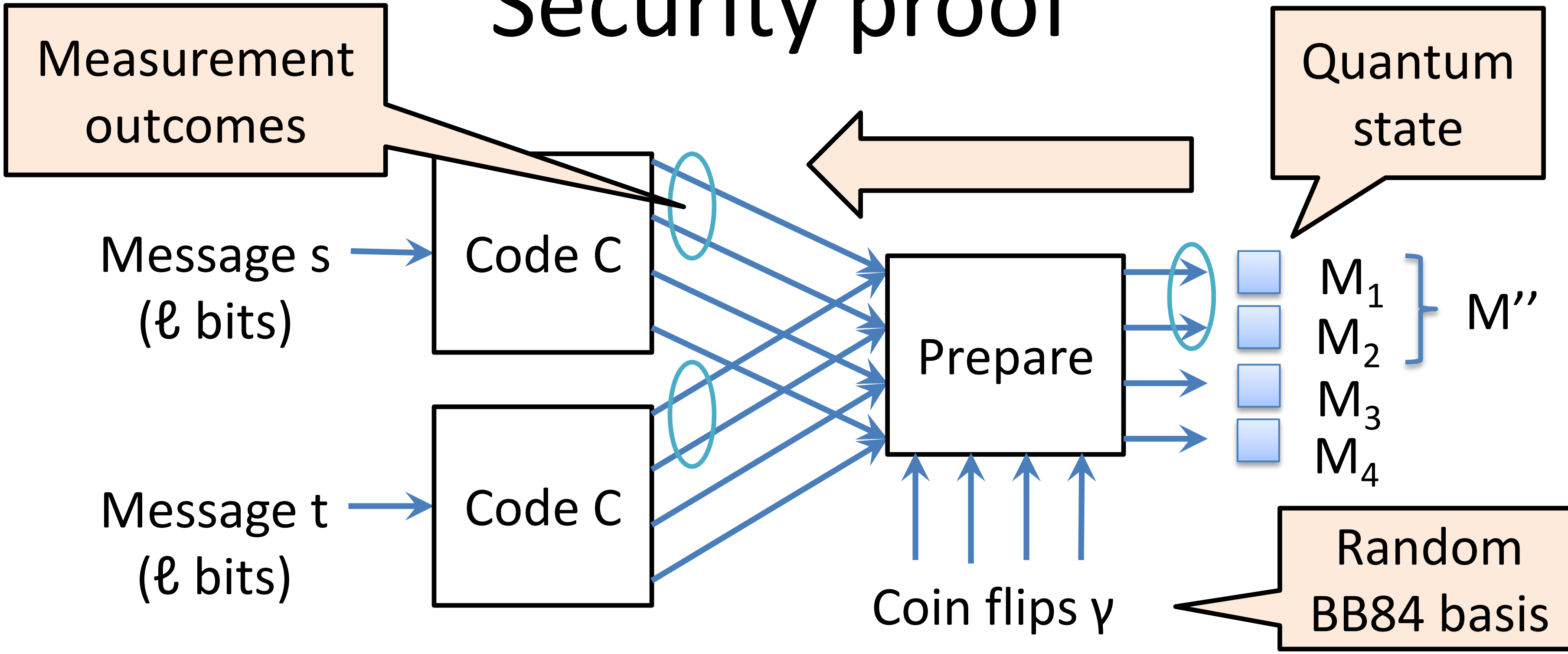
- Wlog, suppose the fictitious adversary A’ measures this subset of qubits first
- When A’ observes M_i for all i in this subset, call this event M’’
- Want to analyze Pr(S,T|M’’)

Security pr



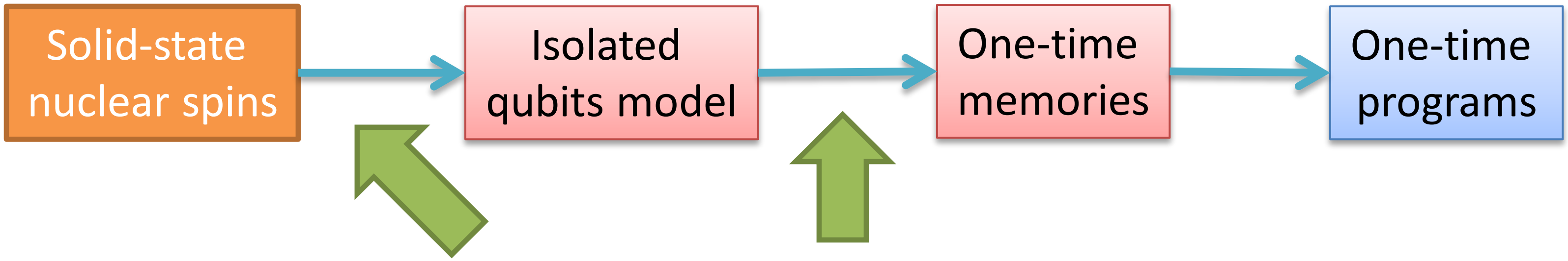
- How to analyze Pr(S,T|M'')?

Security proof



- Now run the experiment backwards…
  - Measuring a quantum state using a sequence of random BB84 bases
- Use an entropic uncertainty relation to lower-bound $H^{\varepsilon}_{\infty}(S,T|M'')$
  - Borrowed from the bounded quantum storage model [Damgard et al, 2006]

Outlook



- We have showed how to construct OTM's based on isolated qubits
  - Instead of isolated qubits, can we use more realistic models of the underlying hardware?
  - Noisy entangling operations?
  - Shallow quantum circuits?

Isolated qubits are fun

- For theorists:
  - Another model, where many interesting cryptographic tasks are possible!
    - Known constructions seem very far from optimal!
    - Based on simple probabilistic constructions, crude bounds
- For experimentalists:
  - Another family of interesting quantum devices that can be realized
    - Very different from quantum repeaters
    - Want long coherence times, good single qubit operations, <u>no</u> entanglement swapping

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Thank you!