Physical Randomness Extractors

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University of Michigan
joint works with Carl Miller (arXiv:1402.0489), Kai-Min Chung and Xiaodi Wu (arXiv:arXiv:1402.4797)

Carl Miller

Kai-Min Chung

Xiaodi Wu

1. Motivation

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- Why is it difficult to get randomness?

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- Why is it difficult to get randomness?
- Why quantum and untrusted quantum devices?

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- Error parameter: deviation of XE from $U_X \otimes E$
- True randomness: error → 0 (as other parameters grow)

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- 1 T bits/day?

Central question

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How do we get true randomness and know that we've got it?

We aren't always getting it: many security vulnerabilities

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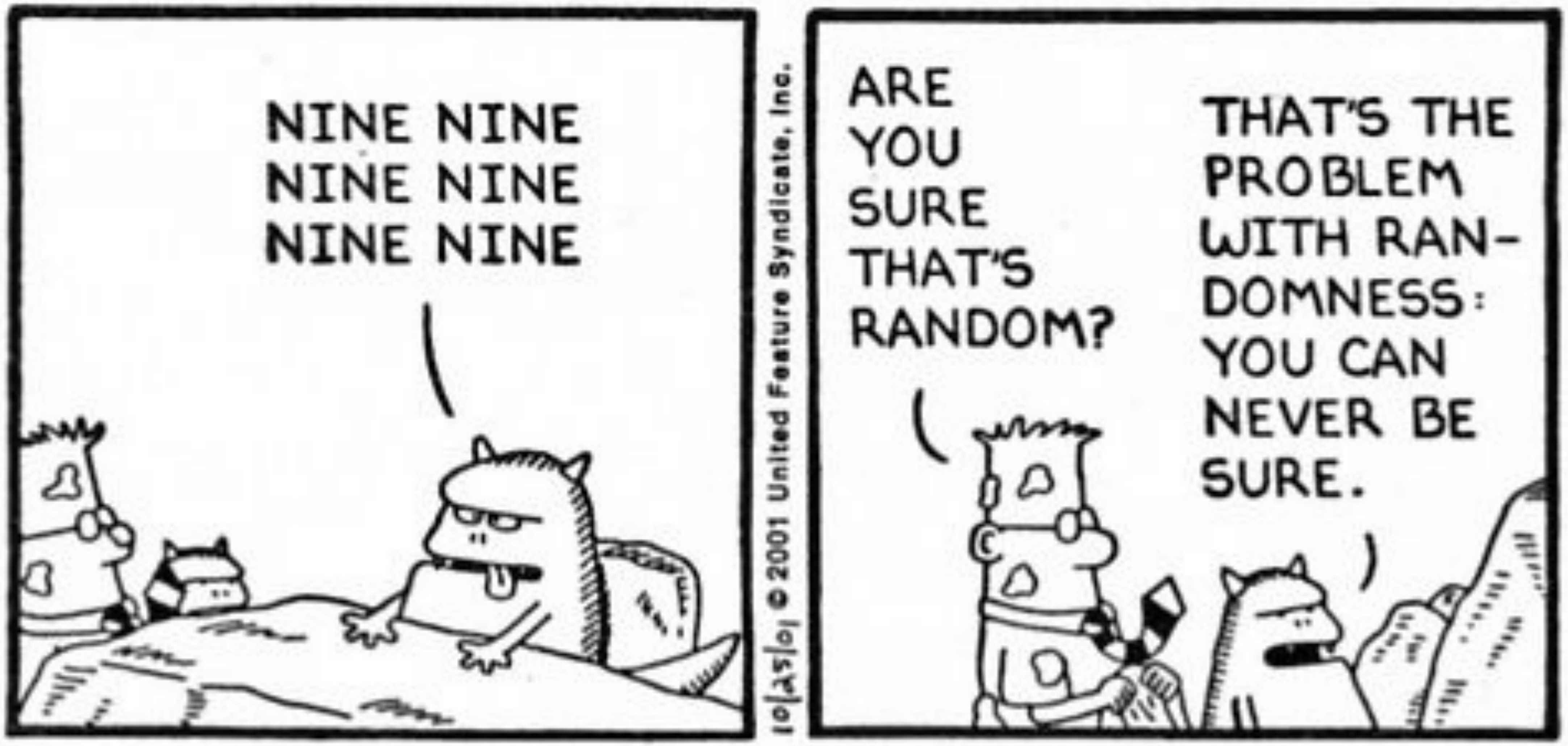
"Ultimately the results of our study should serve as a wake-up call that secure random number generation continues to be an unsolved problem in important areas of practice."

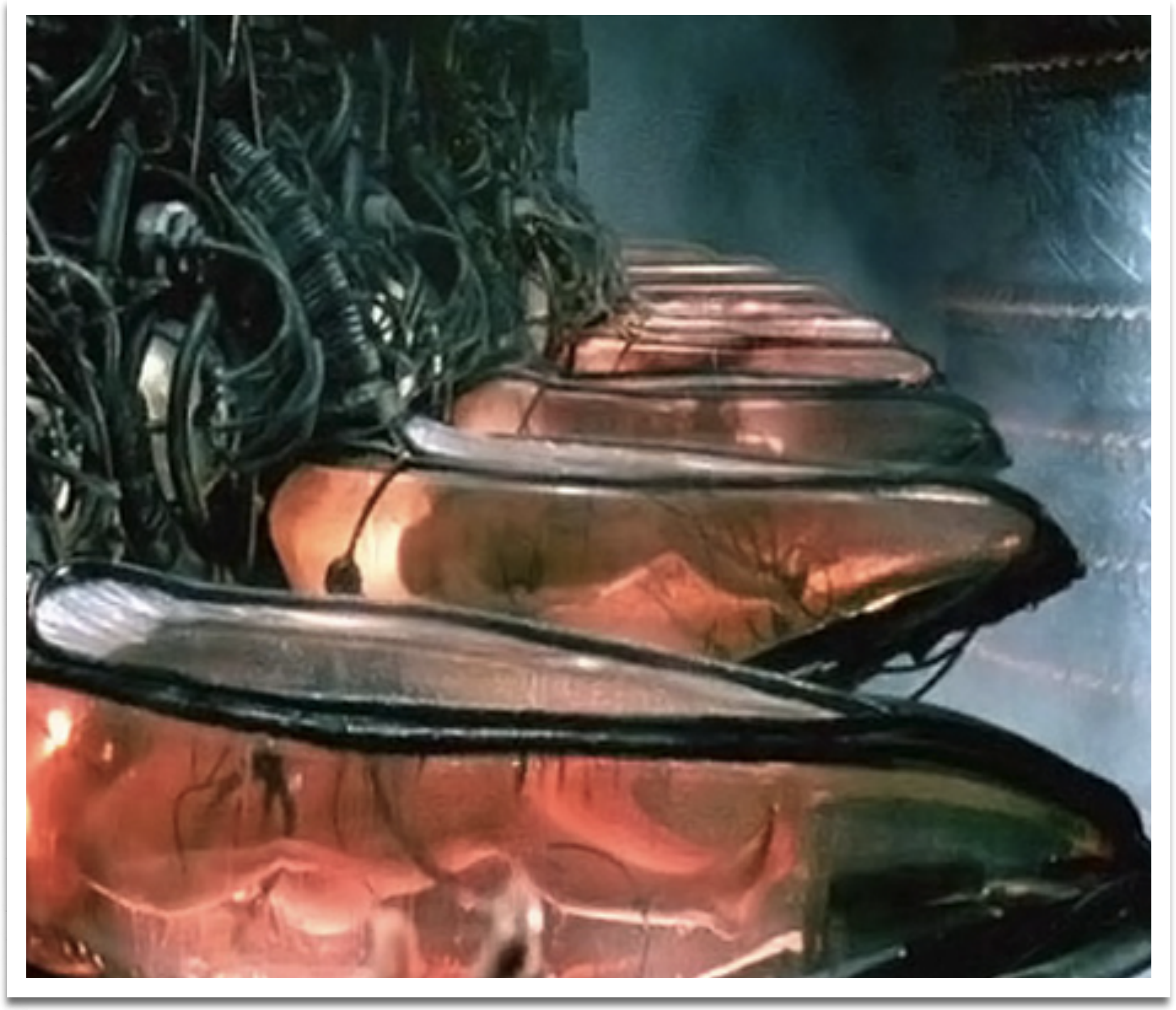
[Heninger+]

How can we be sure it's random?



Motivation::randomness extractors

Also a deep physics question



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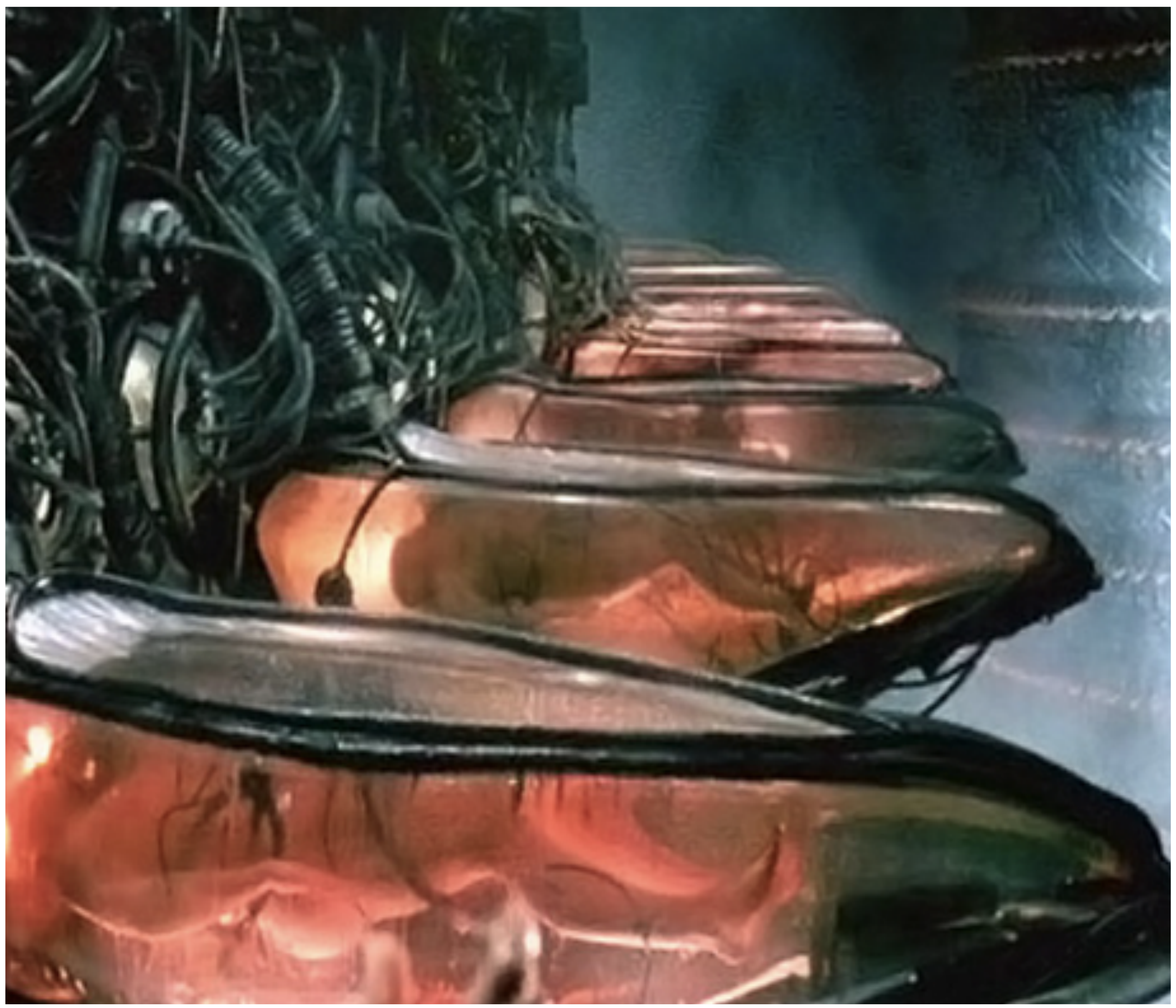

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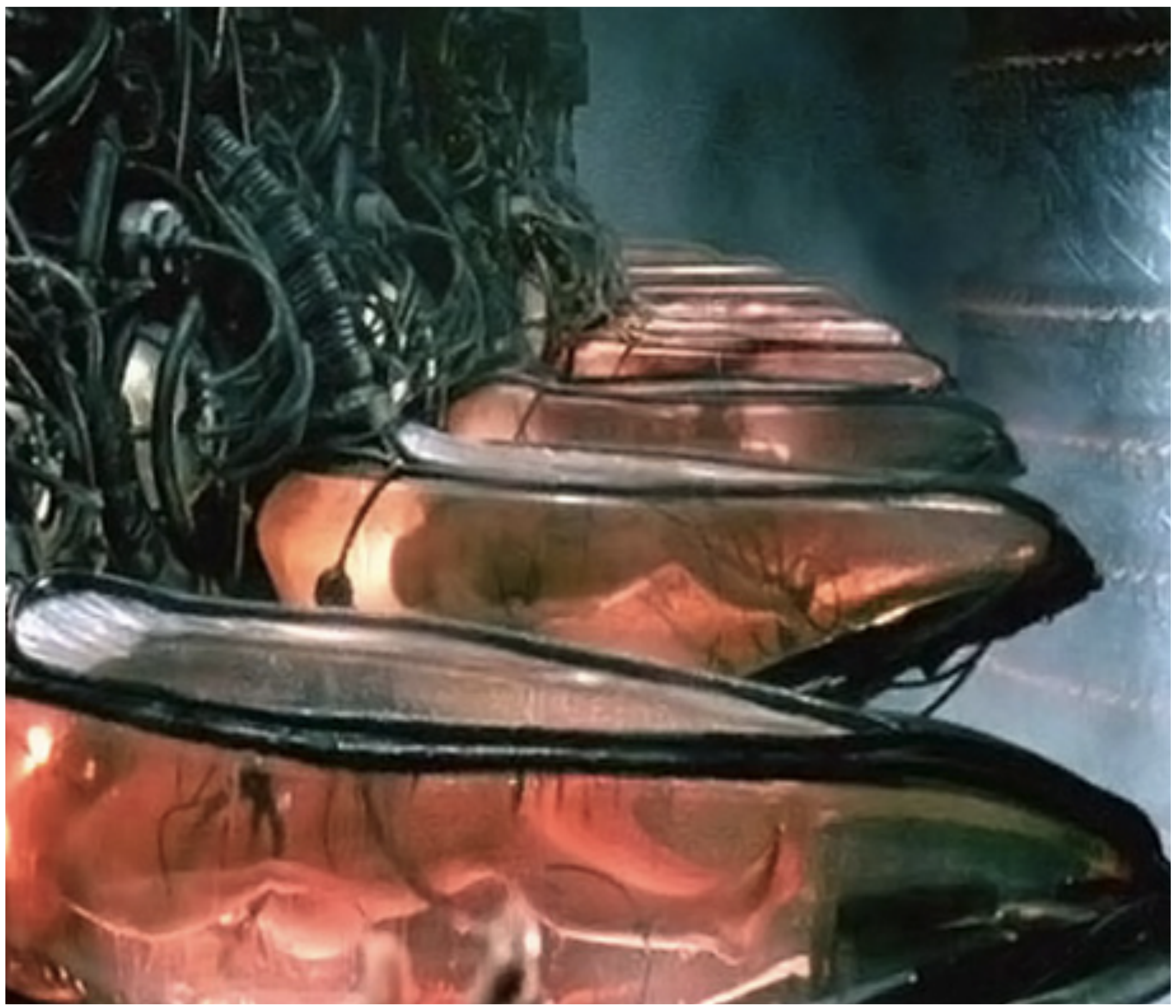

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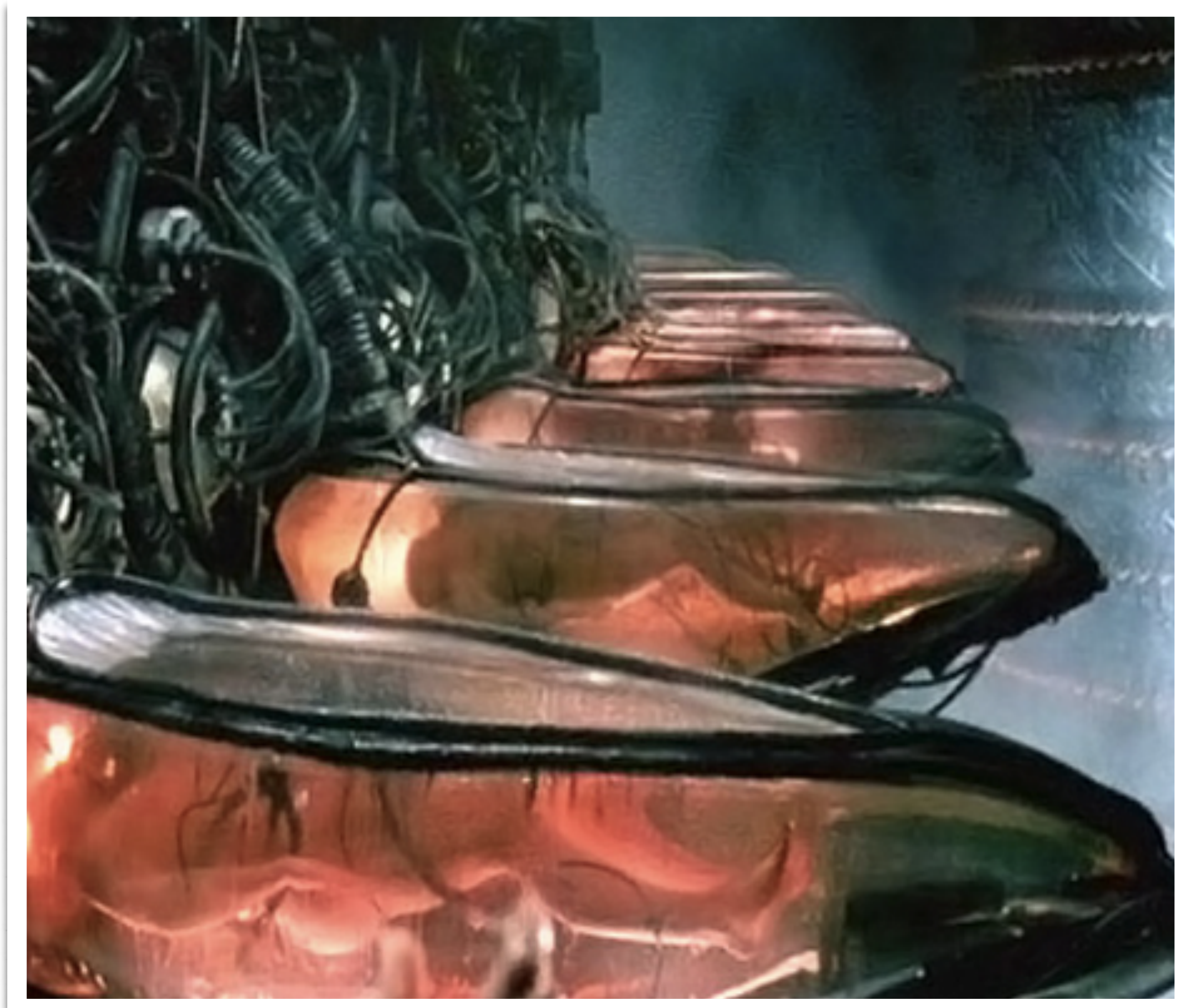

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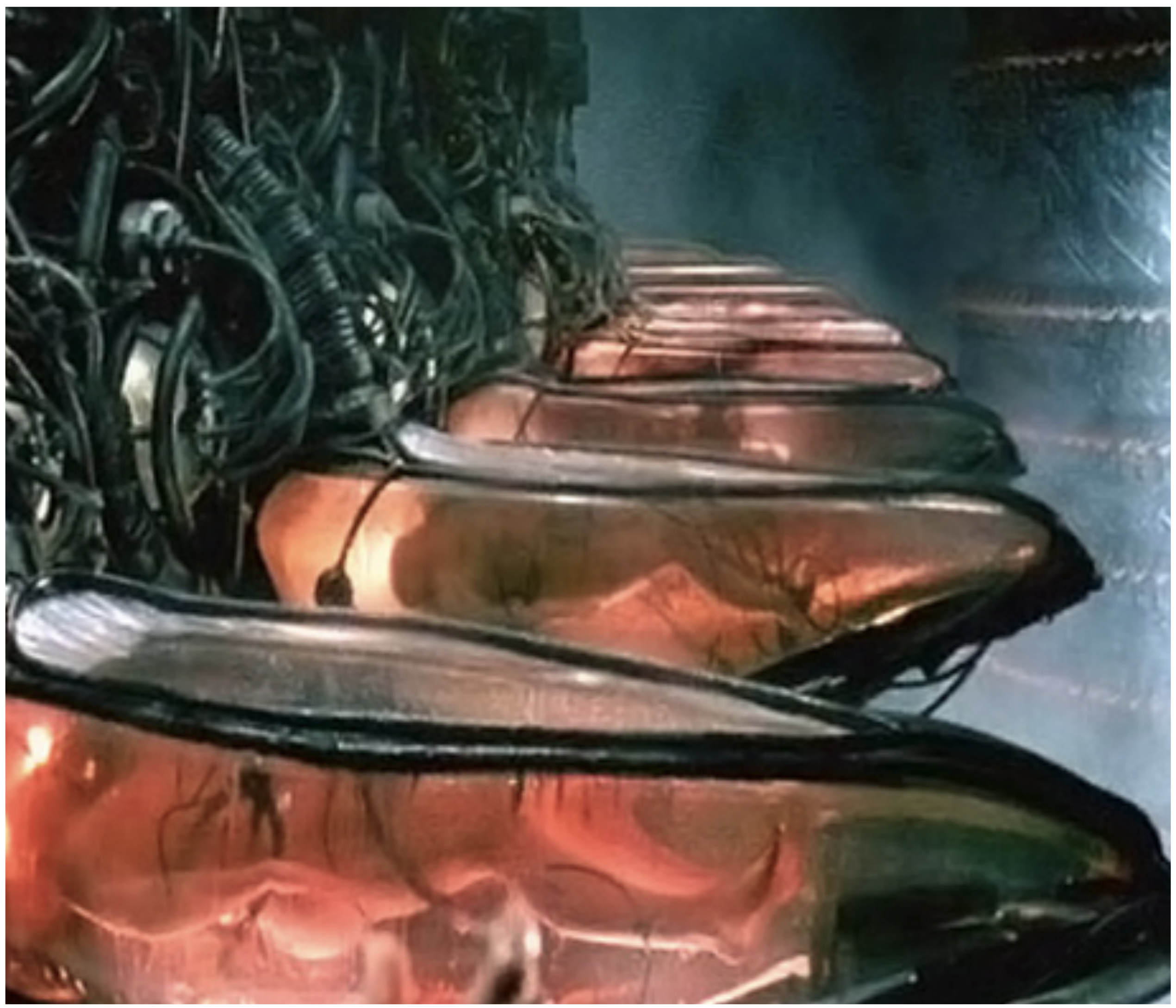

- Does randomness exist at all?
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- Assuming the world is not deterministic,
  Could there be almost perfect randomness?
- Or, are we stuck with weak randomness?

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- Model weak source by min-entropy
- Turn weak sources to true randomness
  - Ensure randomness whenever assumptions are met
  - Excellent constructions for seeded extraction (i.e. one source is uniform)

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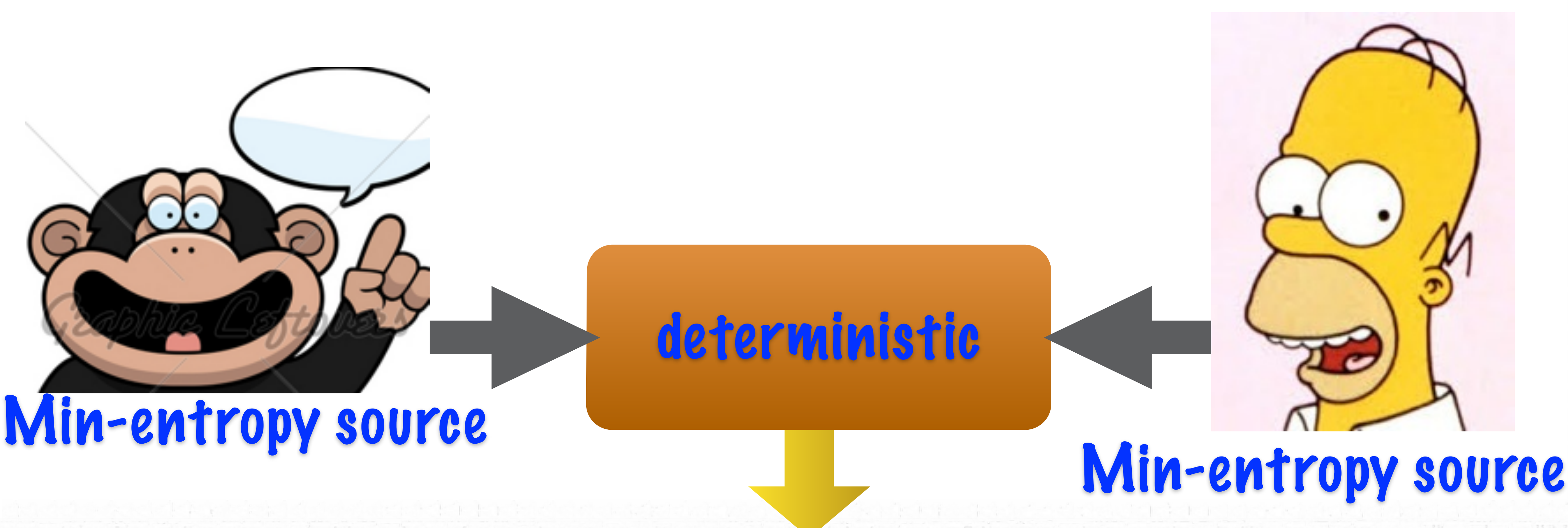

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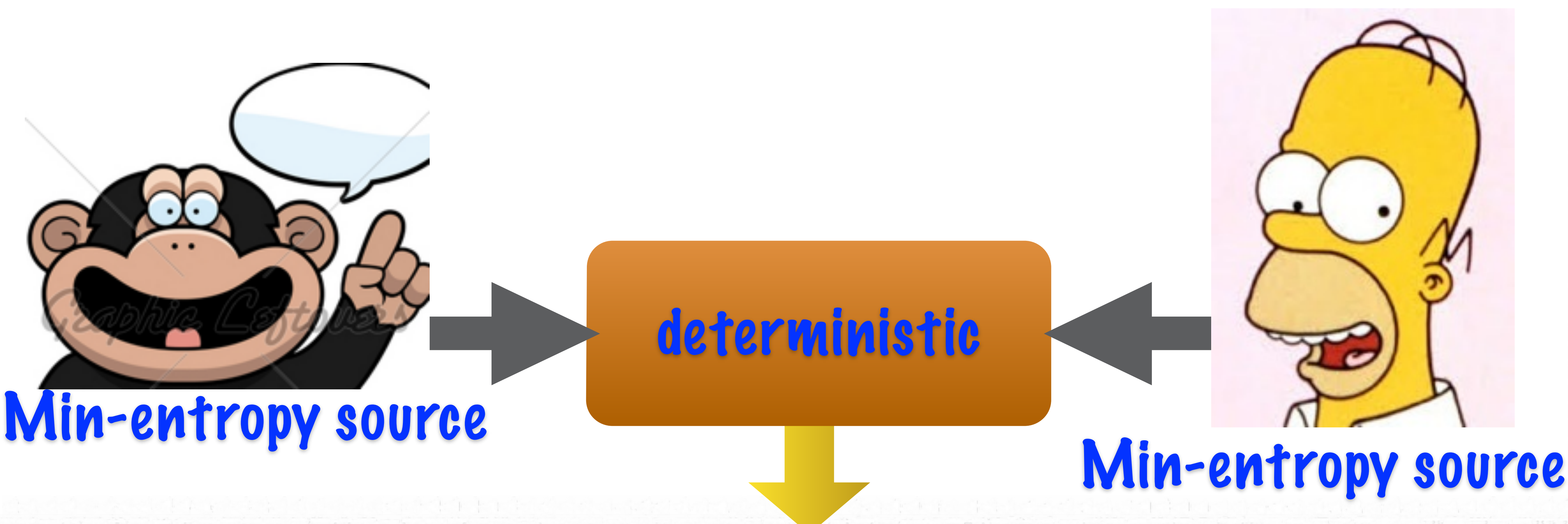

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Quantum solution with trusted devices

- True quantum randomness (passes all randomness tests)
- High bit rate of 4Mbits/sec
- Affordable, compact and reliable
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OEM

QUANTIS IS OFFICIALLY CERTIFIED

QUANTIS has been evaluated and certified by the Swiss Fed
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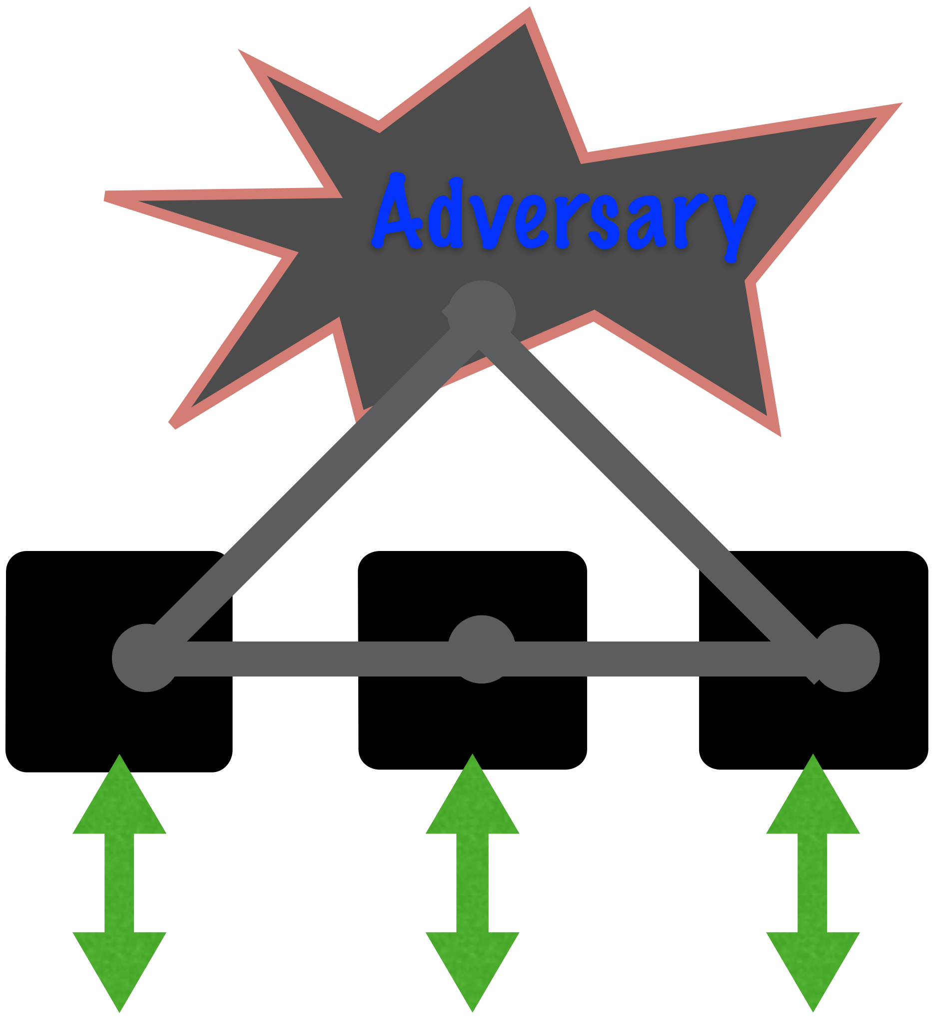
- As classical beings, we cannot sense quantum directly
- Are we willing to trust the manufacturer or the certifying agency?
- Even yes, devices may not be reliable.
    - Current technologies are prone to "noise"

Untrusted Quantum Devices



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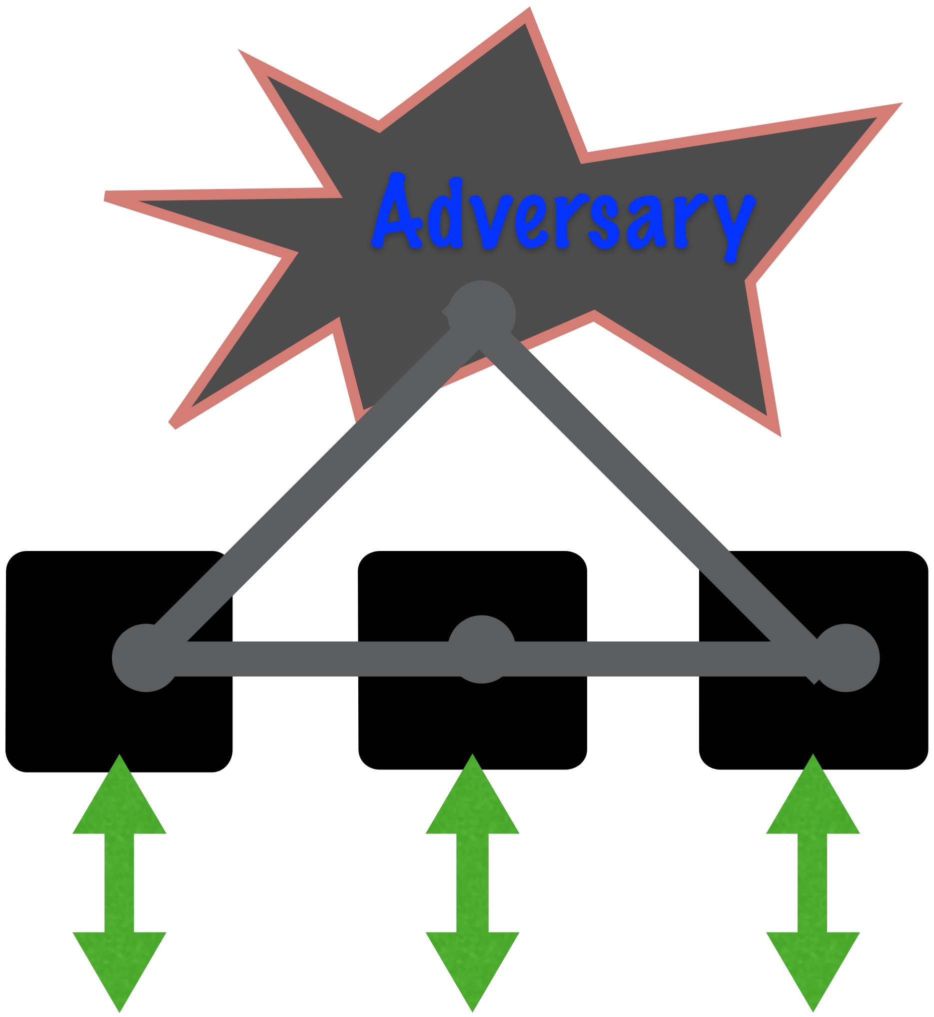
Untrusted Quantum Devices



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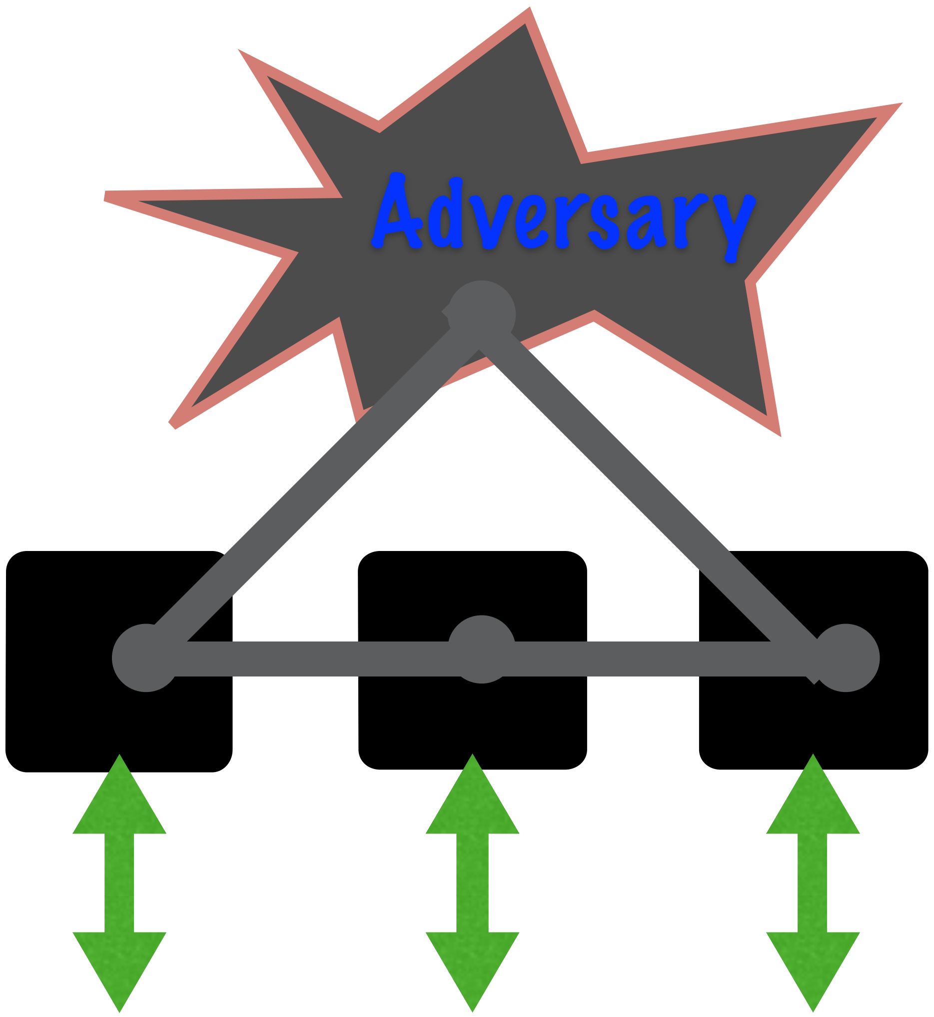

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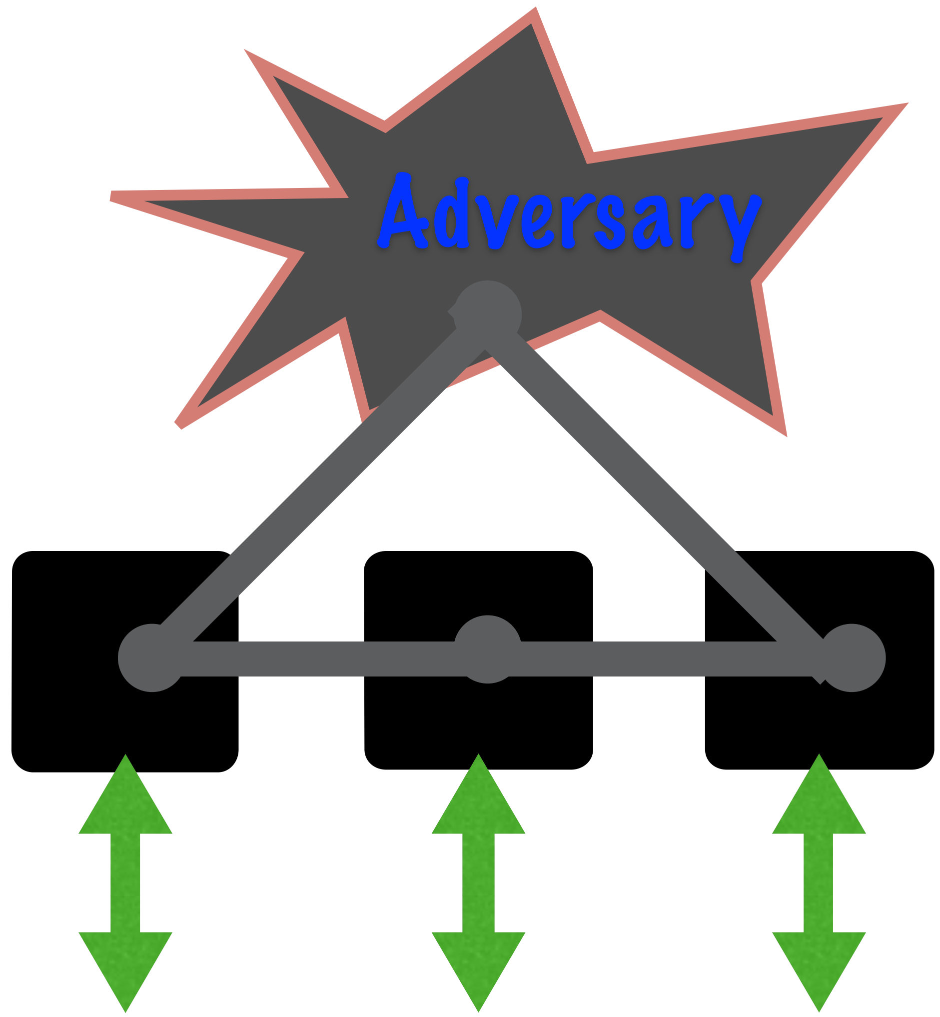

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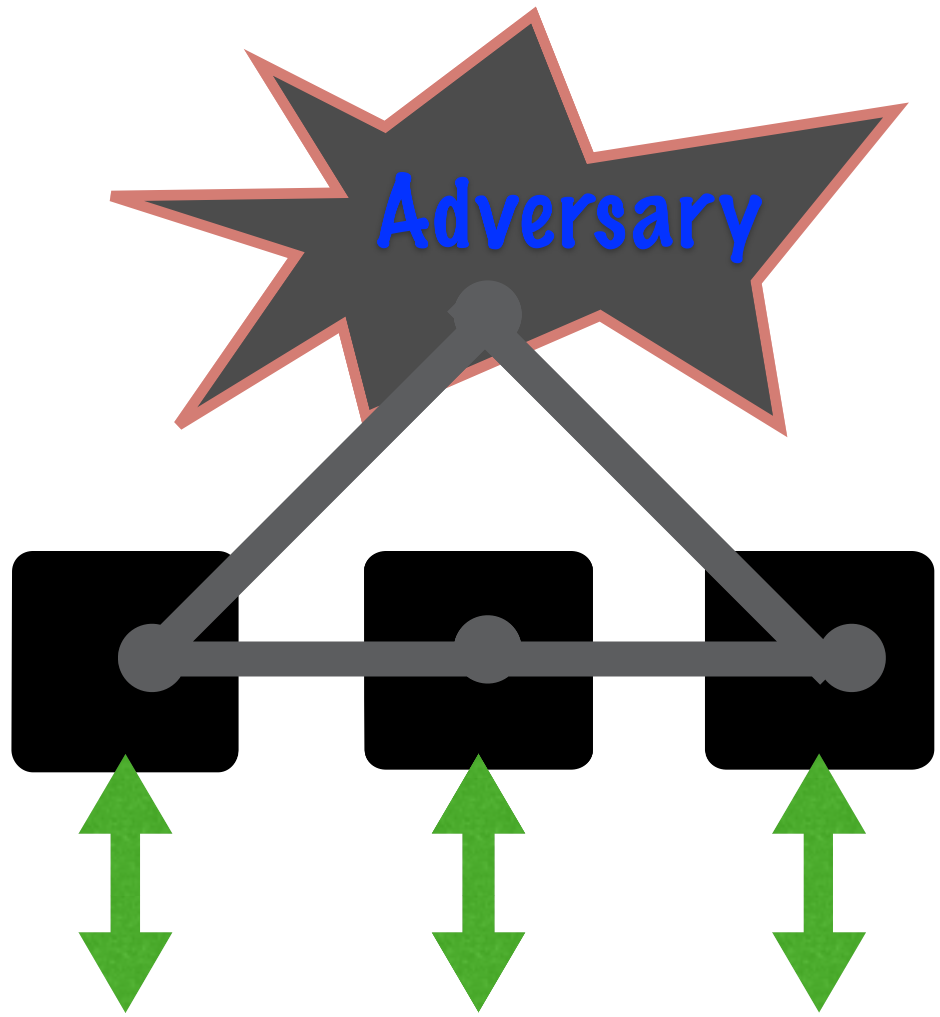

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  - Device can be imperfect or even malicious
  - May be in quantum correlation with the adversary and each others

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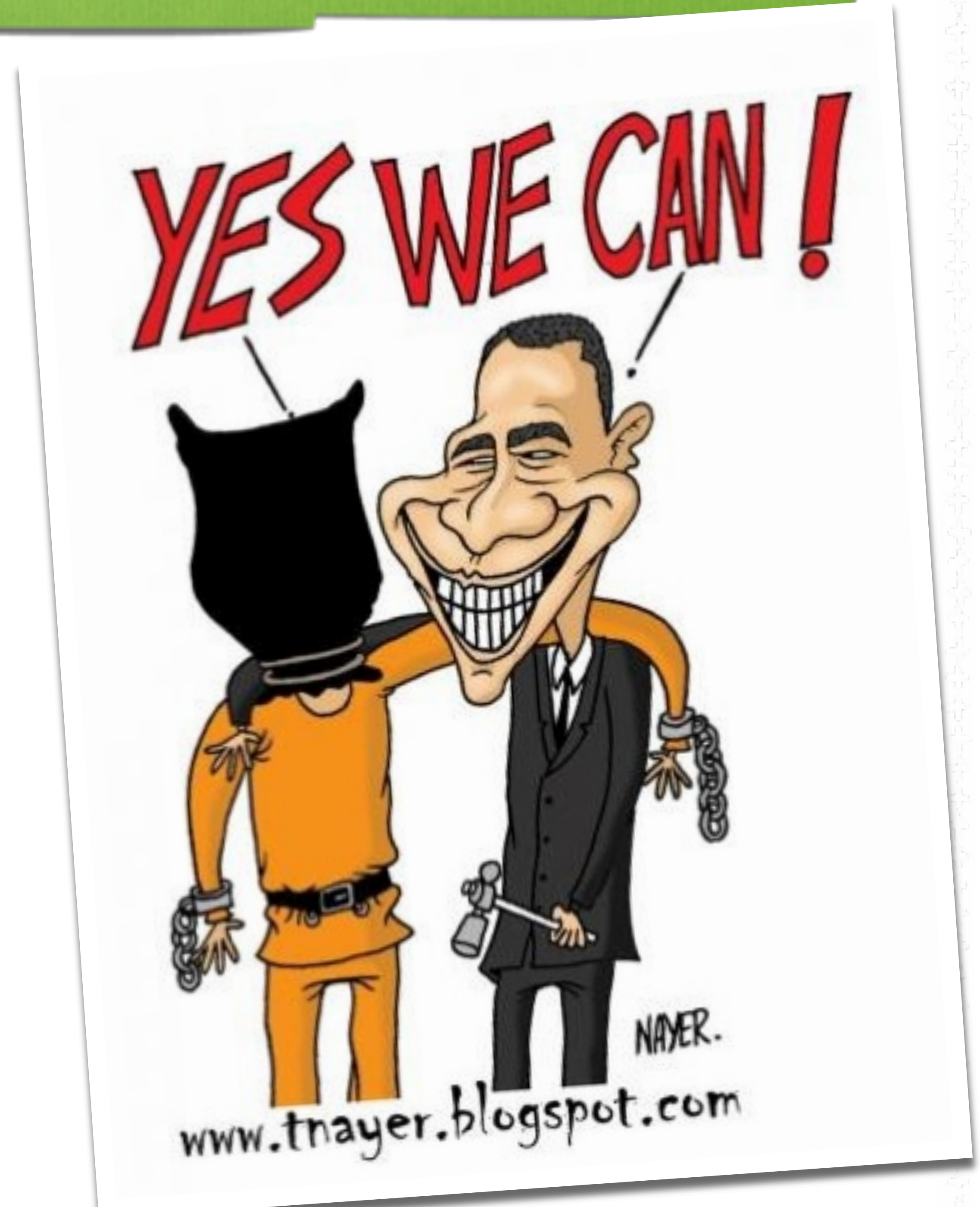

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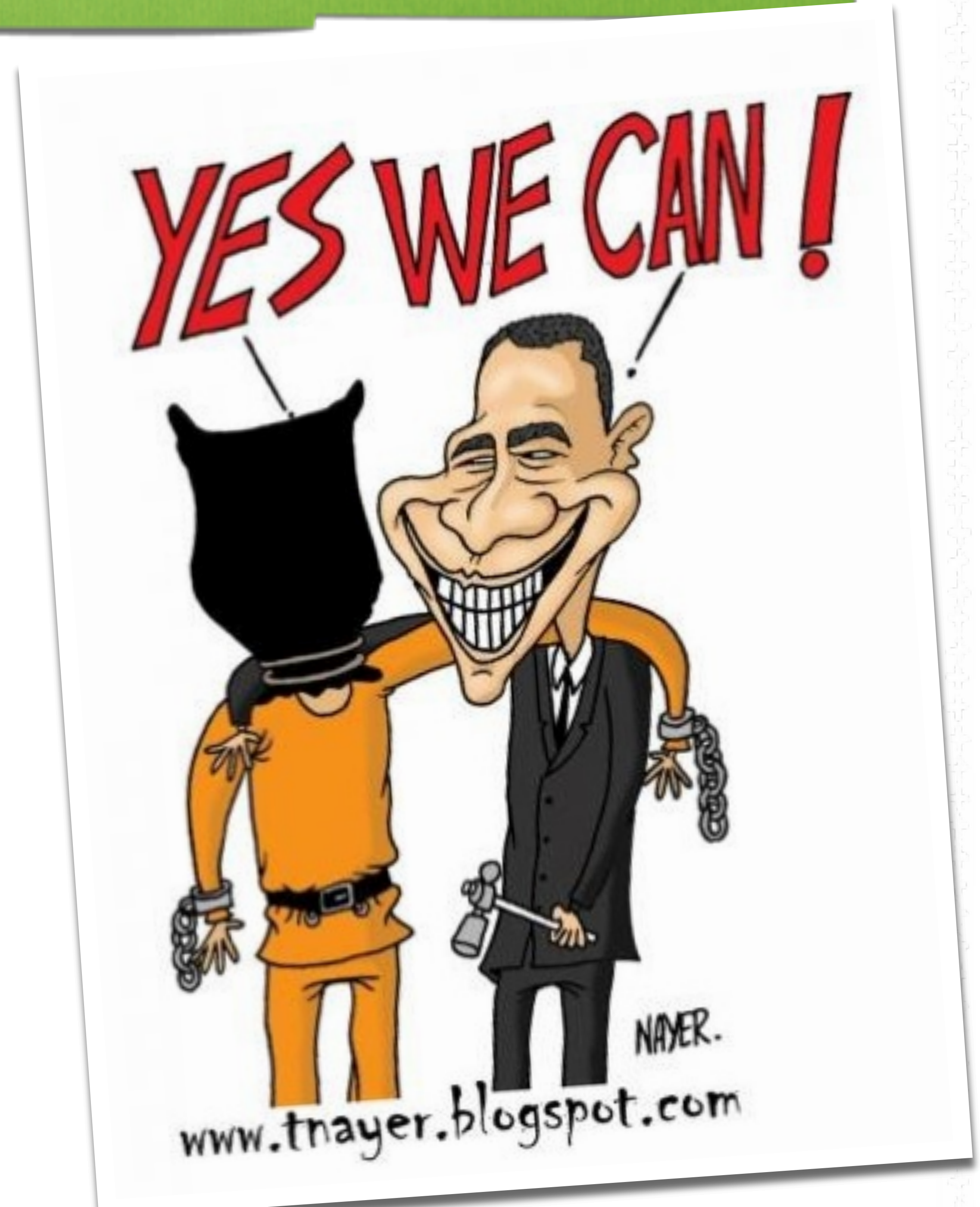
- Untrusted-device quantum cryptography



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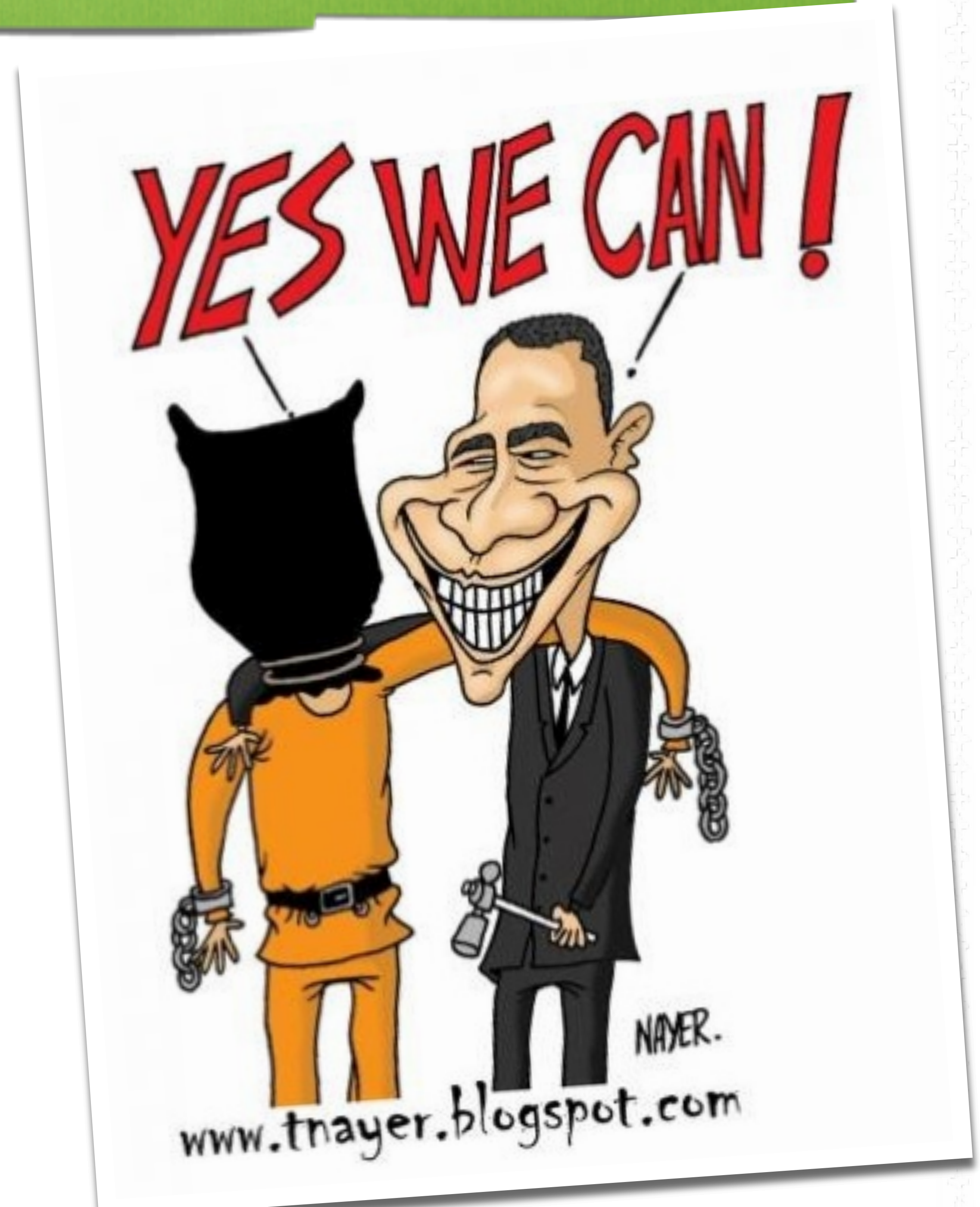

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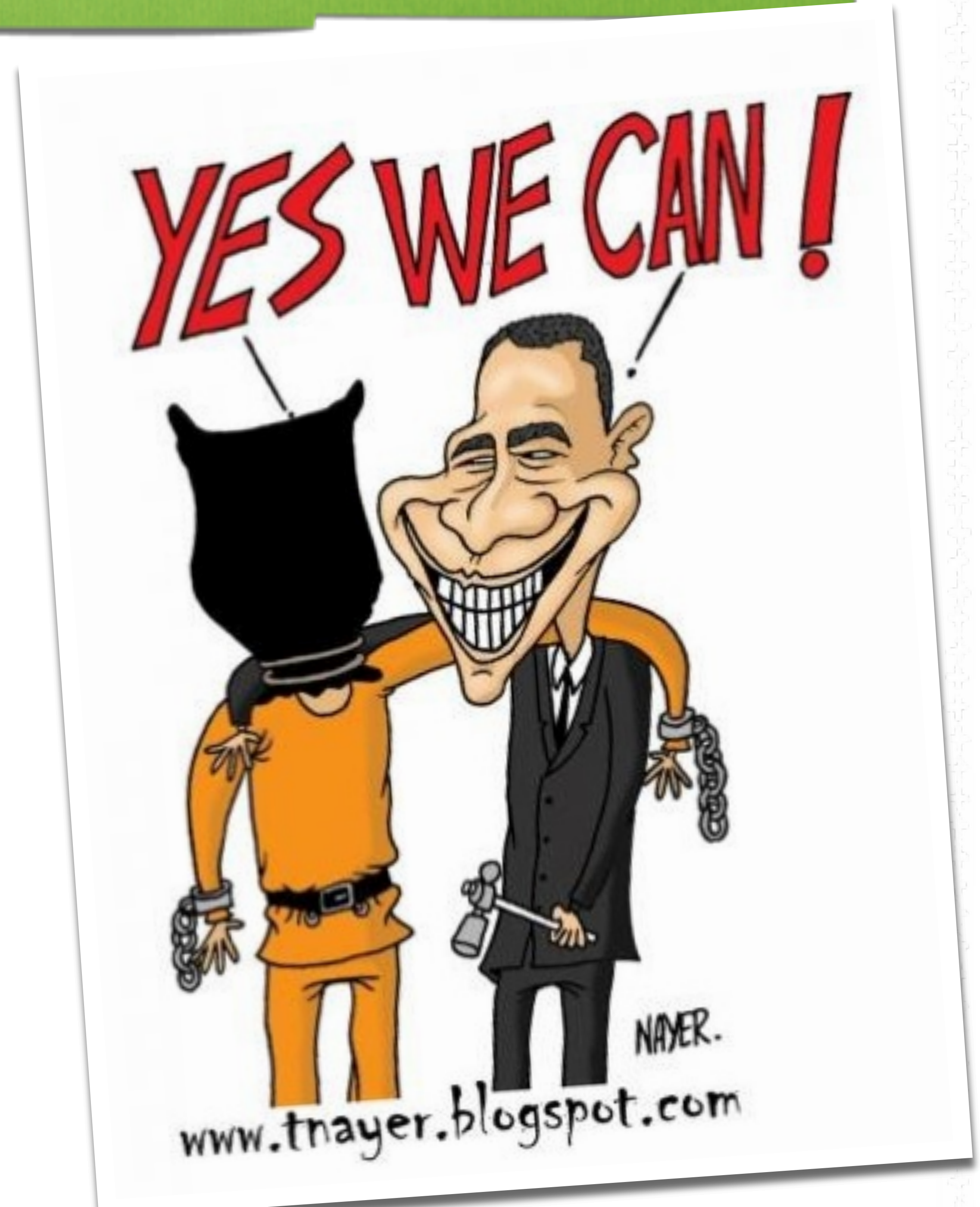
  - Many recent works



Goal

Create and expand true randomness using a single classical source and untrusted quantum devices

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- What's been done?
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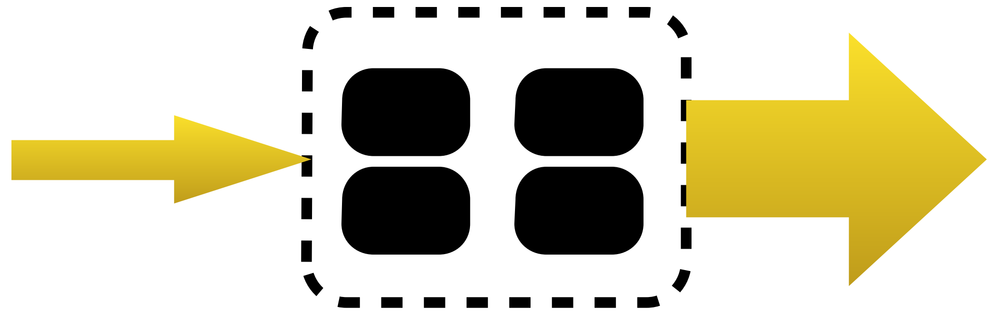
- Our results [Miller-Shi, Chung-Shi-Wu]

Randomness Expansion [Colbeck'06, Colbeck-Kent'11]

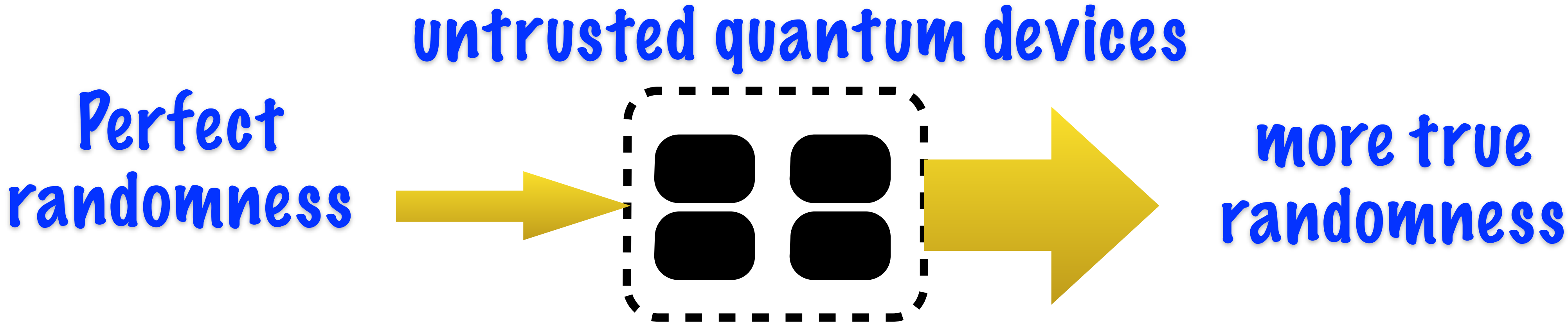
untrusted quantum devices

Perfect randomness



more true randomness

Randomness Expansion [Colbeck'06, Colbeck-Kent'11]



- Turn an initial (uniform) seed to a longer true randomness

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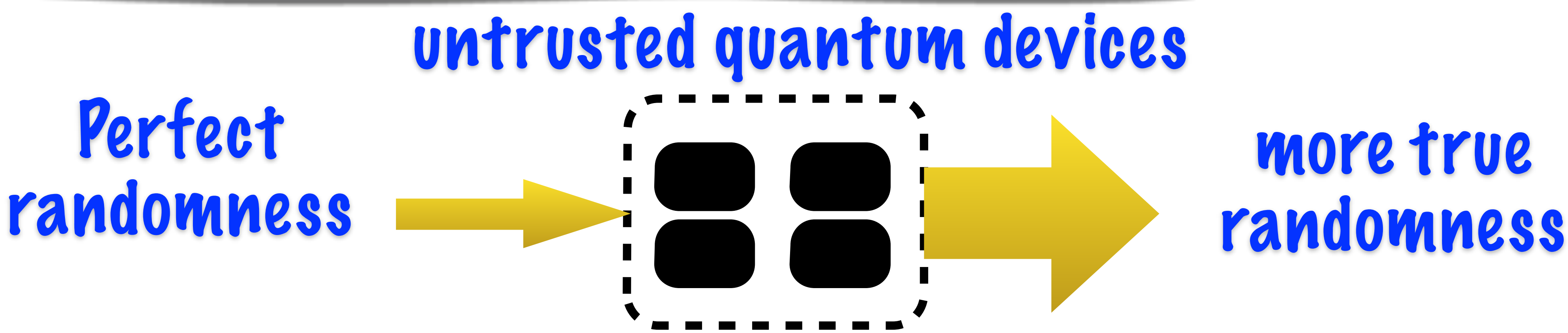

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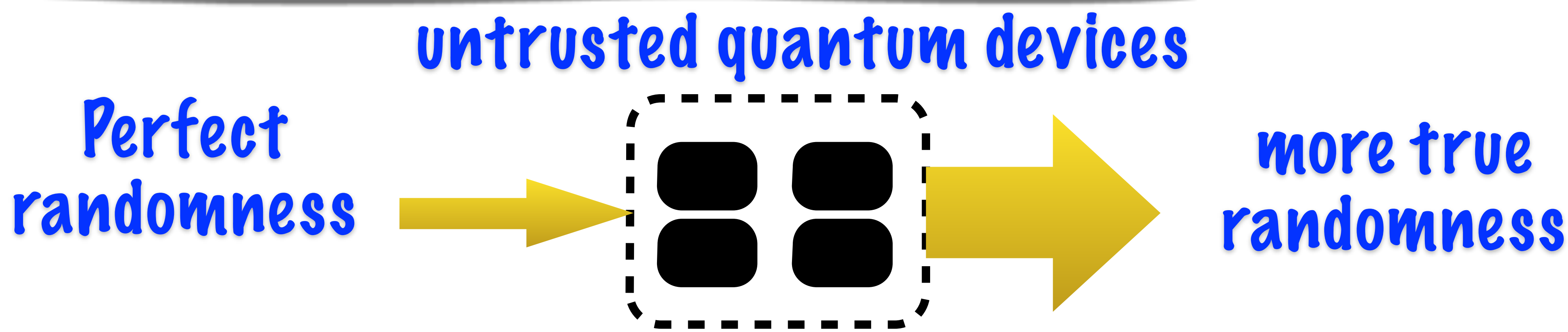

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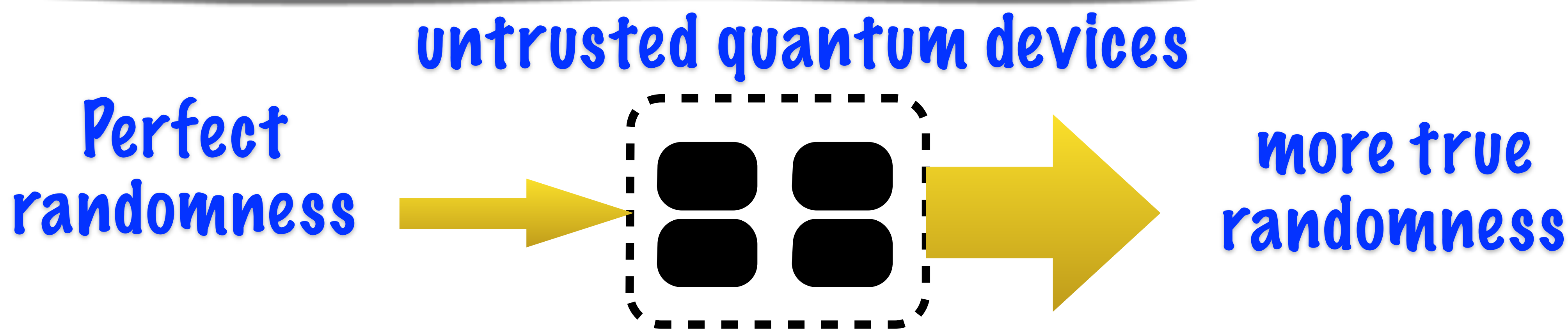

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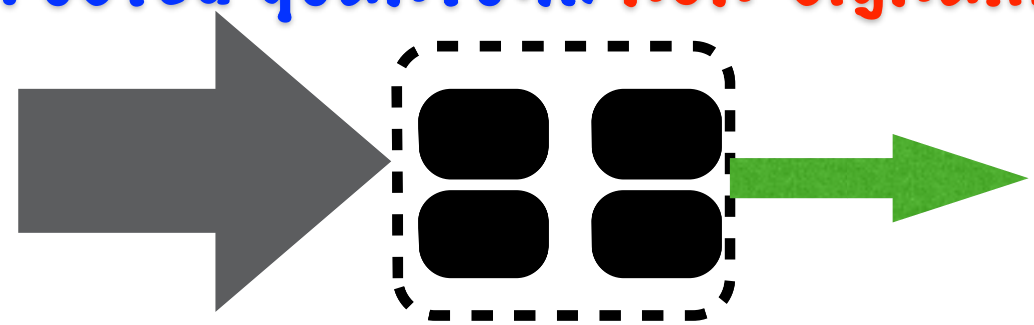
- Quantum security proved by [Vazirani-Vidick'12]
  - Also exponentially expanding: k bits -> exp(k^c) bits

Randomness Amplification [Colbeck-Renner'12]

untrusted quantum/non-signaling devices

SV-source



one true random bit

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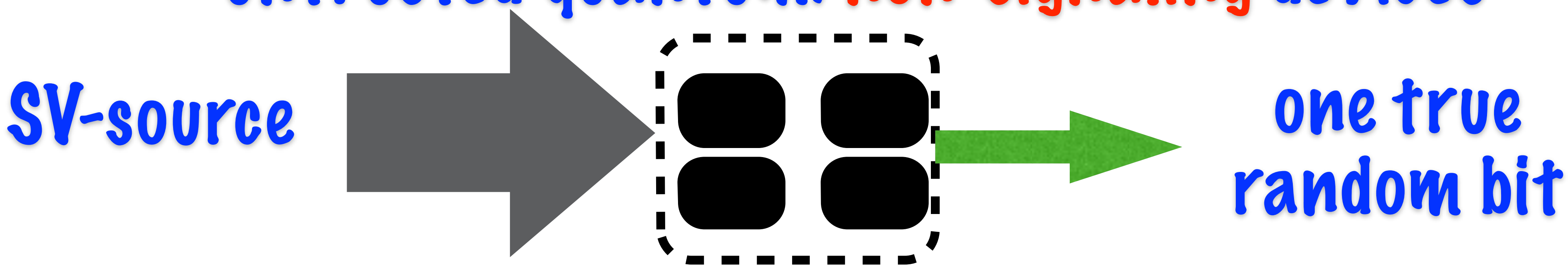

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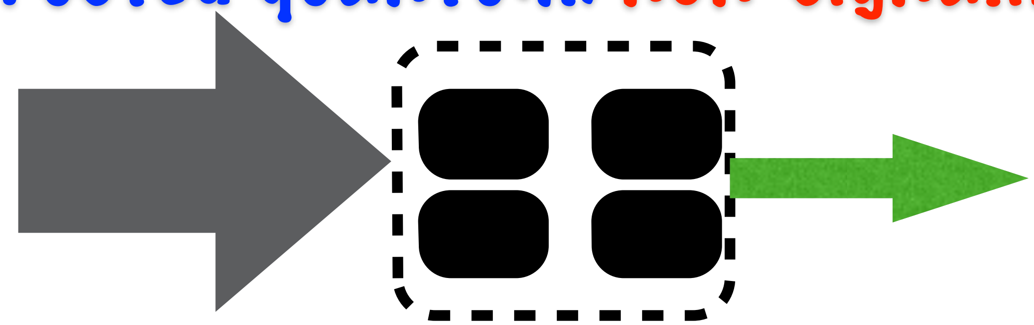

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- Model weak randomness as an Santha-Vazirani (SV) source: $x_1, x_2, \ldots., x_n$, s.t. for a constant ϵ and any adversary's side information e, $\mathrm{Prob}[x_i = 1 \mid x_1, x_2, \ldots, x_{i-1}, e] \in [1/2-\epsilon, 1/2+\epsilon]$.

Model and Results::history

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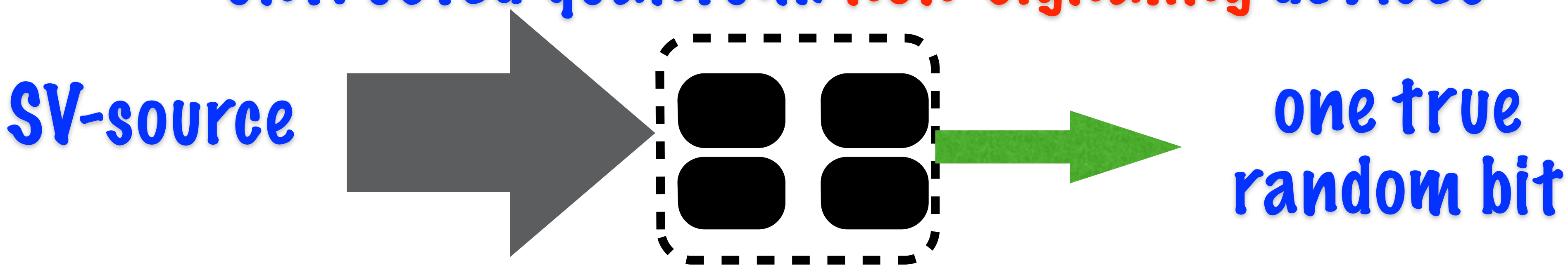

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- [Colbeck-Renner'12]: sufficiently small ϵ; [Gallego+'13]: any $\epsilon<1/2$; [Brandao'14]: constant number of devices

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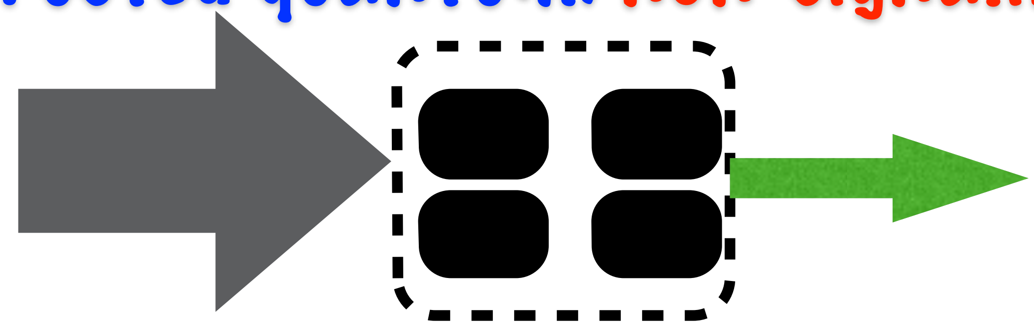

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- [Colbeck-Renner'12]: sufficiently small ϵ;
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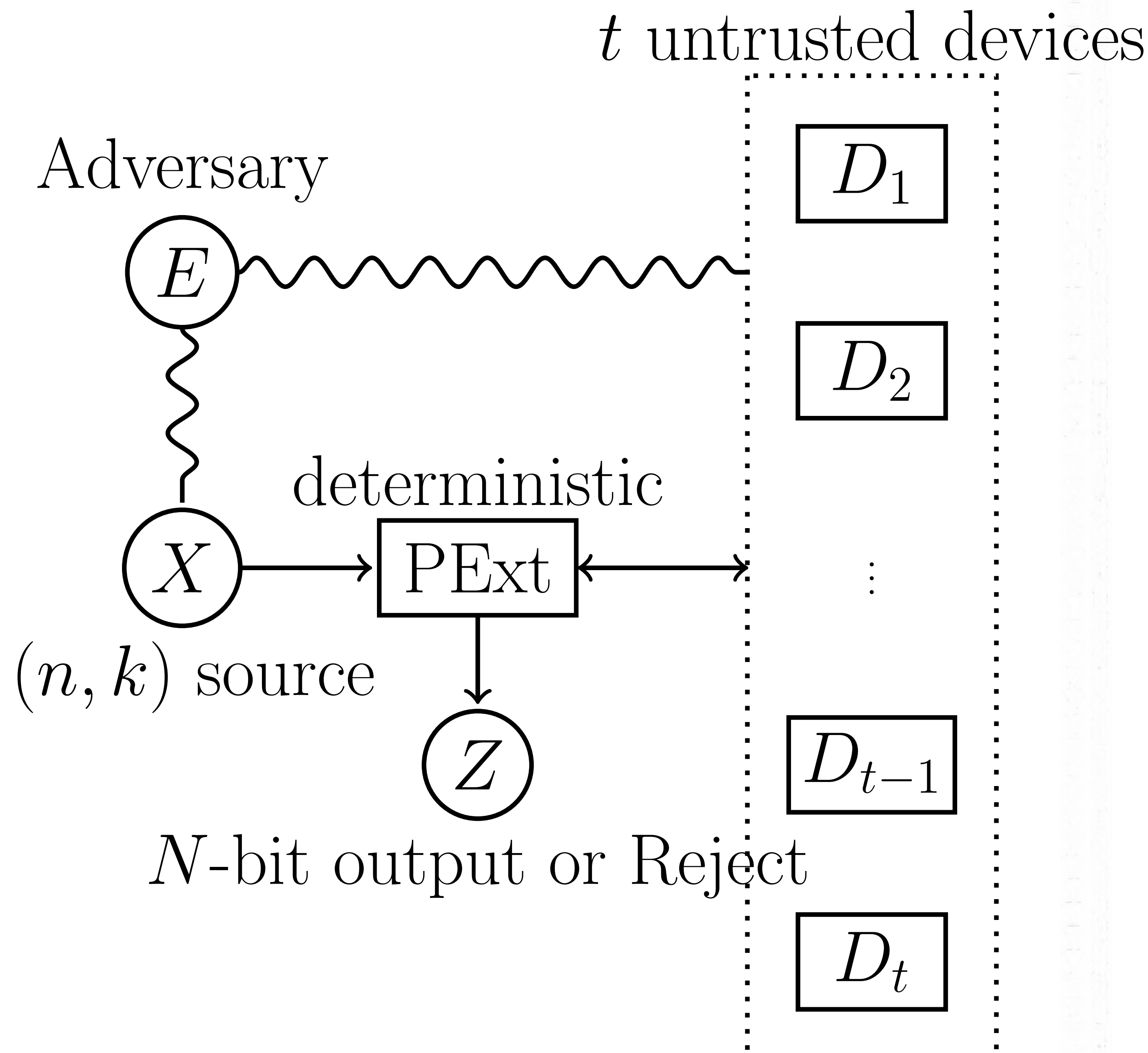
  - All assume independence of the SV-source and the device conditioned on e.

Model and Results::history

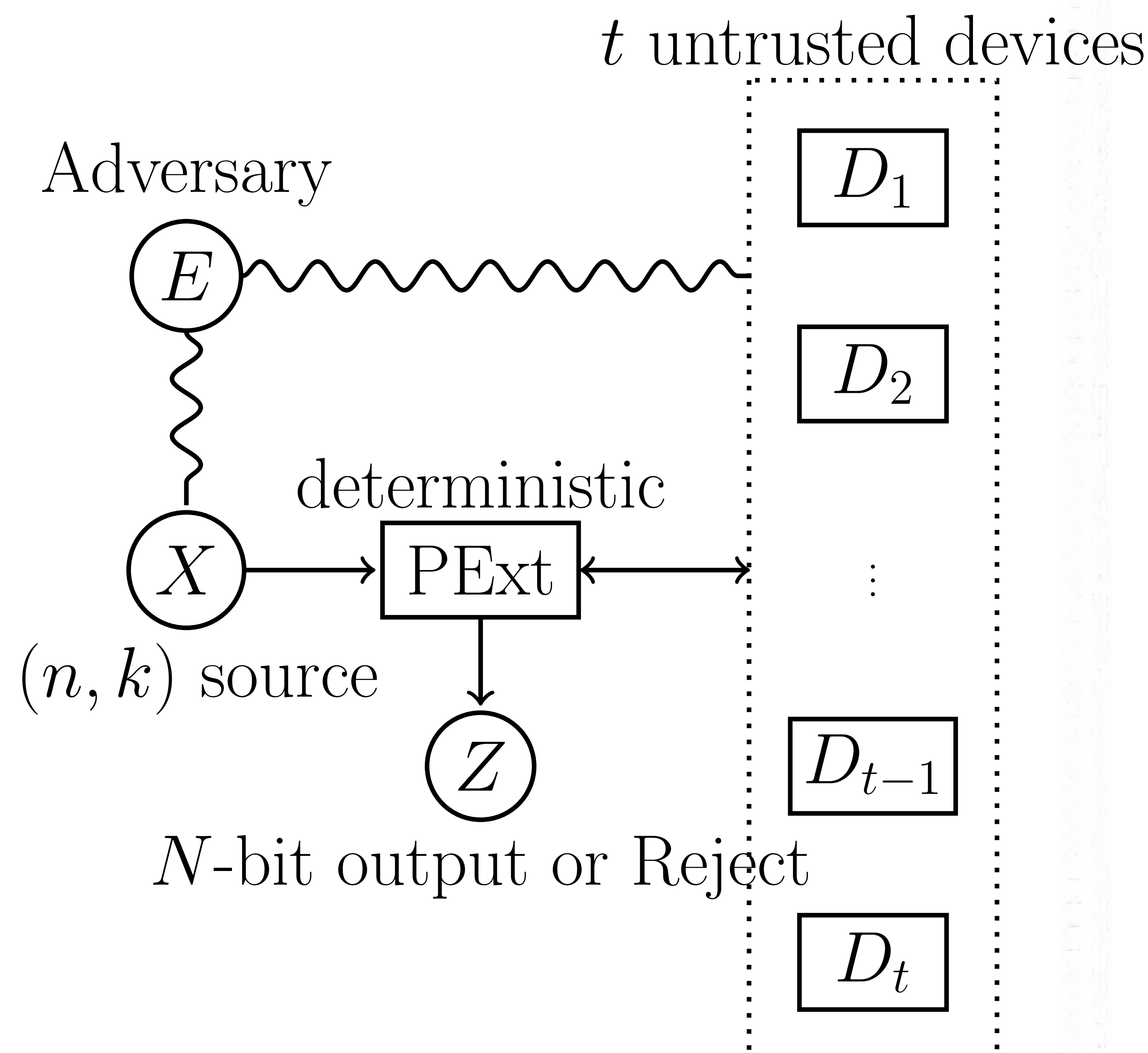
Physical Extractors: a unifying quantum framework



Model and Results::unifying model

Physical Extractors: a unifying quantum framework

- **Protocol: deterministic**



Physical Extractors: a unifying quantum framework

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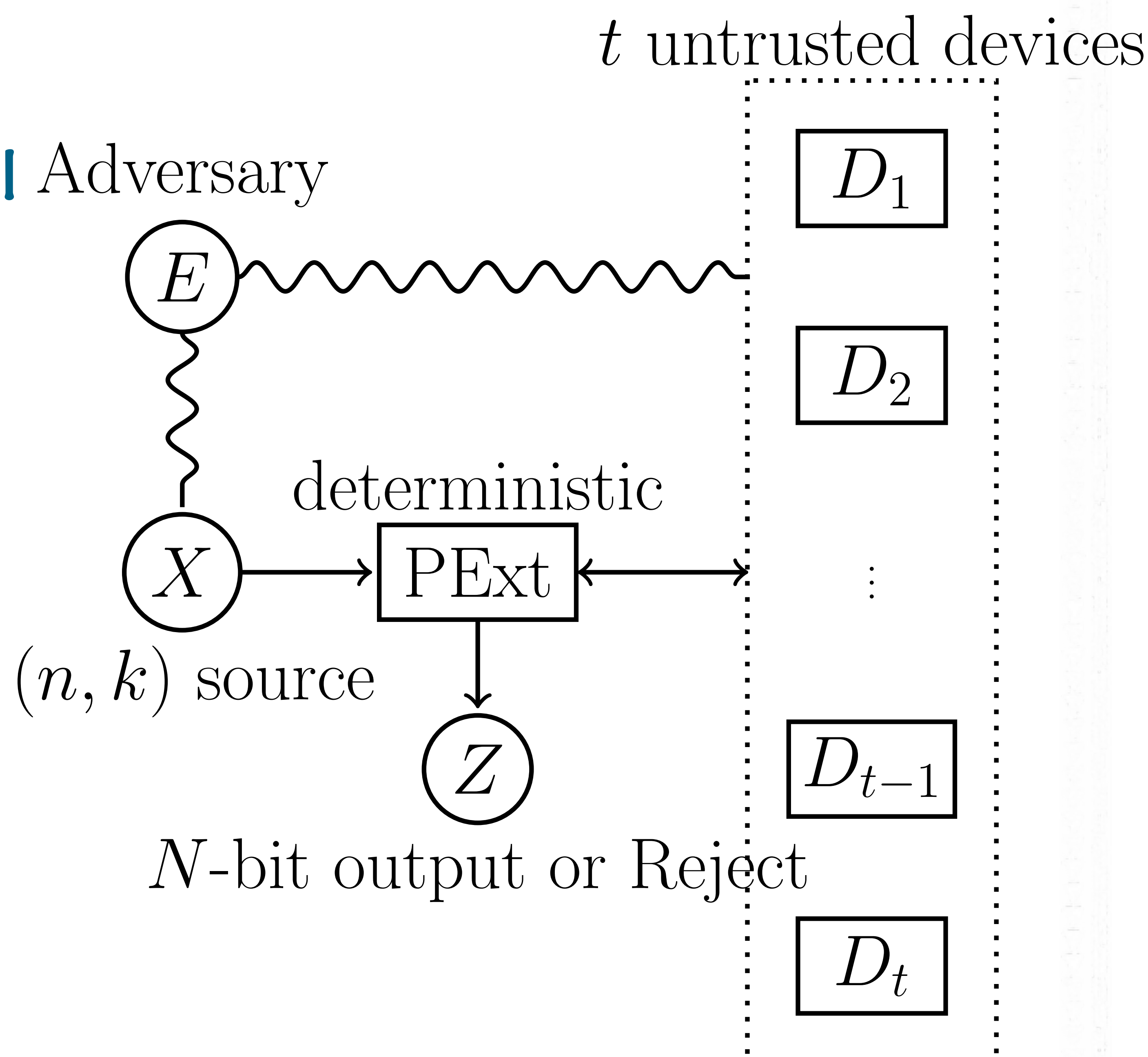

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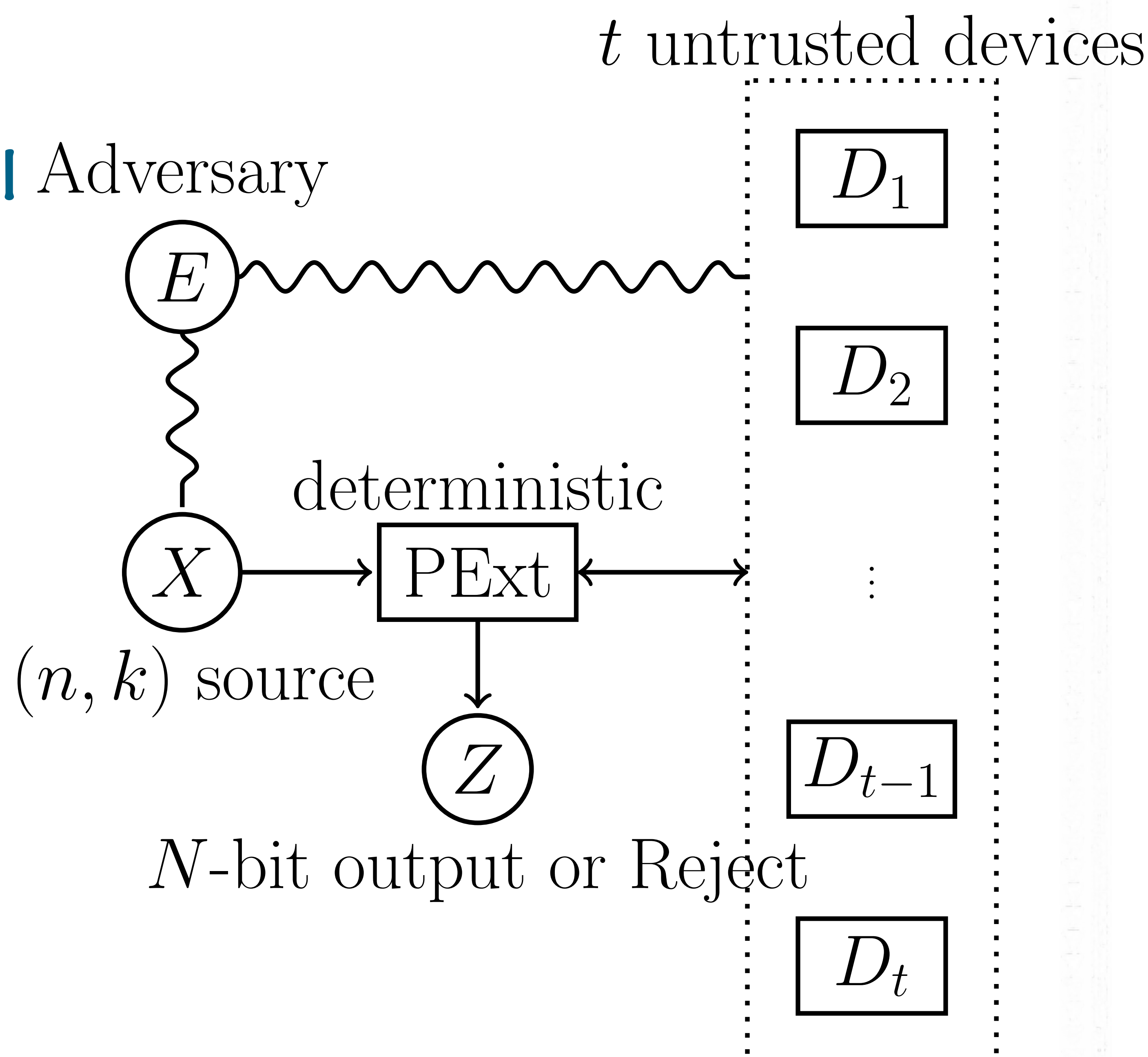

Model and Results::unifying model

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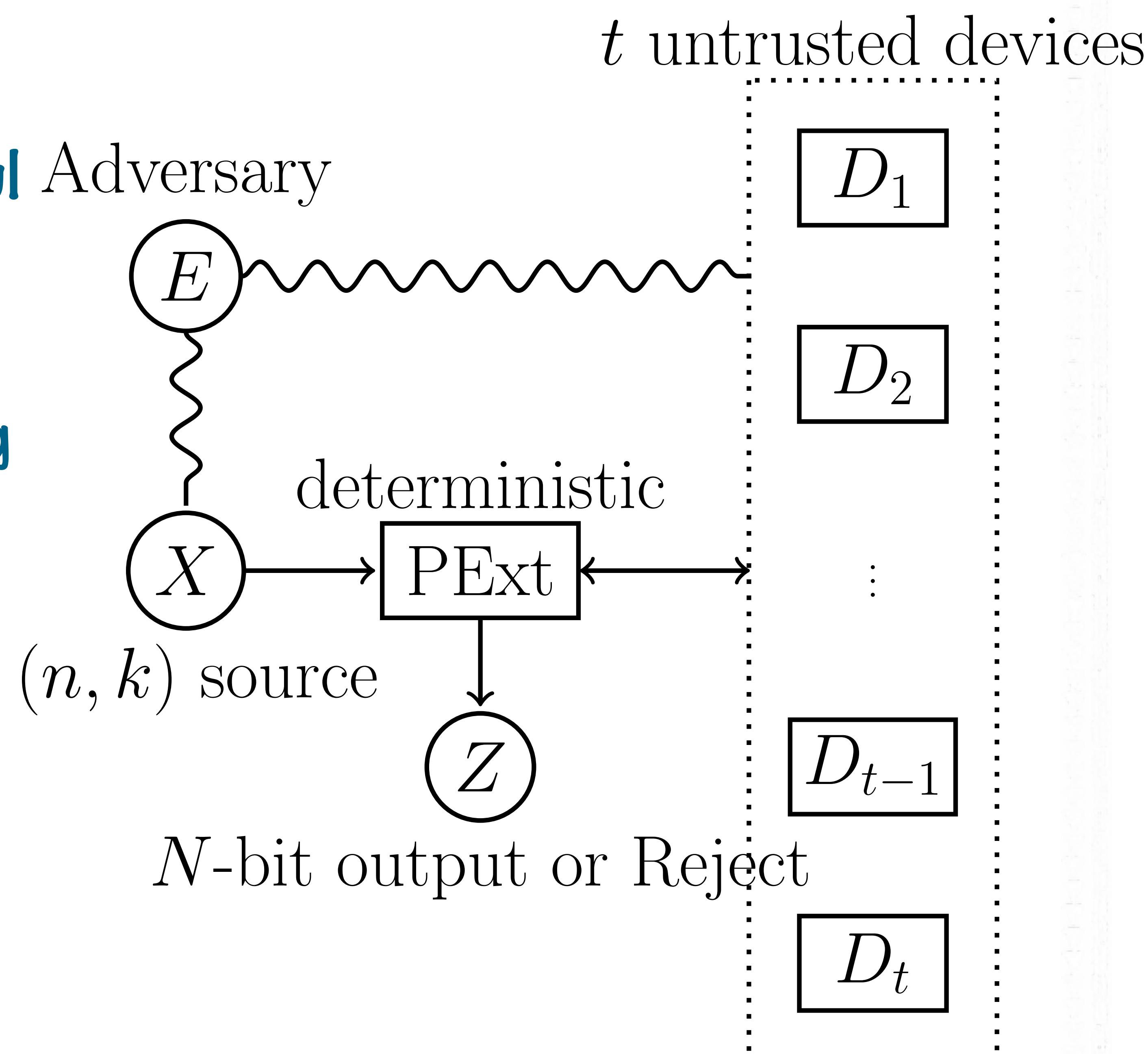

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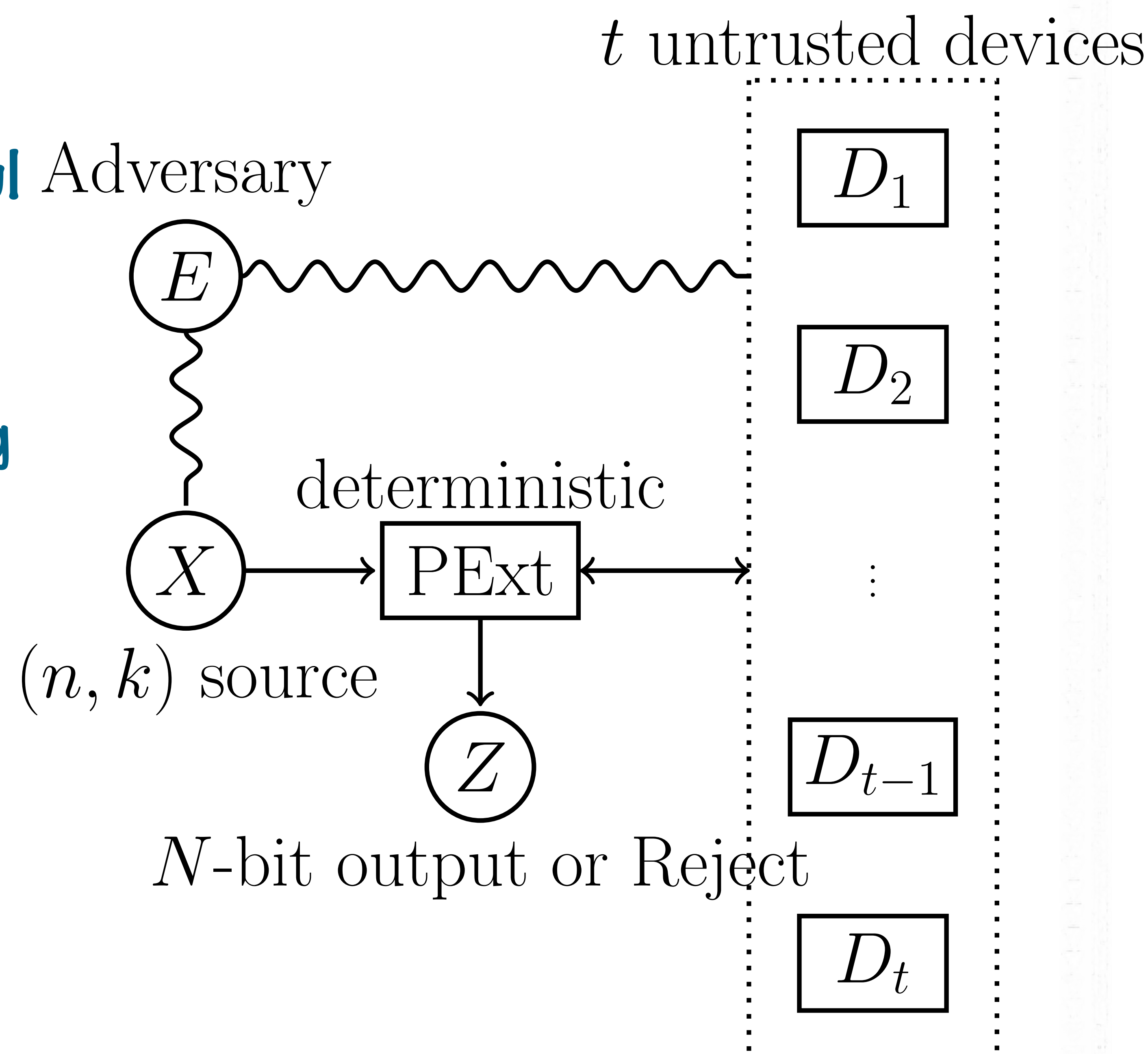

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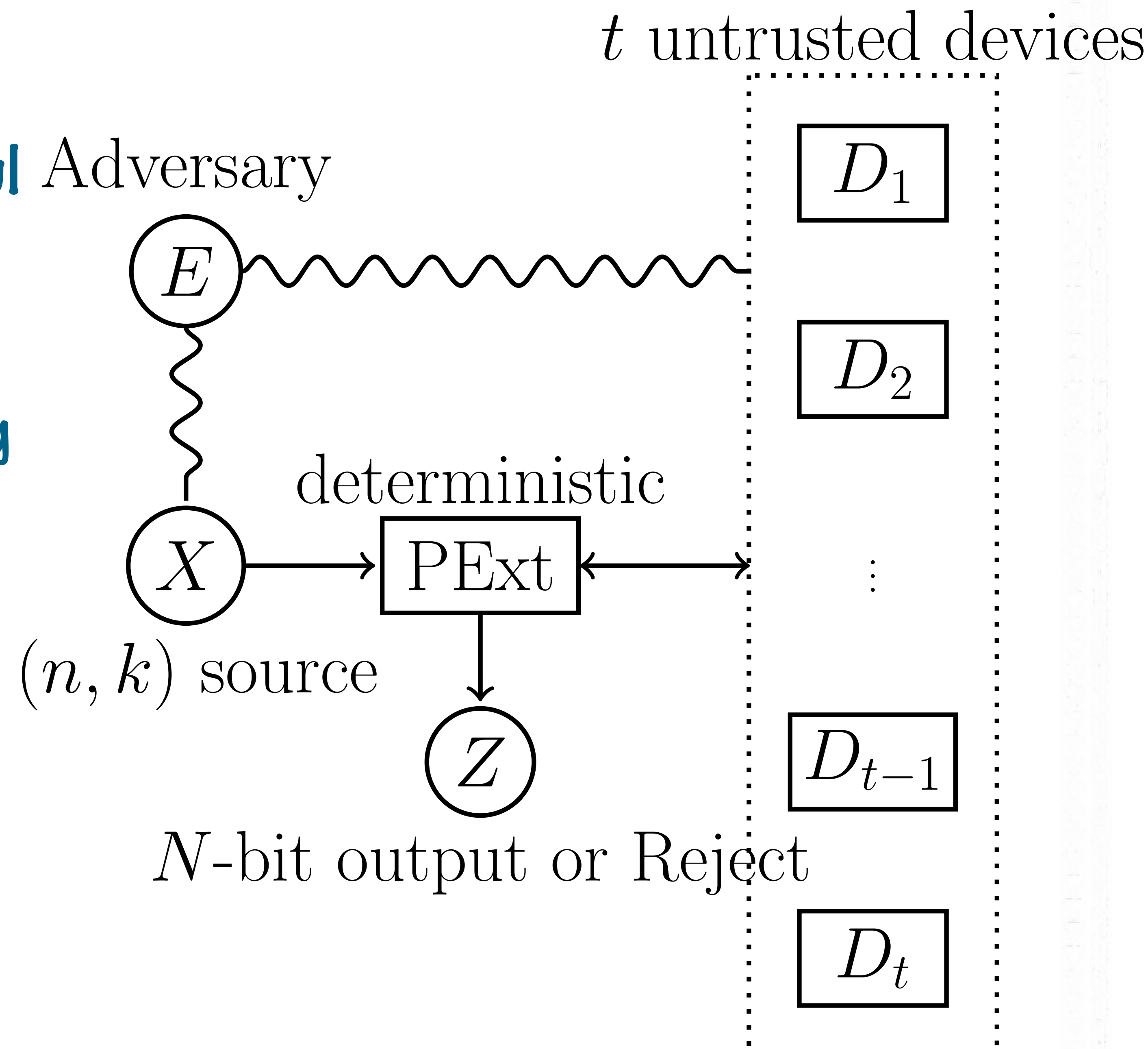

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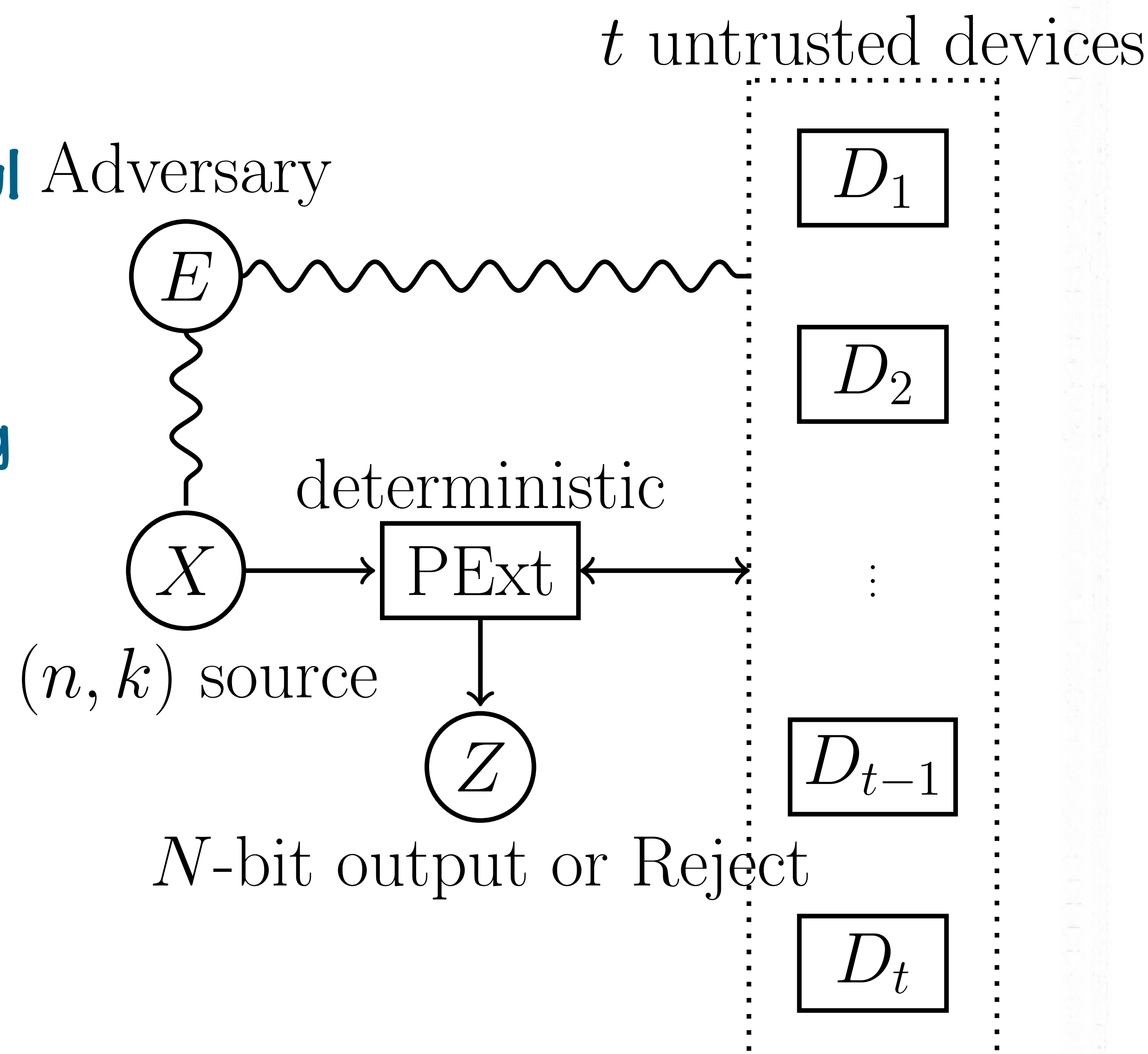
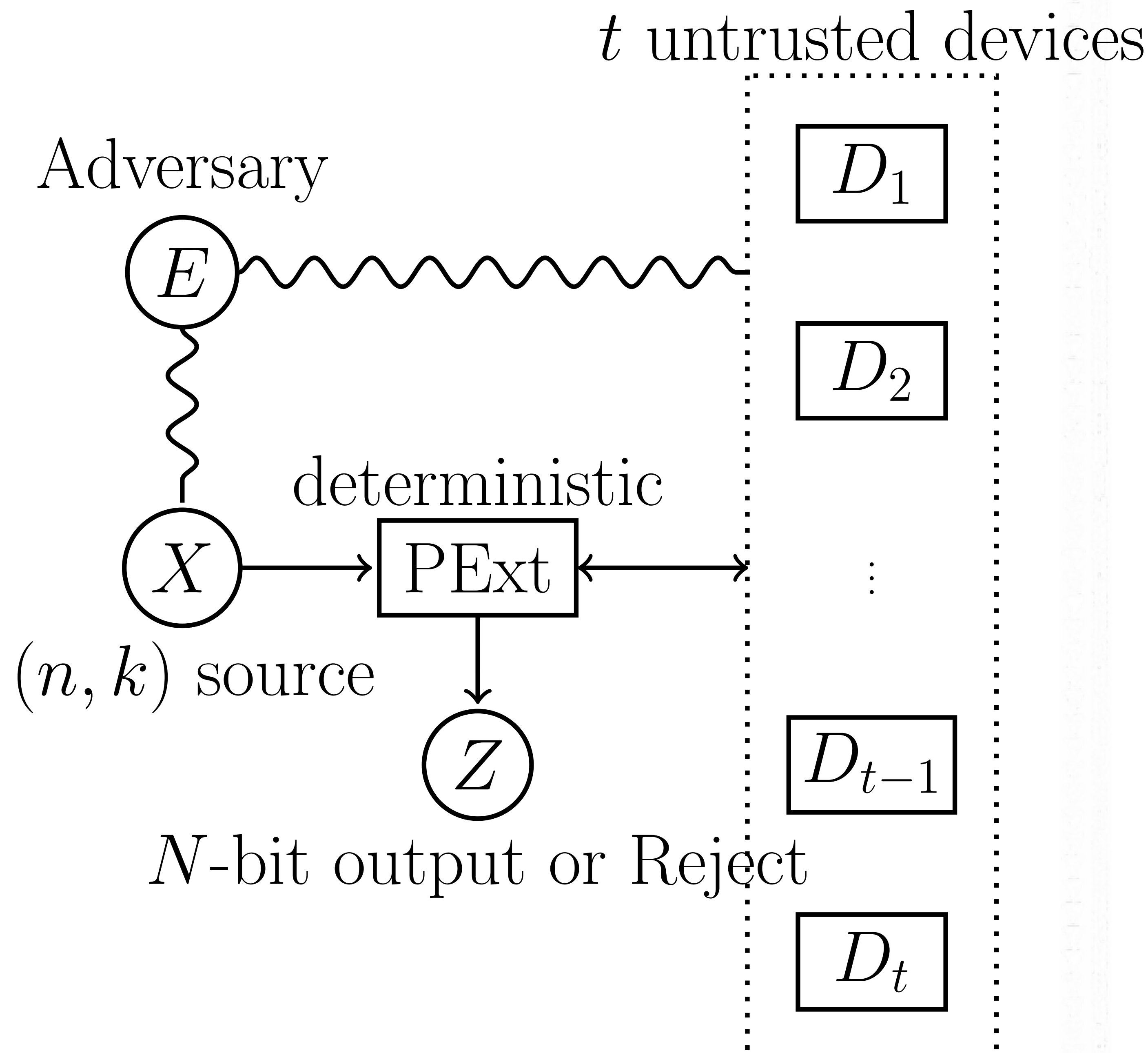
  - Min-entropy w.r.s.t. Adversary +Device



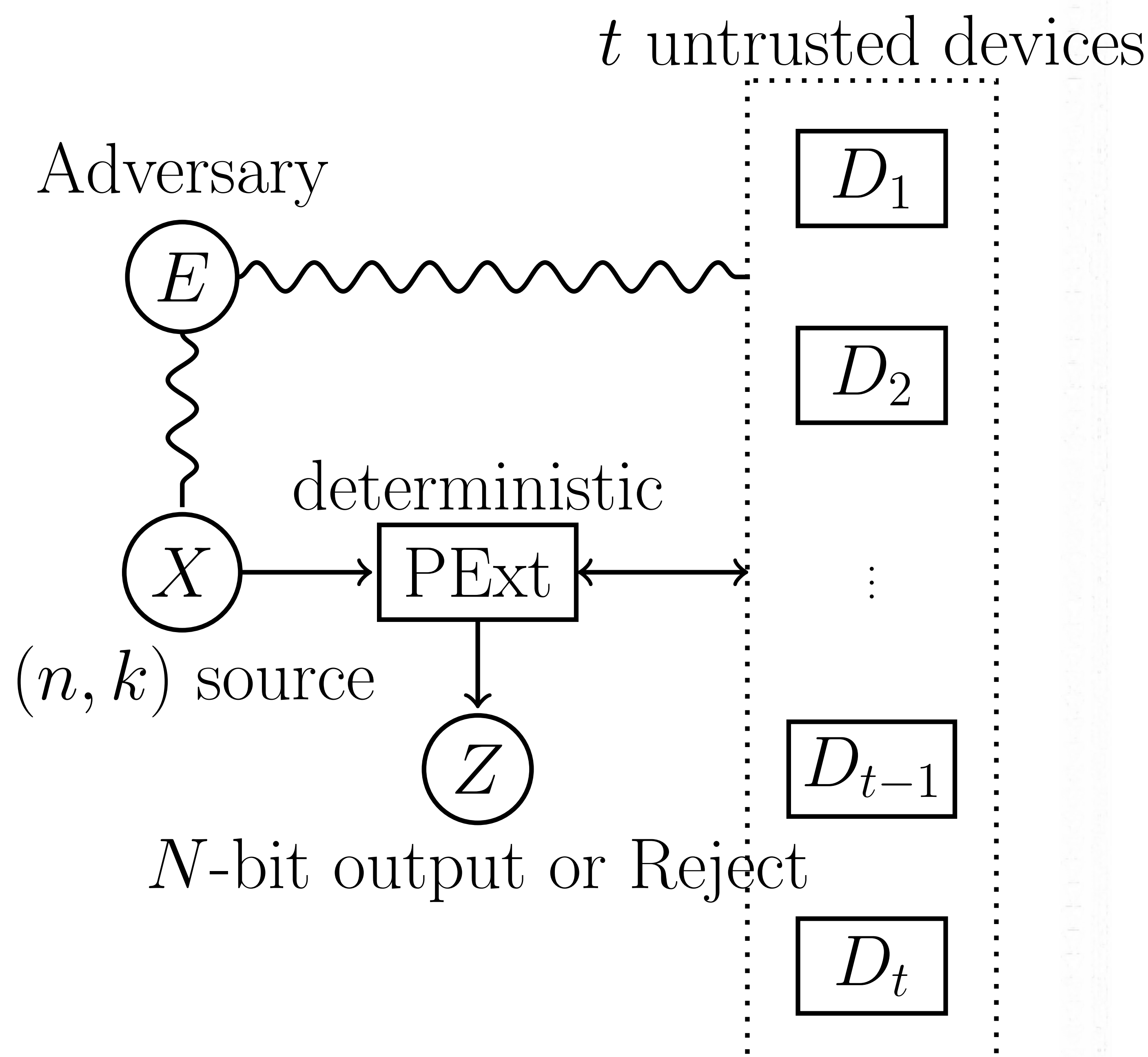
Expansion and amplification as physical extractors



Model and Results::a unifying framework

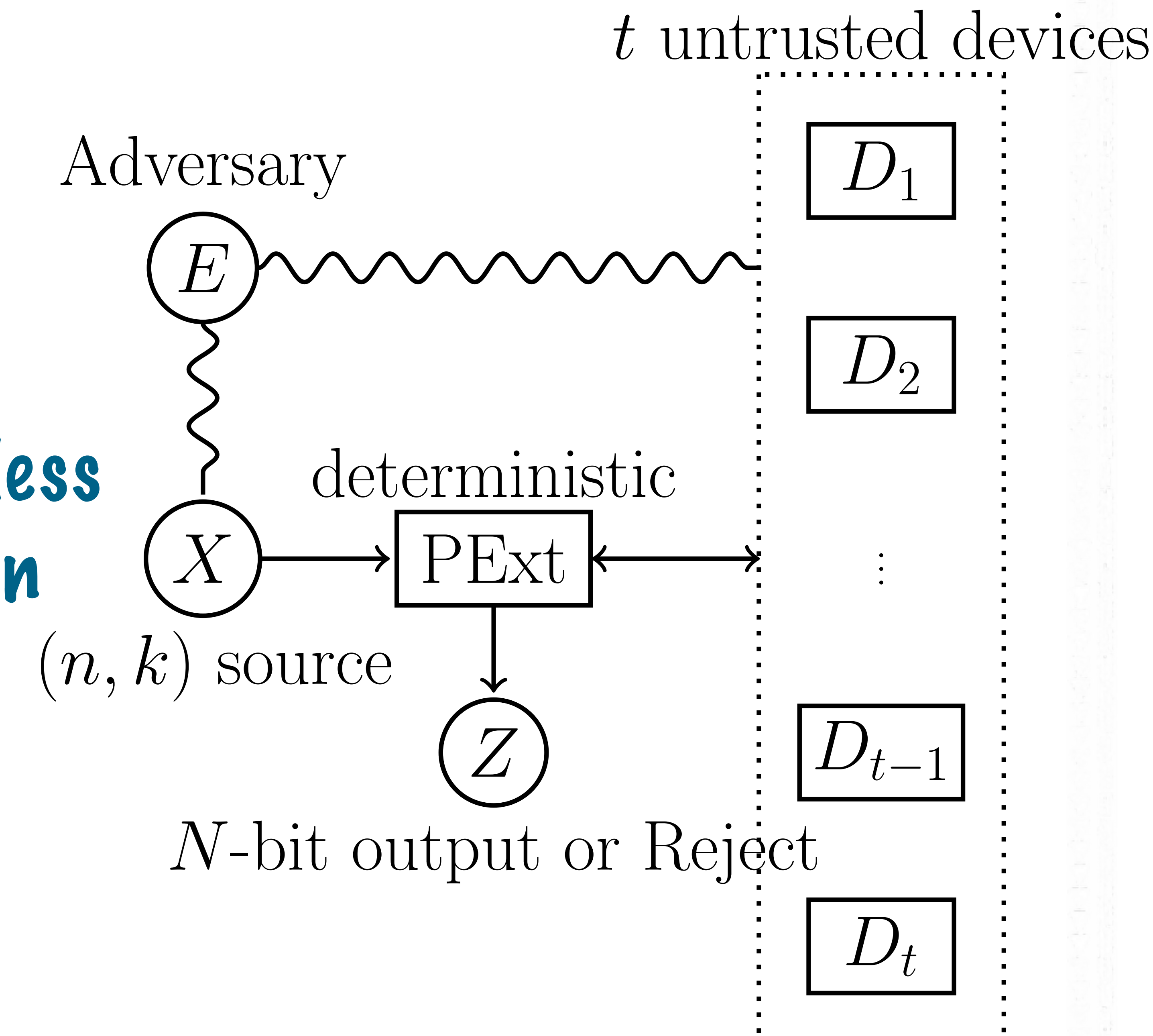
Expansion and amplification as physical extractors

- **Expansion: seeded extraction**



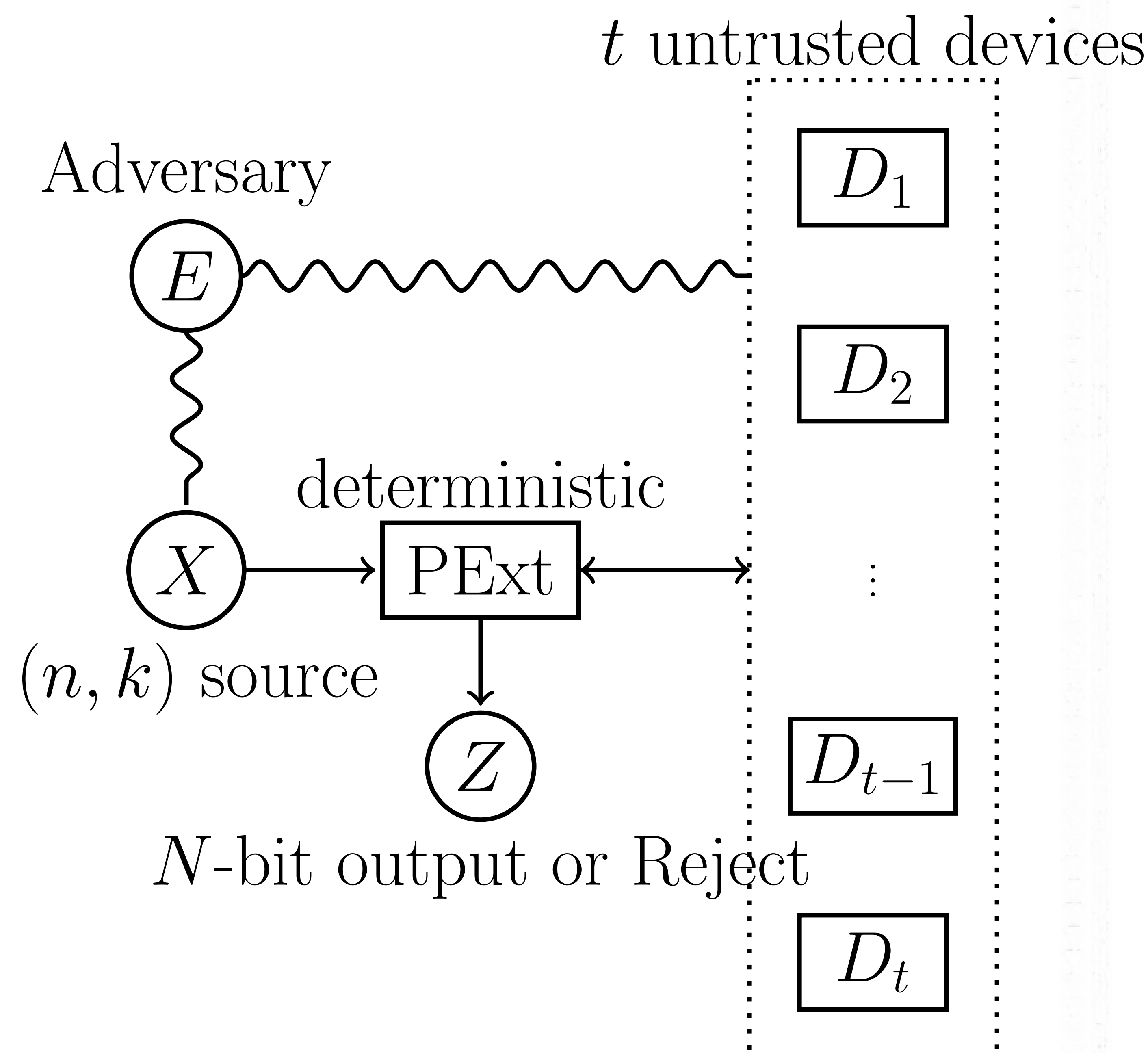
Expansion and amplification as physical extractors

- **Expansion: seeded extraction**
- **Amplification: 1-bit seedless extraction with a certain strong SV-source and conditional-independent input-device**



Model and Results::a unifying framework

Goals for physical extractors



Goals for physical extractors

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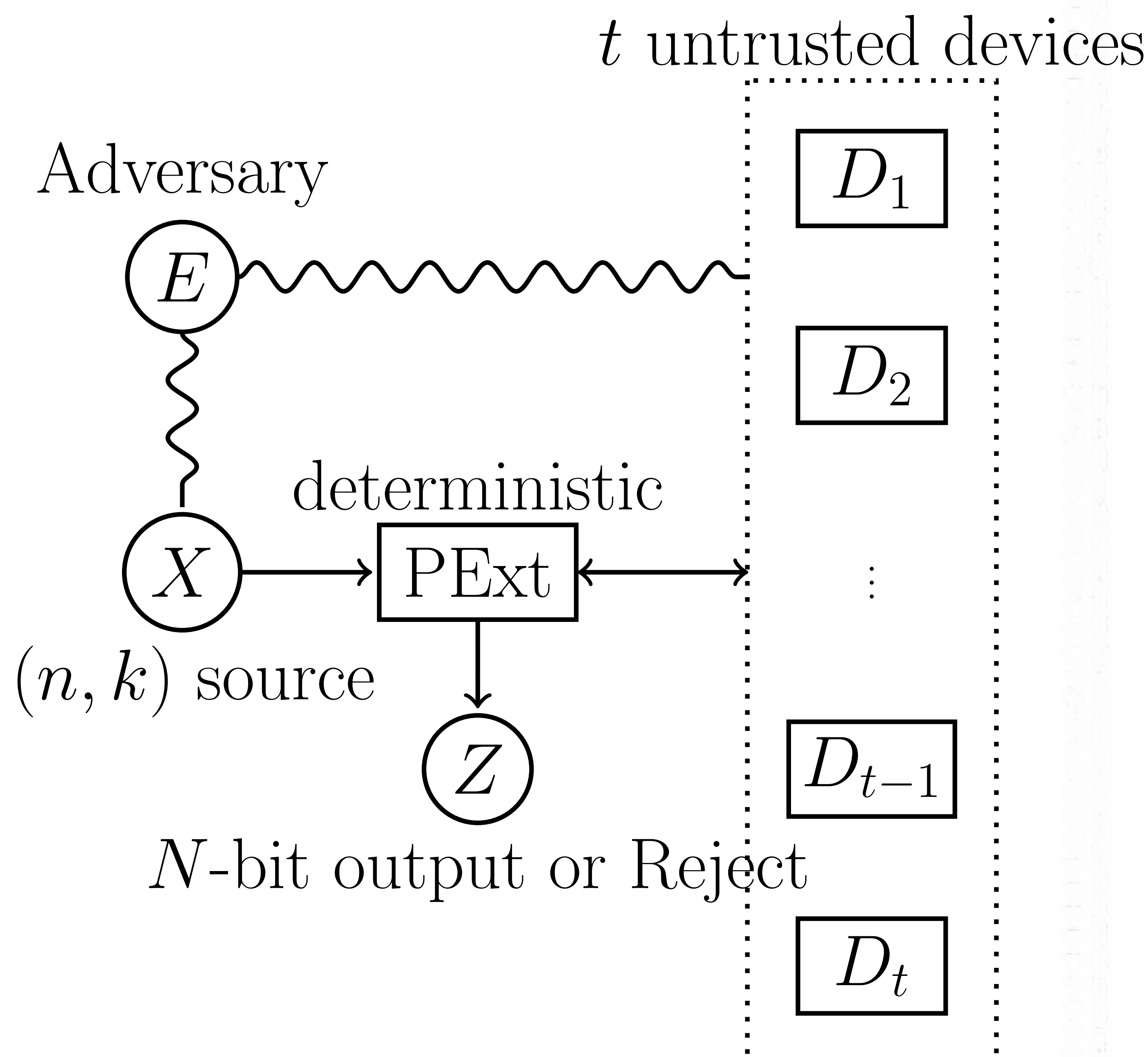
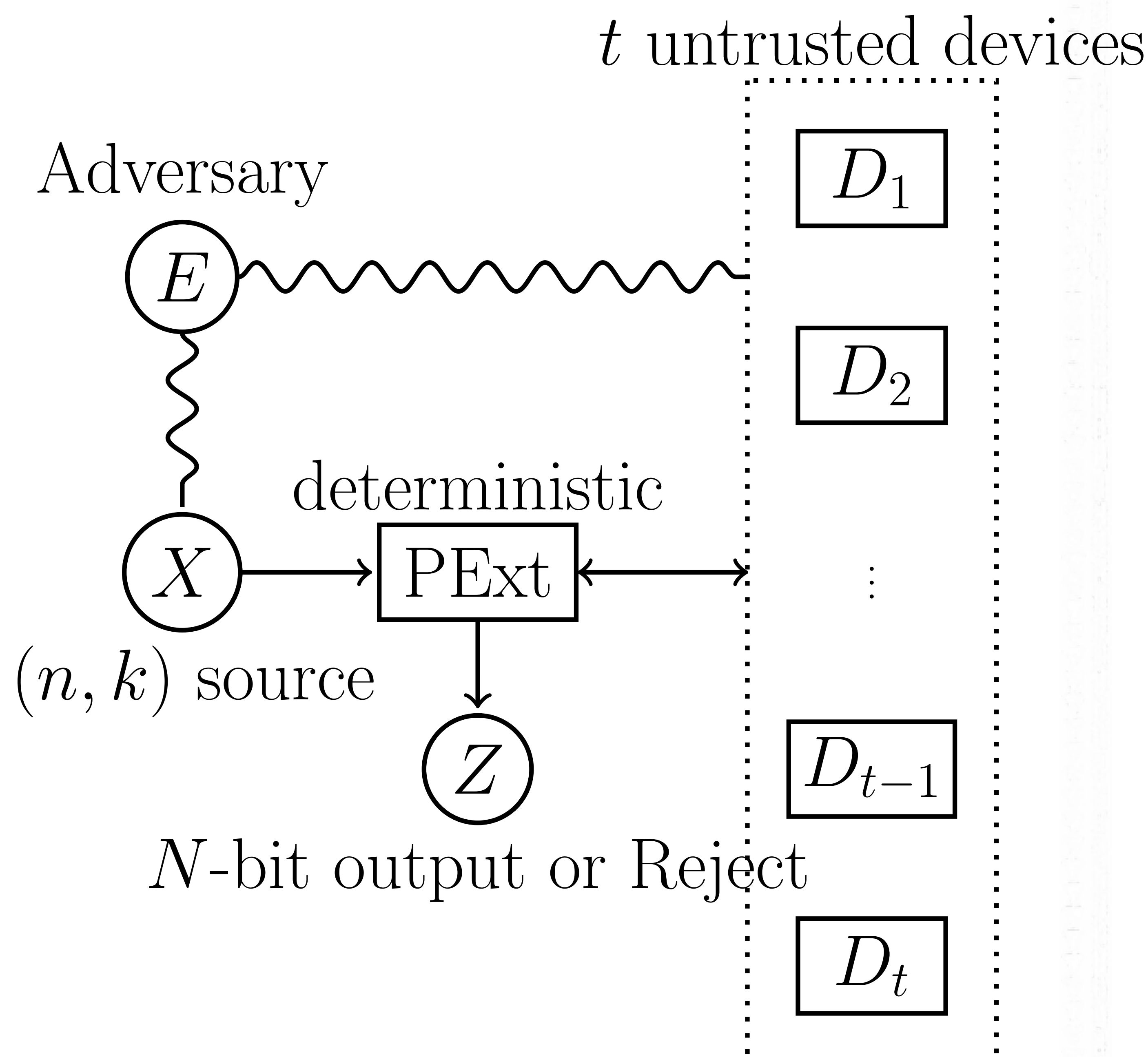


Goals for physical extractors

1. Security: quantum
2. Quality: small errors



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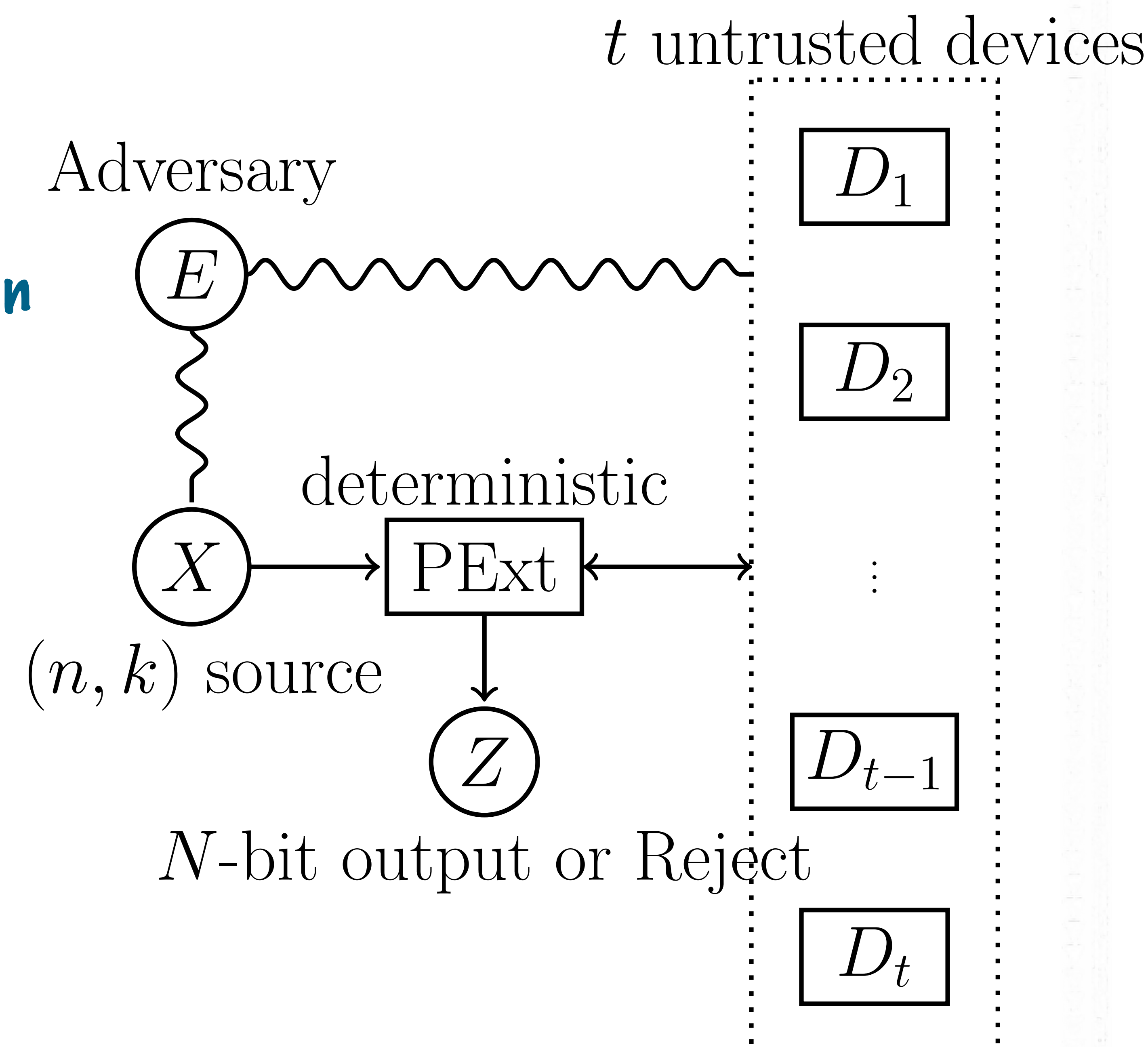

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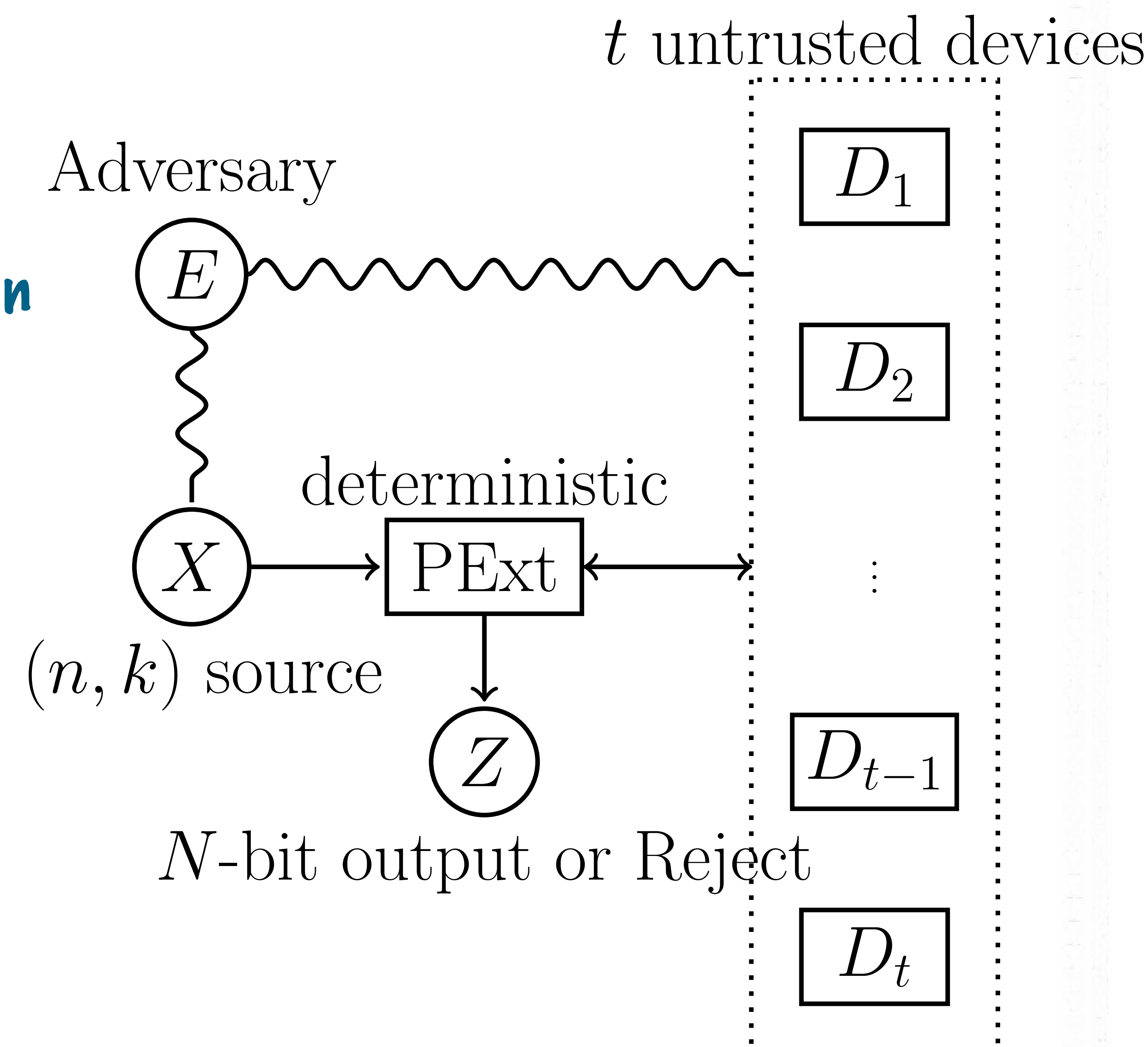

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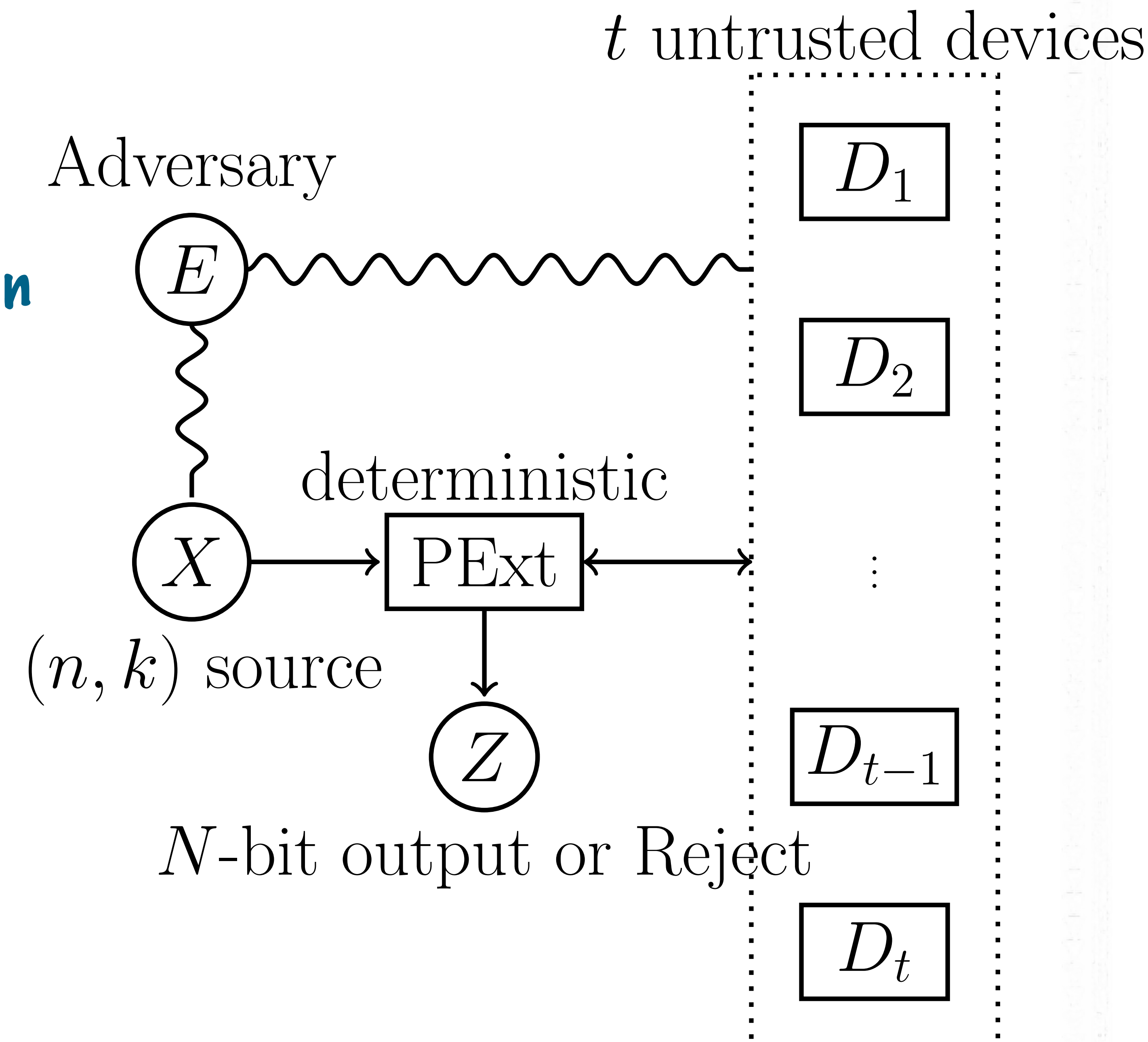

Goals for physical extractors

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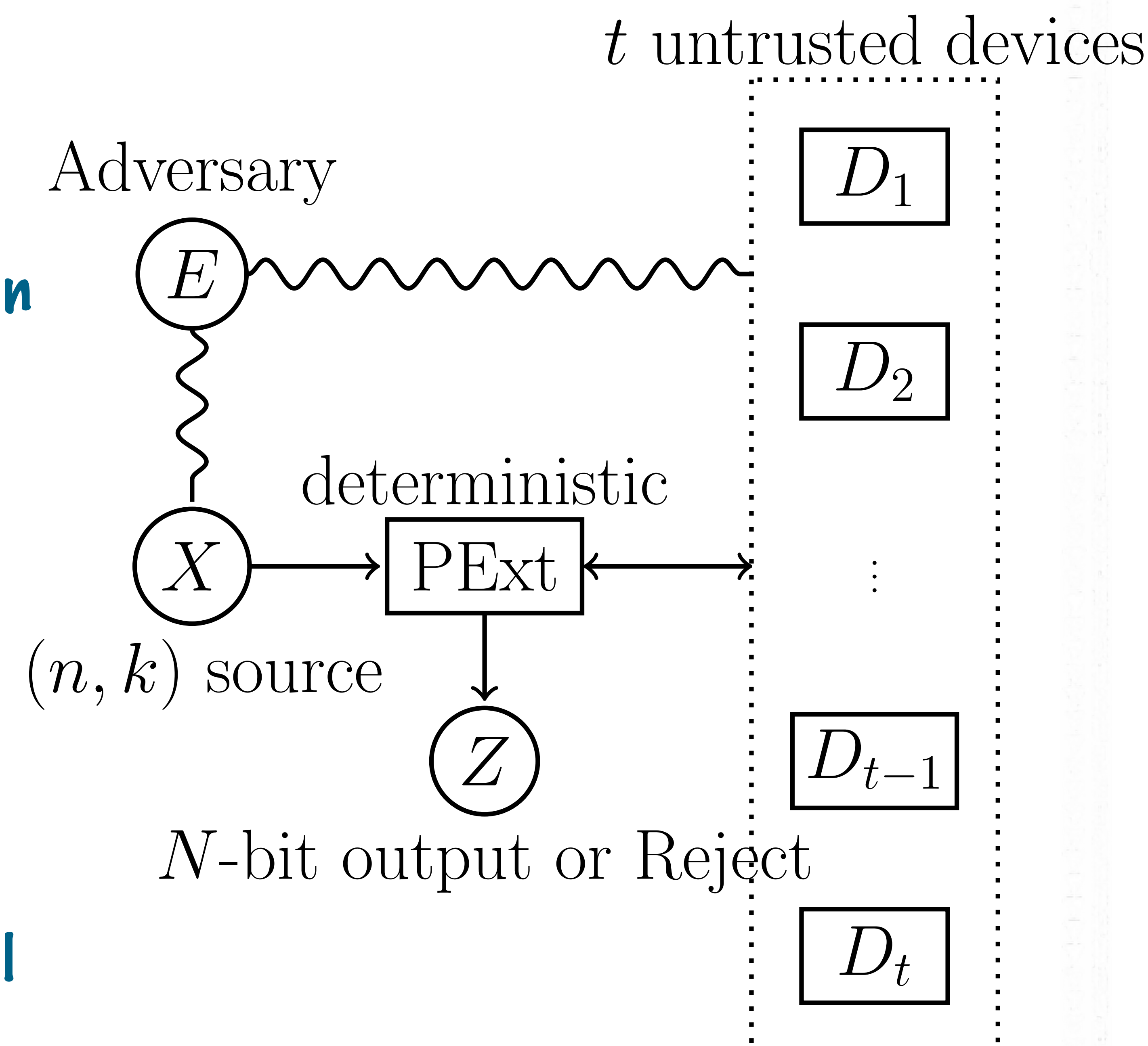
6. Efficiencies: Quantum memory, number of devices, computational complexity



Open problems::Physical Extractors

Result: seeded extraction [Miller-Shi]

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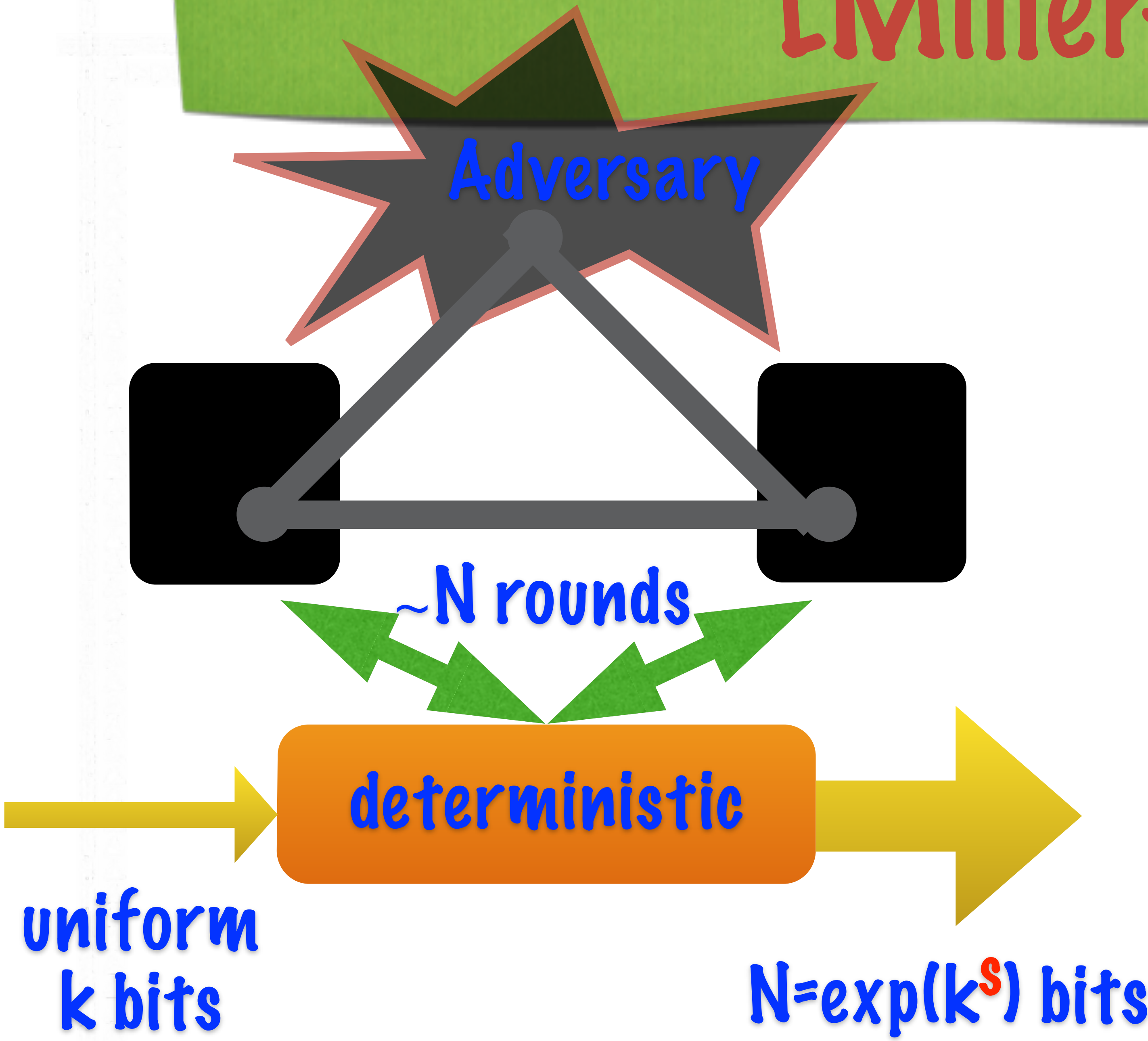

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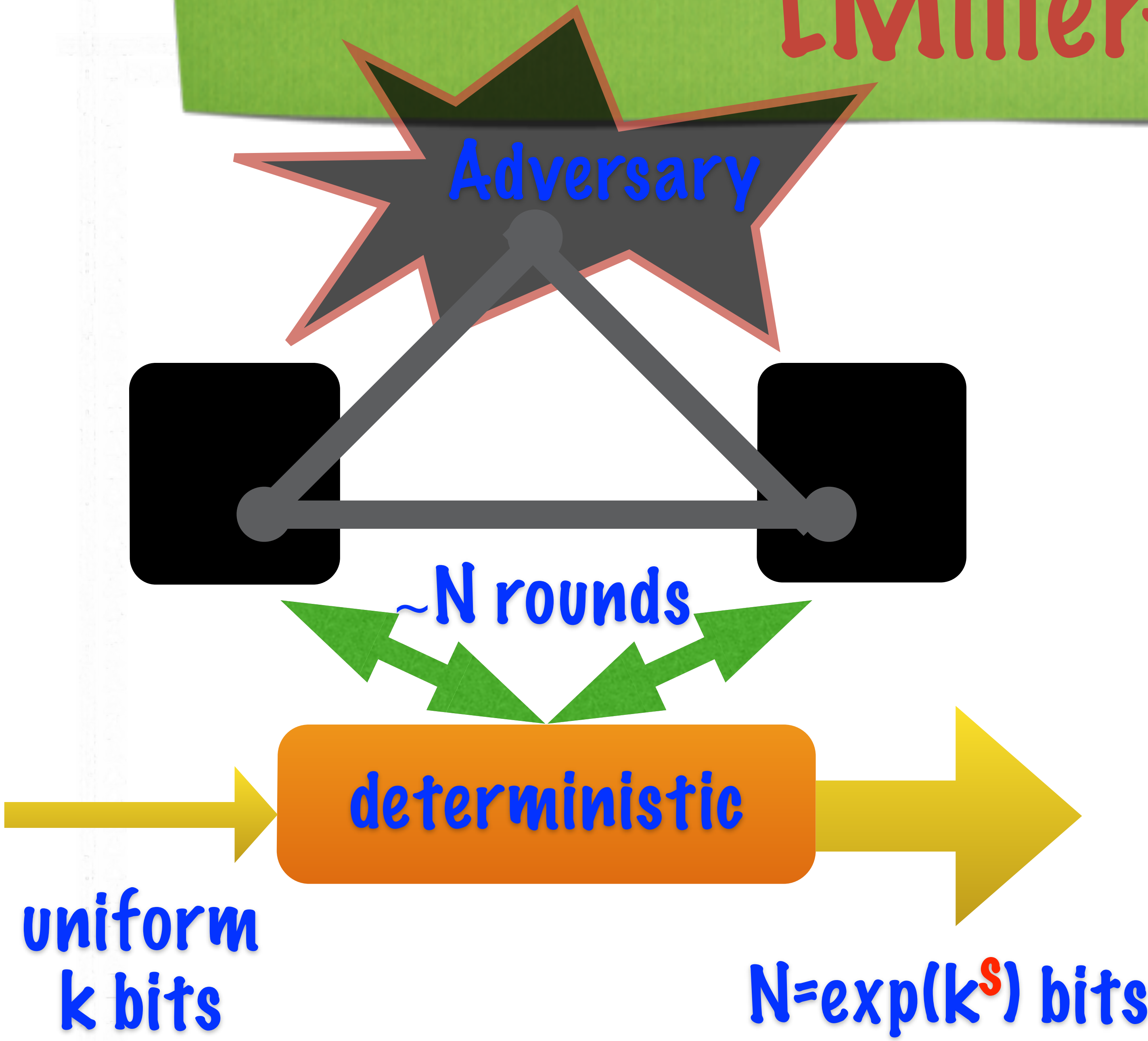

- 2 devices, exponential expanding, quantum security (match VV'12)

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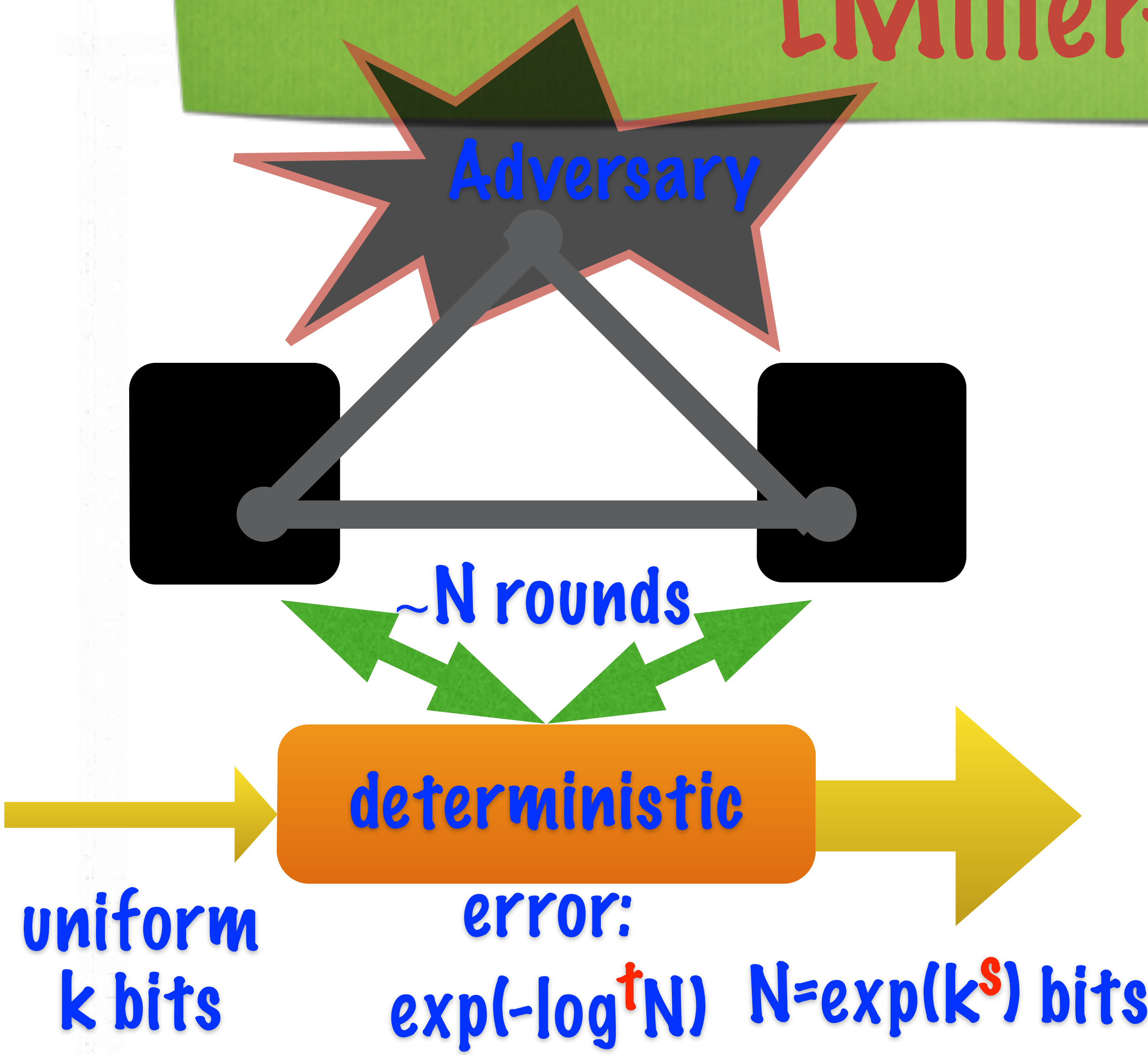

- 2 devices, exponential expanding, quantum security (match VV'12)
- Cryptographic security: errors = negligible in running time

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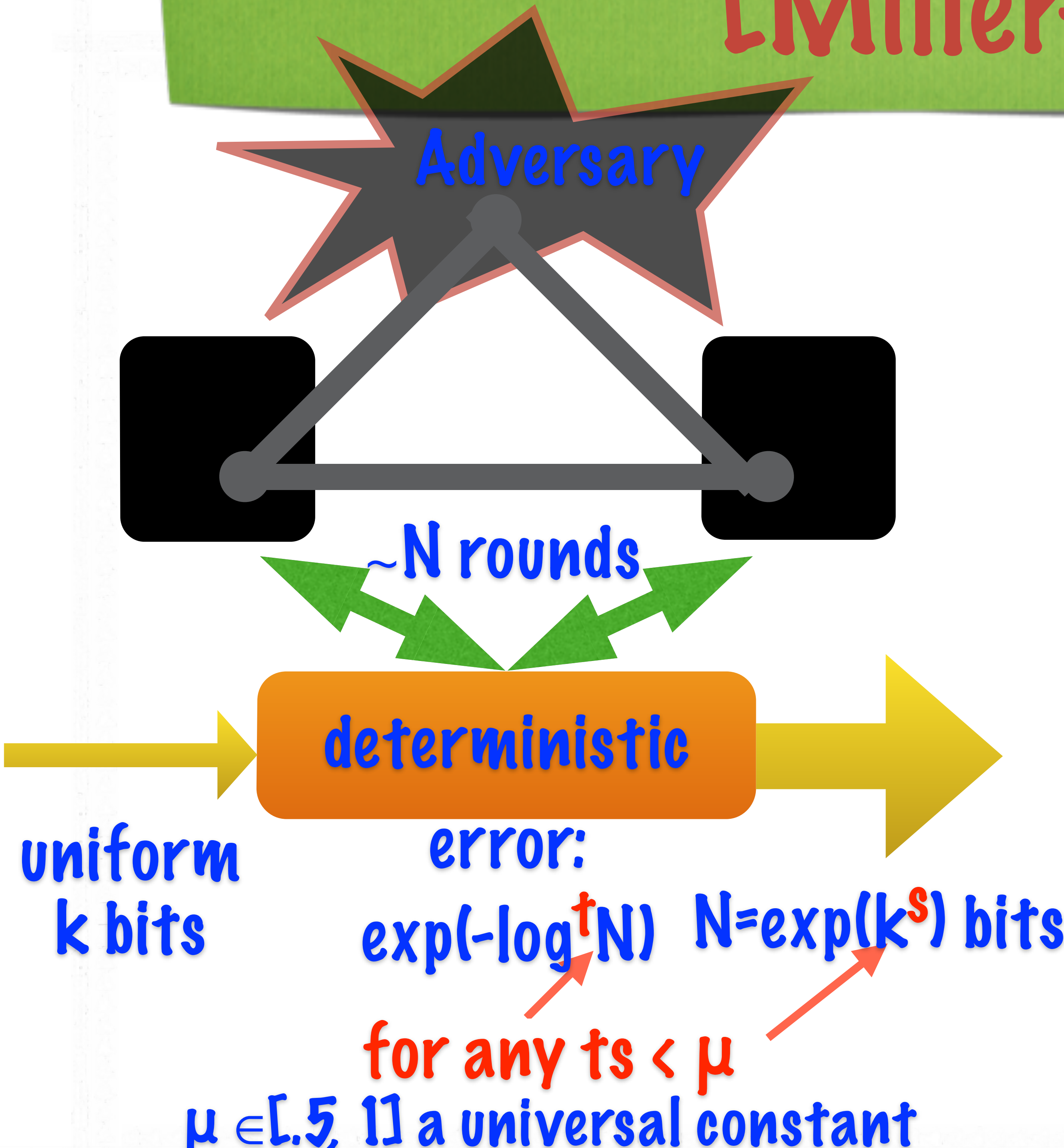

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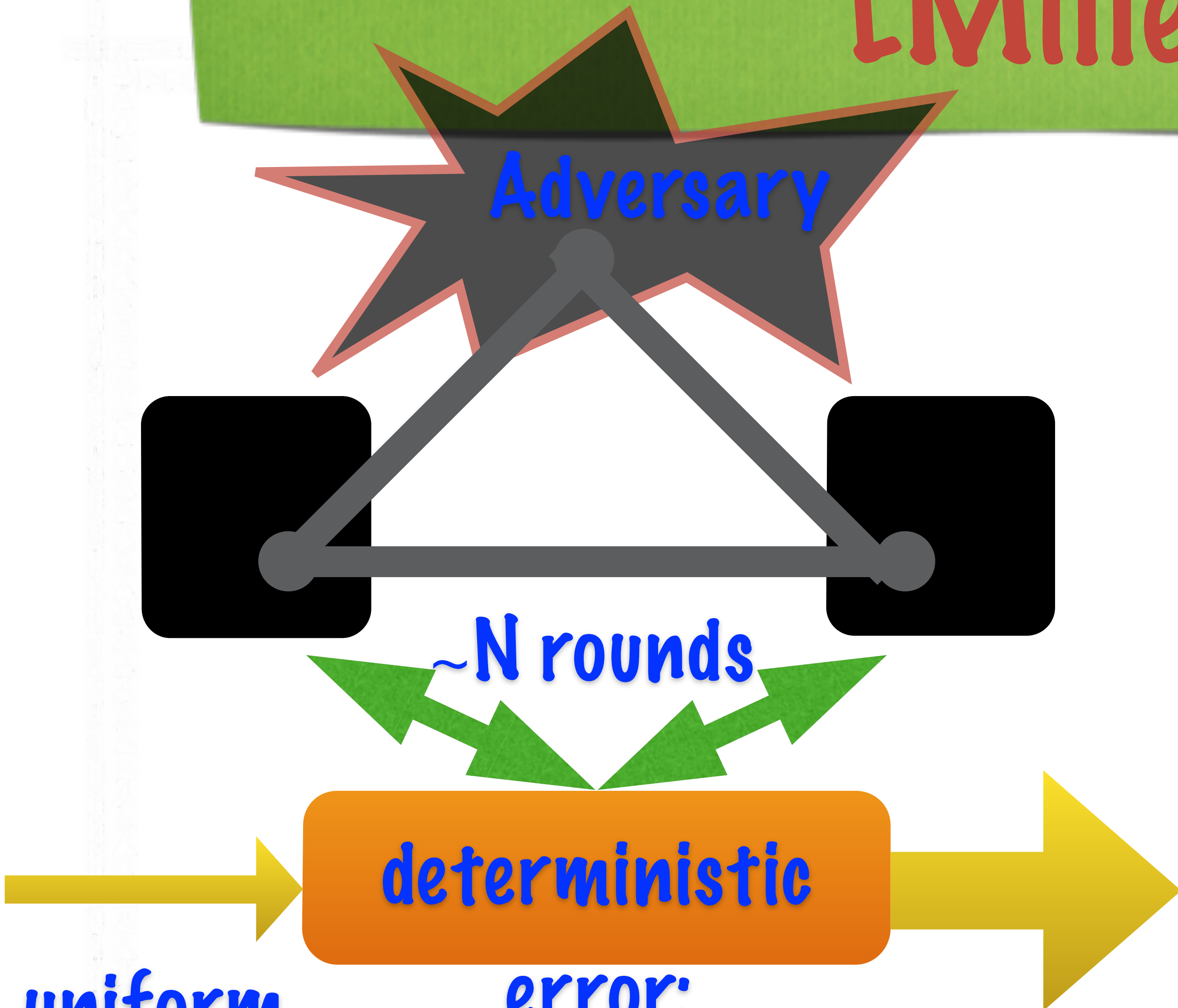

uniform k bits

error: $\exp(-\log^t N)$

$N = \exp(k^s)$ bits

for any $ts < \mu$

$\mu \in [.5, 1]$ a universal constant

- 2 devices, exponential expanding, quantum security (match VV'12)
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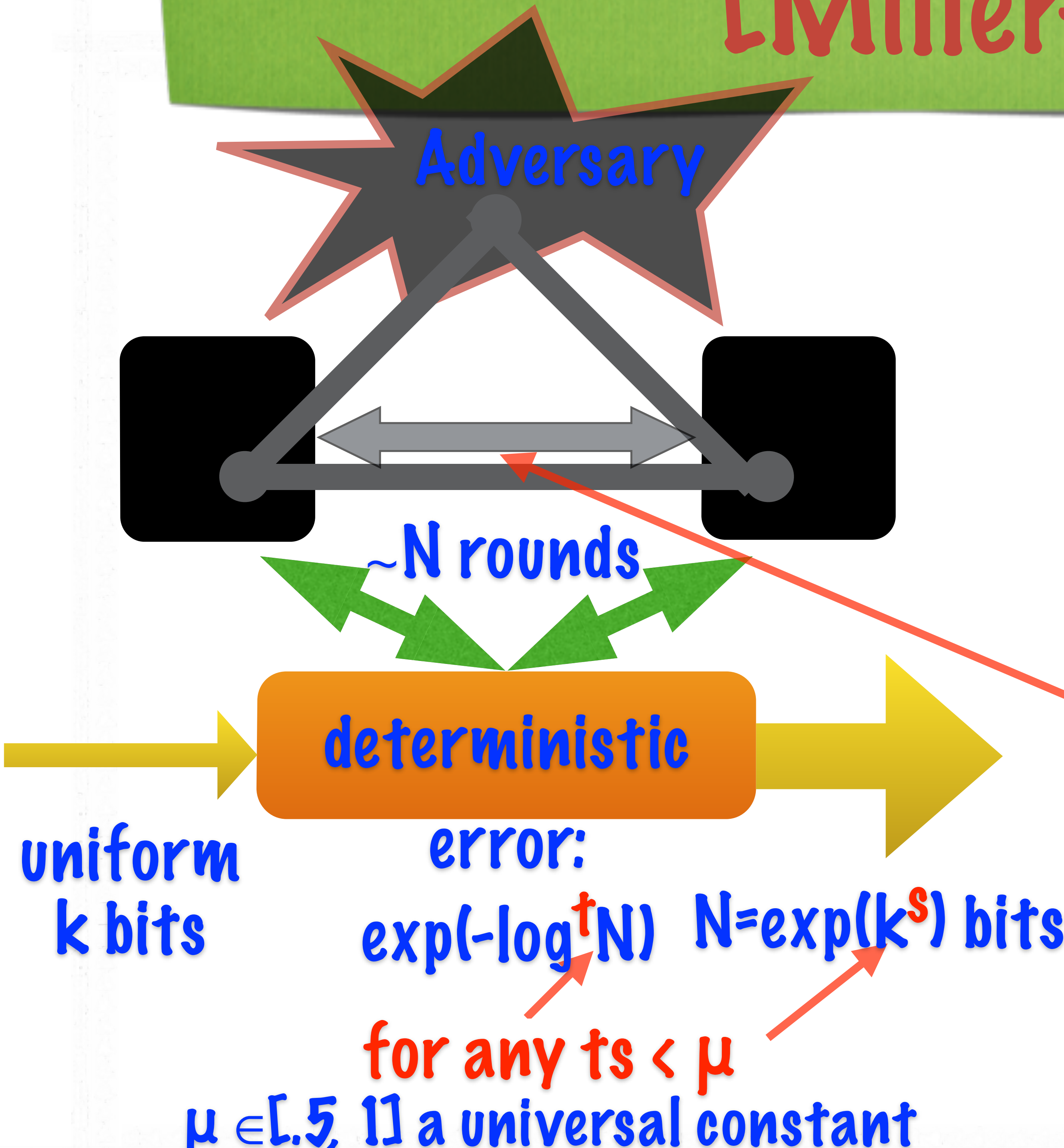

- 2 devices, exponential expanding, quantum security (match VV'12)
- **Cryptographic security**: errors = negligible in running time
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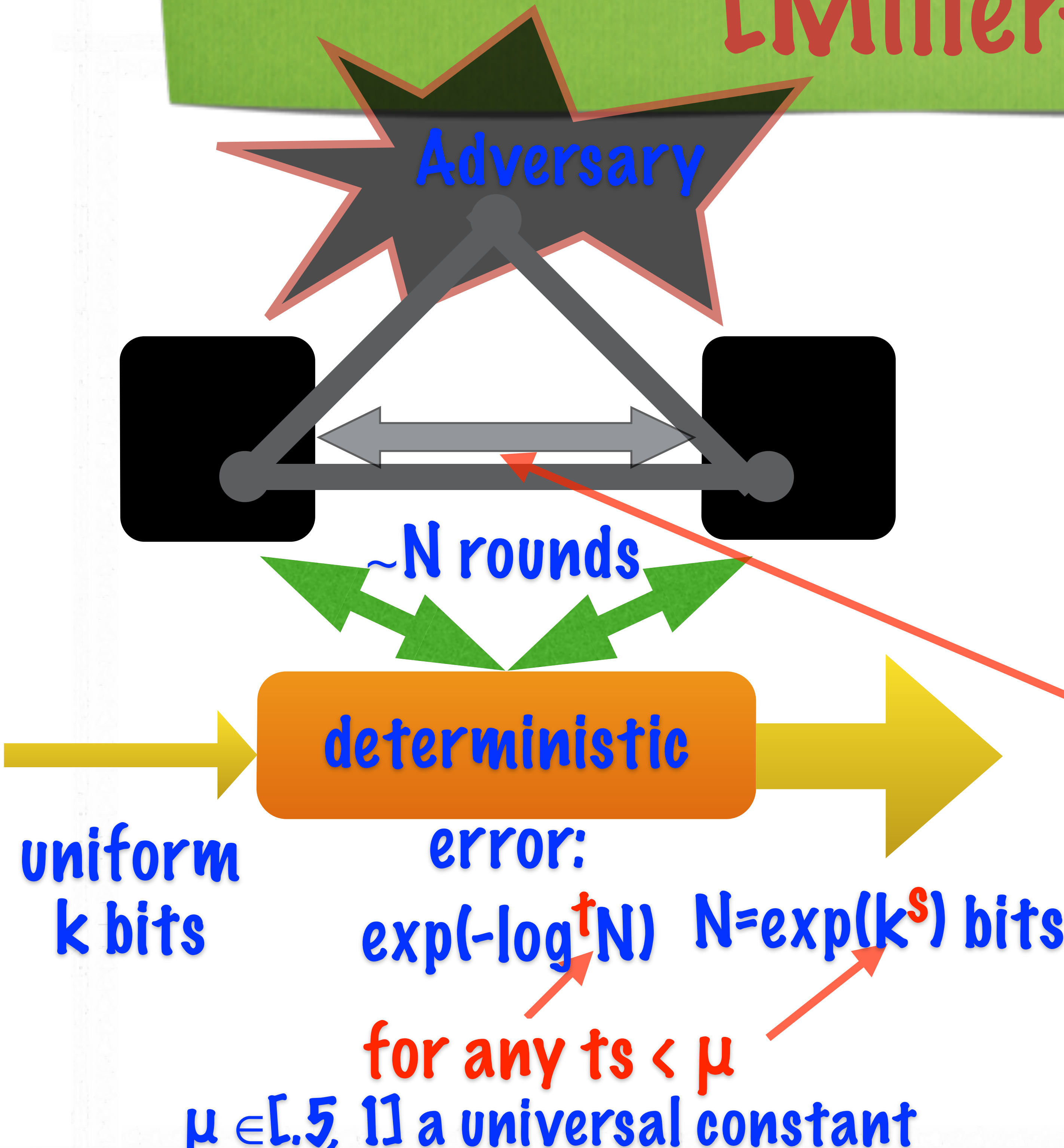
- **Unit size quantum memory**: allow in-between-rounds of communication

Model and Results::seeded extraction

Result: seeded extraction [Miller-Shi]



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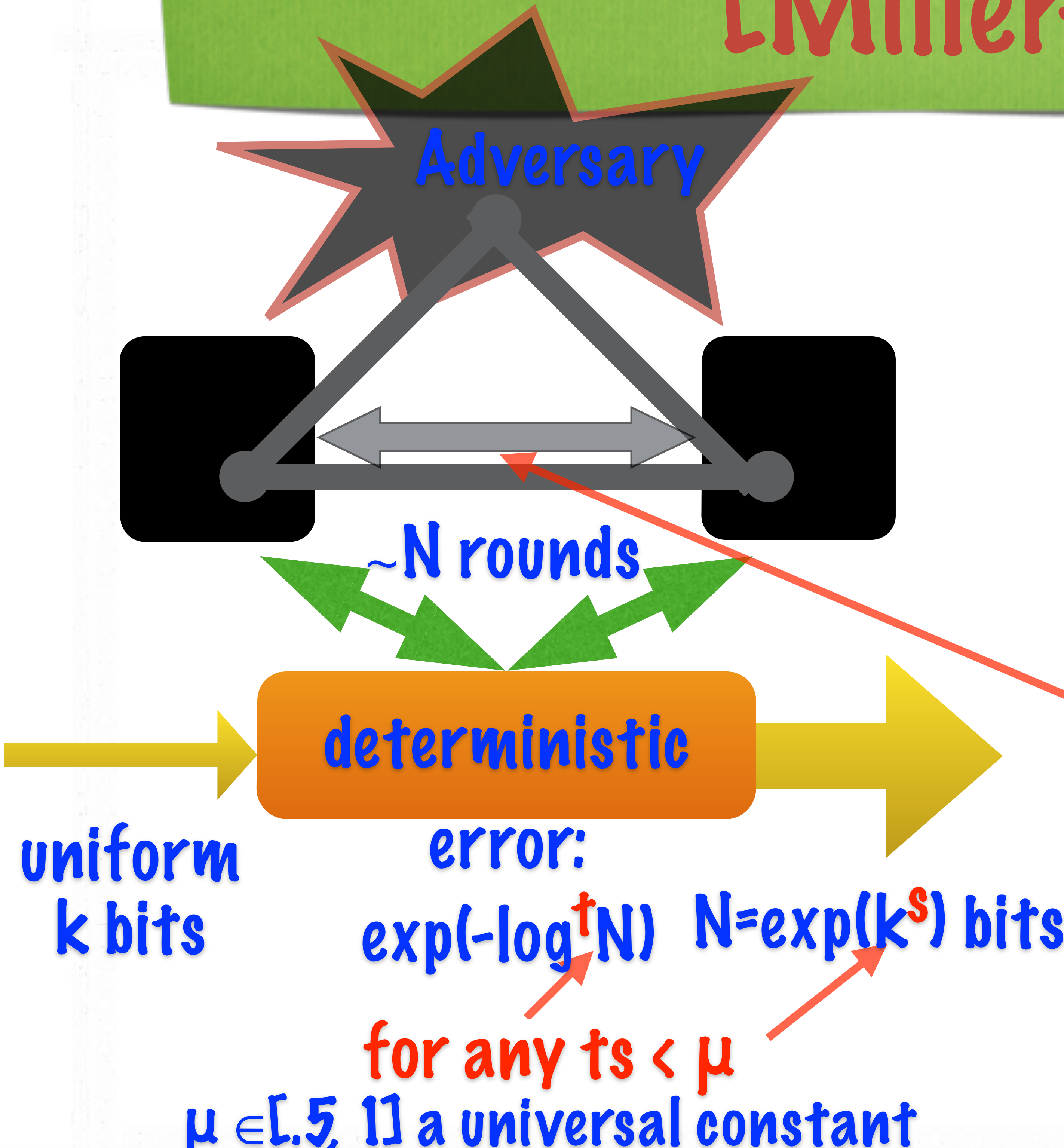

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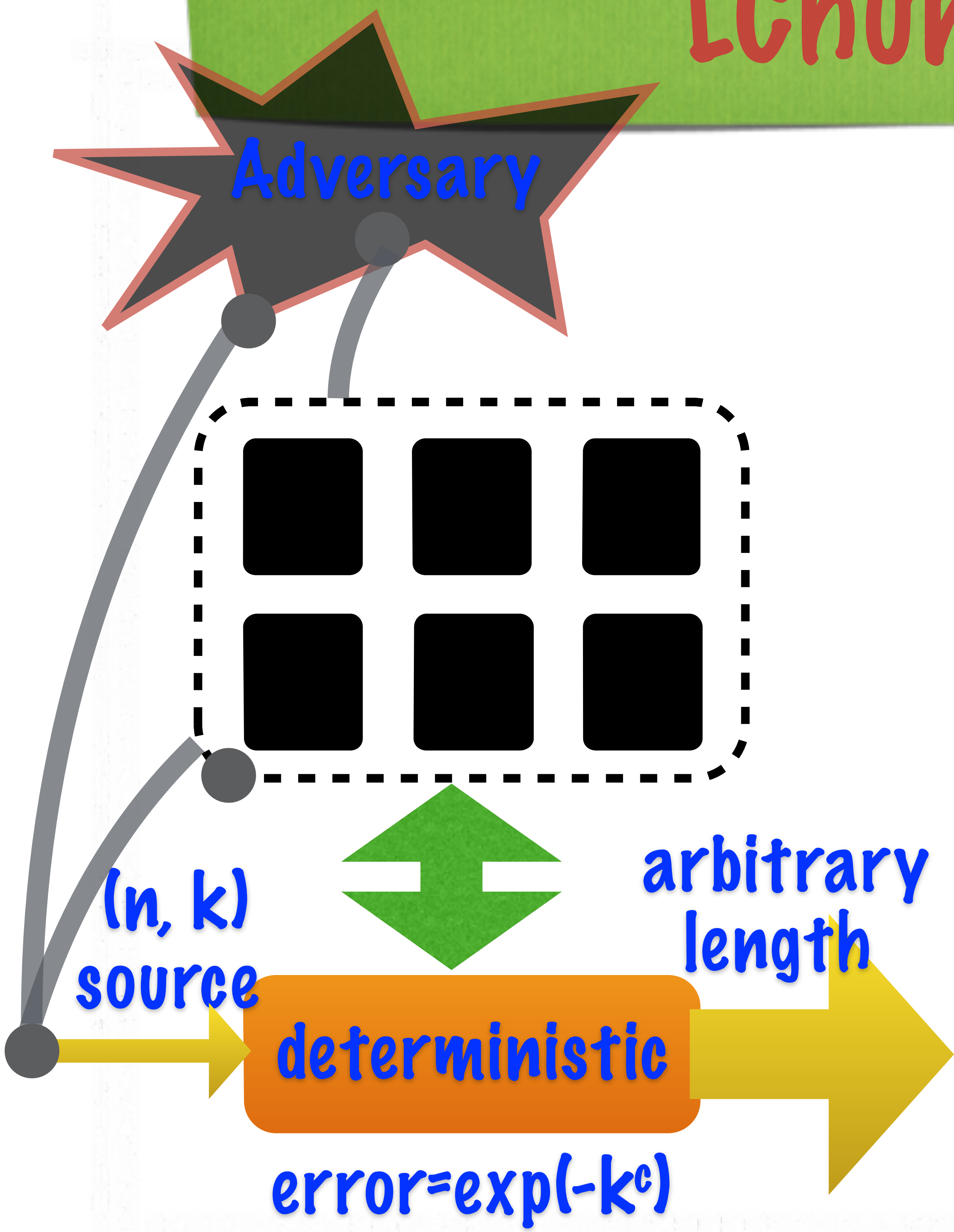
- Cryptographic security: errors = negligible in running time
- Robustness: constant level of noise
- Unit size quantum memory: allow in-between-rounds of communication
- Flexibility in building-blocks
- New proof techniques

Model and Results::seeded extraction

Result: seedless extraction [Chung-Shi-Wu]



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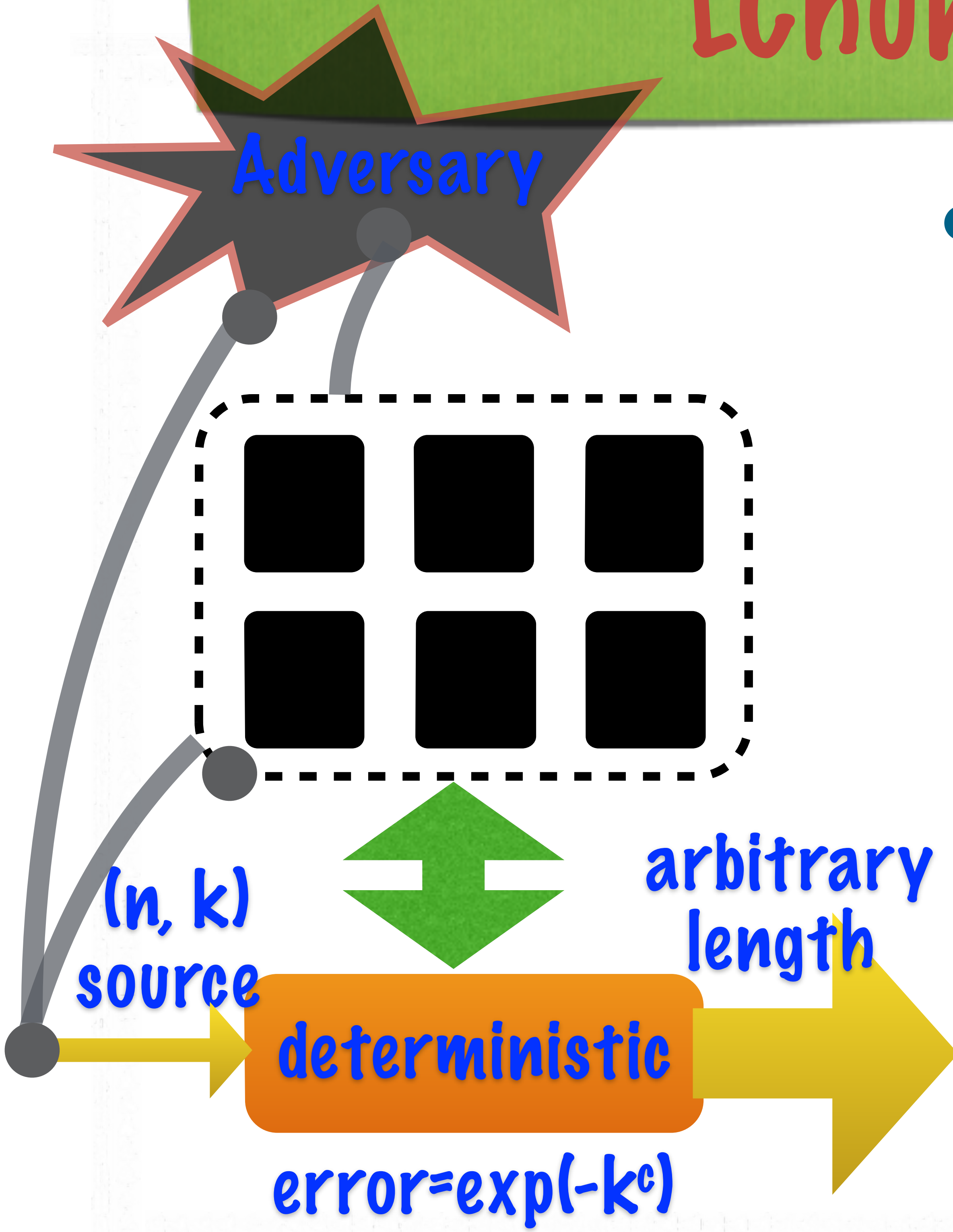
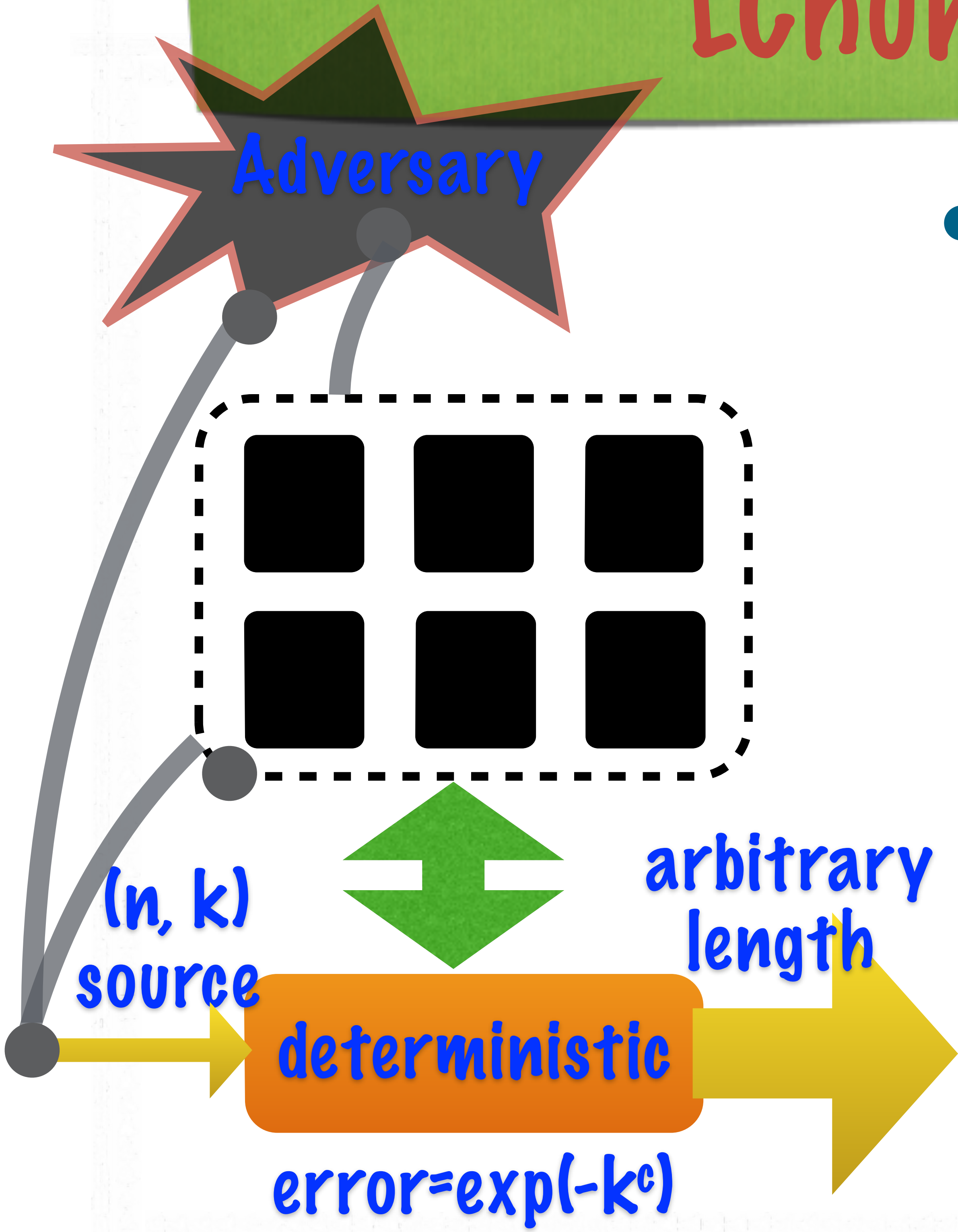


- Use just one min-entropy source

Result: seedless extraction [Chung-Shi-Wu]



- Use just one min-entropy source
  - can be known to Adversary: min-entropy w.r.t. devices

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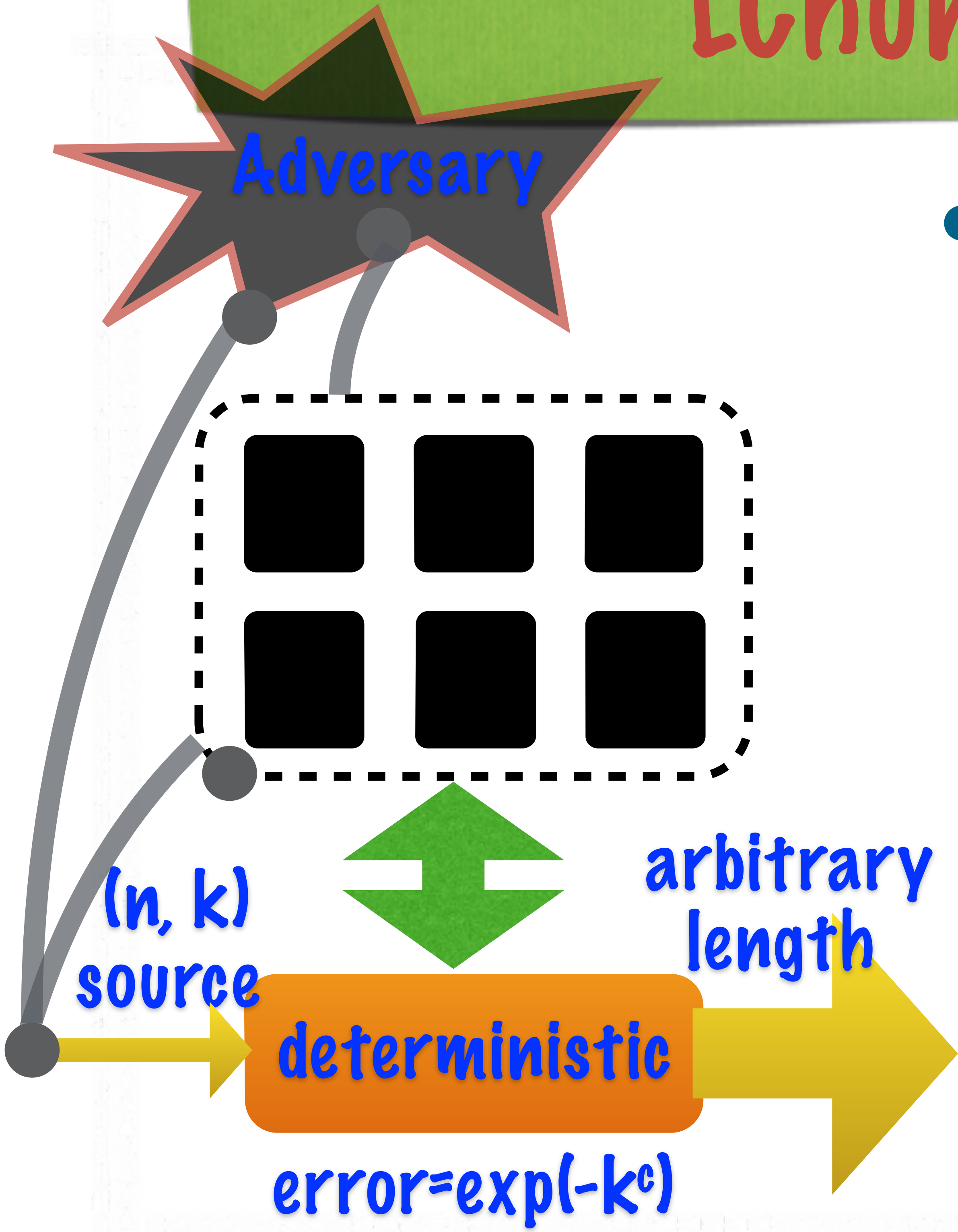

- Use just one min-entropy source
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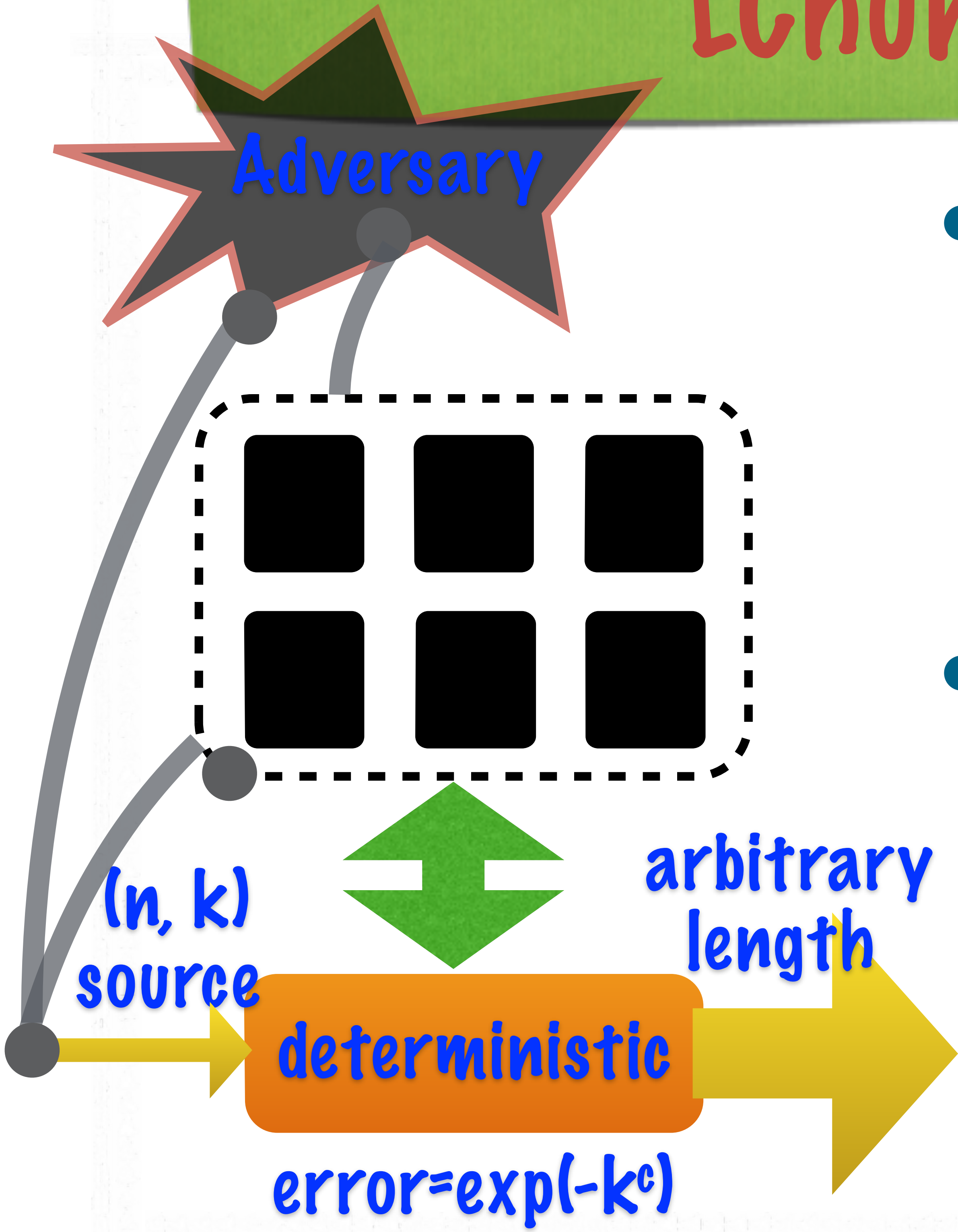

- Use just one min-entropy source
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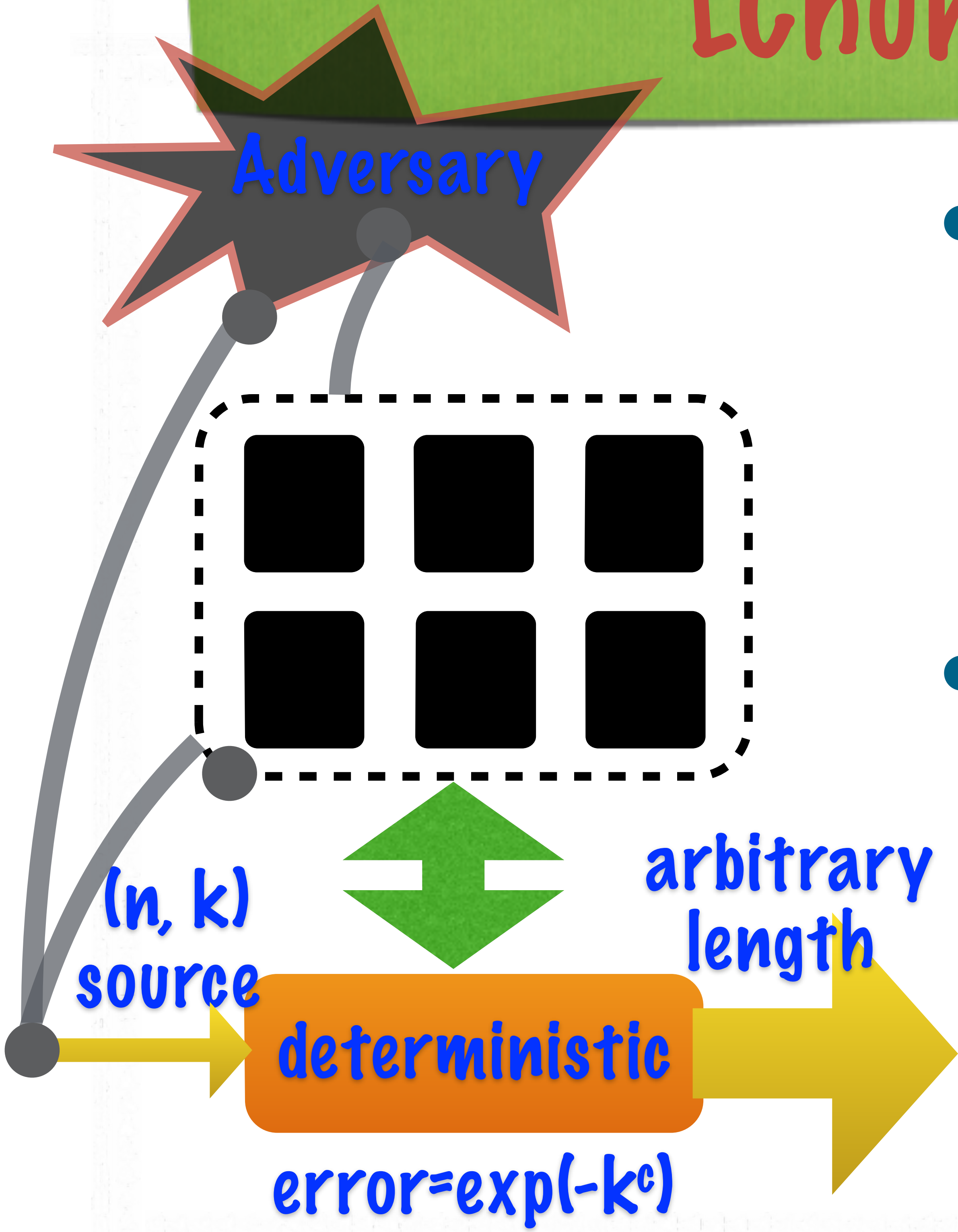

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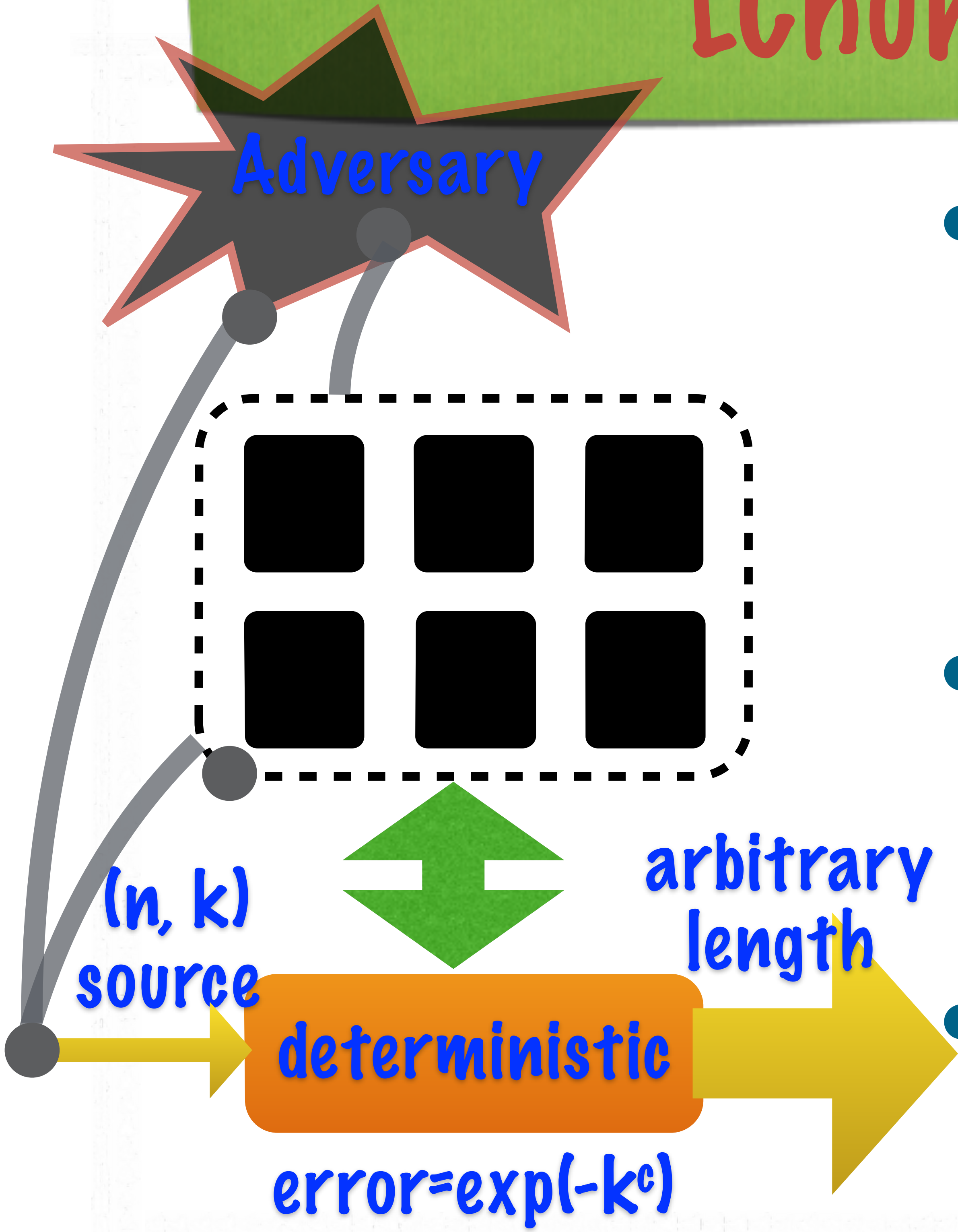

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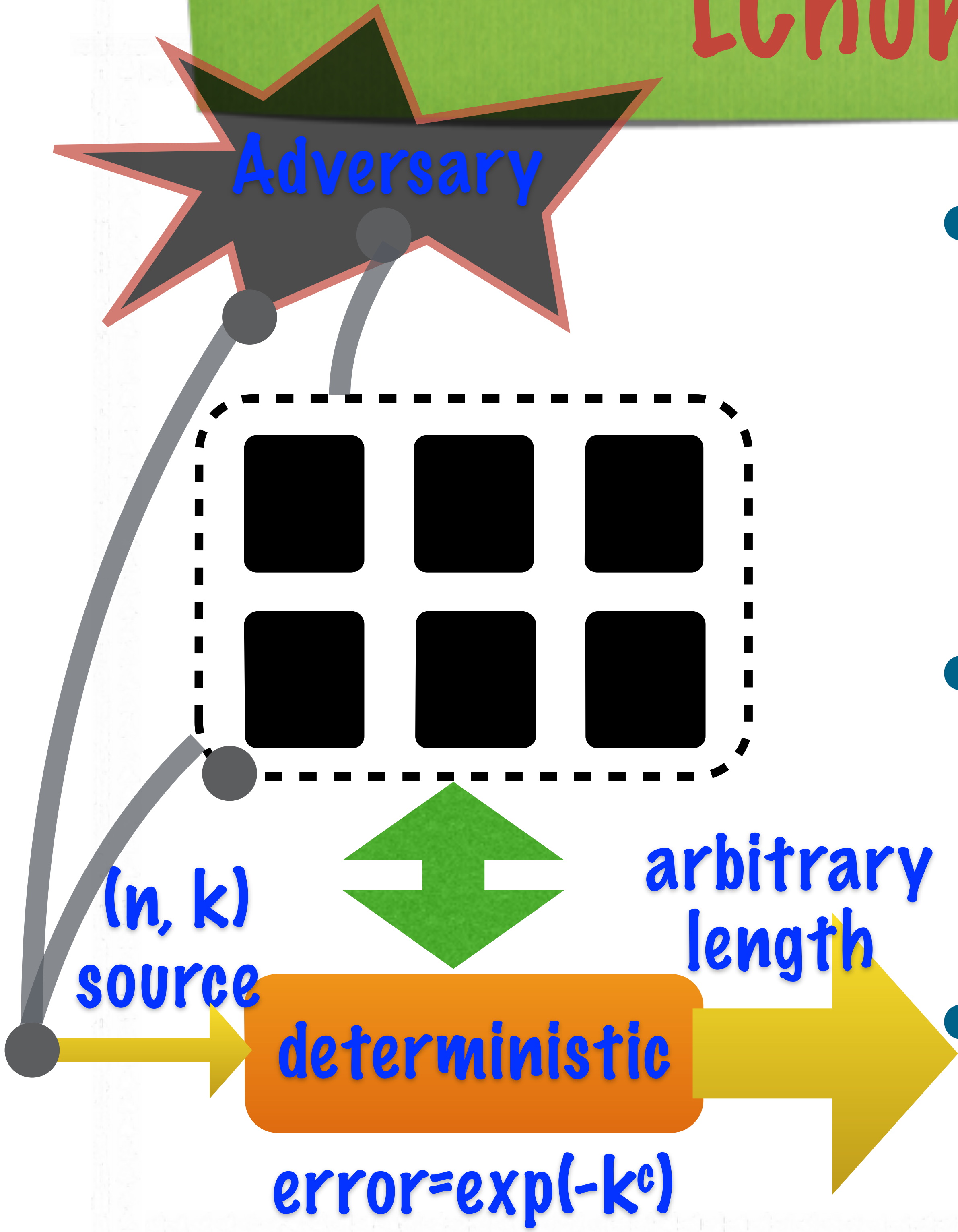

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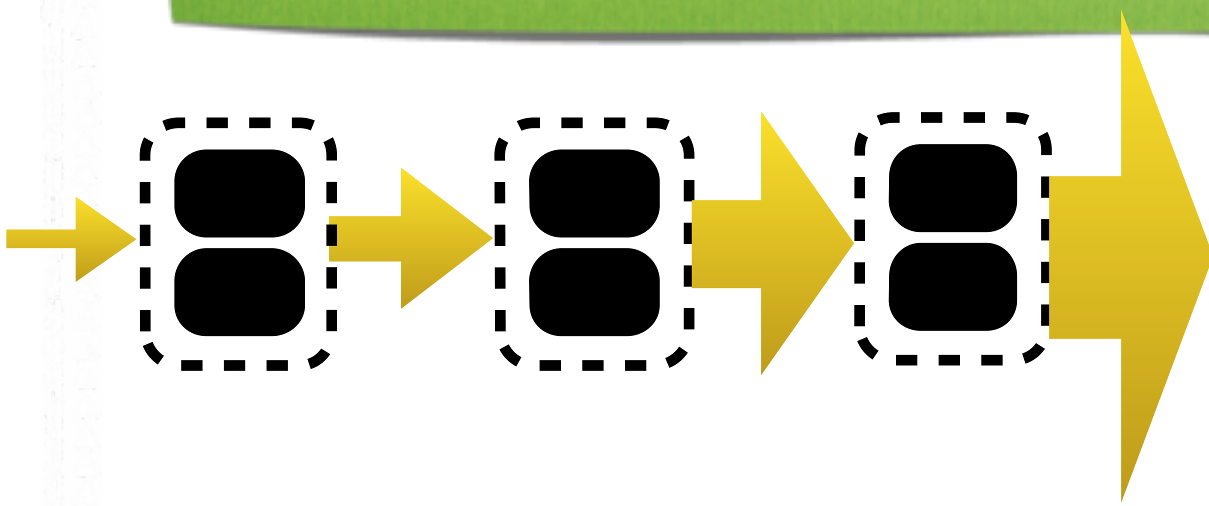
  - Tolerate constant noise by using Miller-Shi or Vazirani-Vidick (qkd)
- Tradeoff between error and #devices
  - Error can be made close to optimal: $\exp(-k^c)$ (lower bound: 2^{-k})

Application: robust unbounded expansion [CSW+MS]



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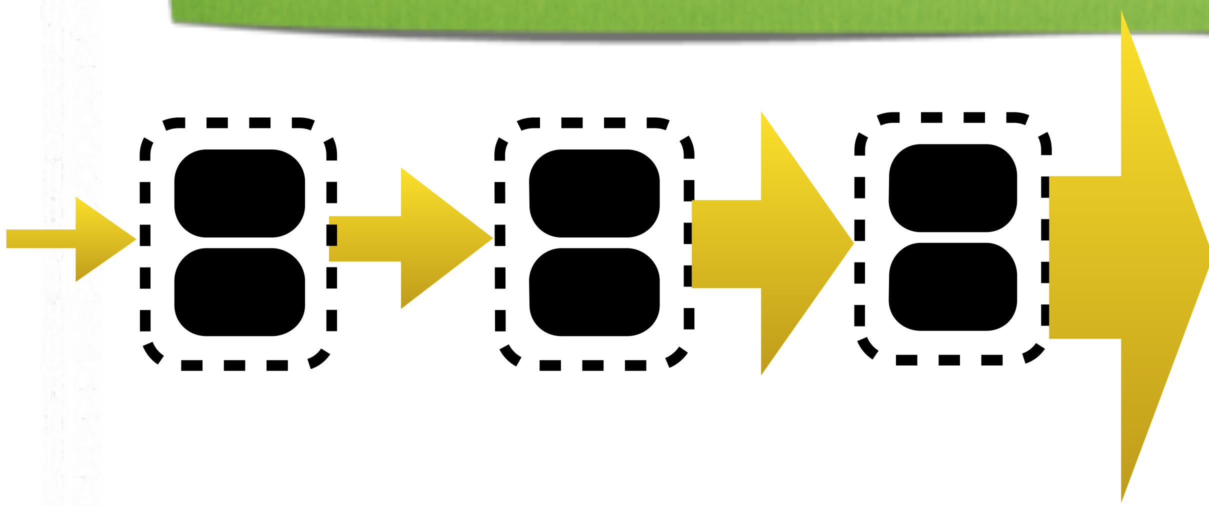

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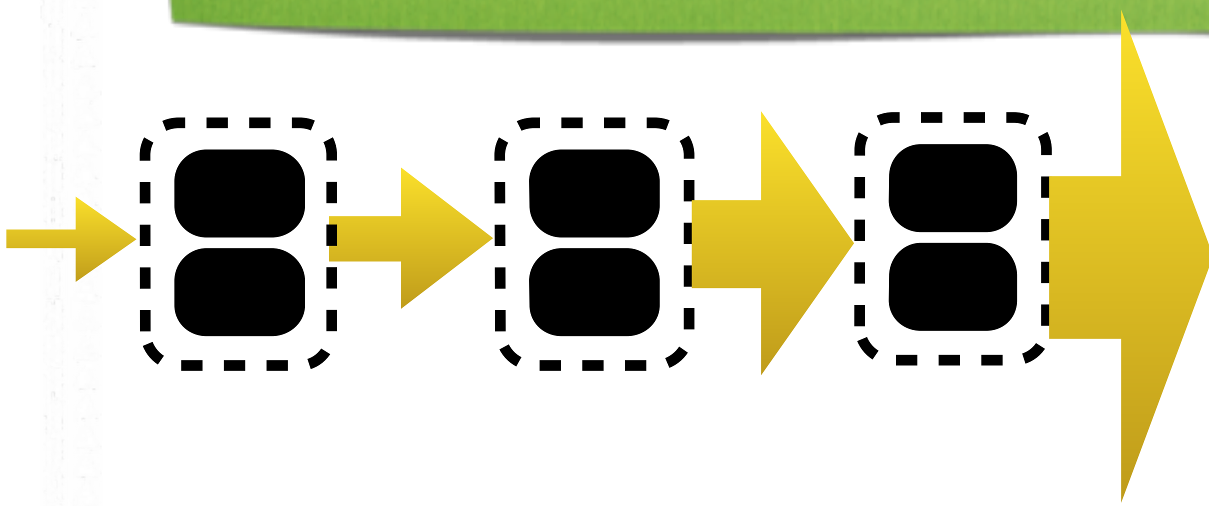

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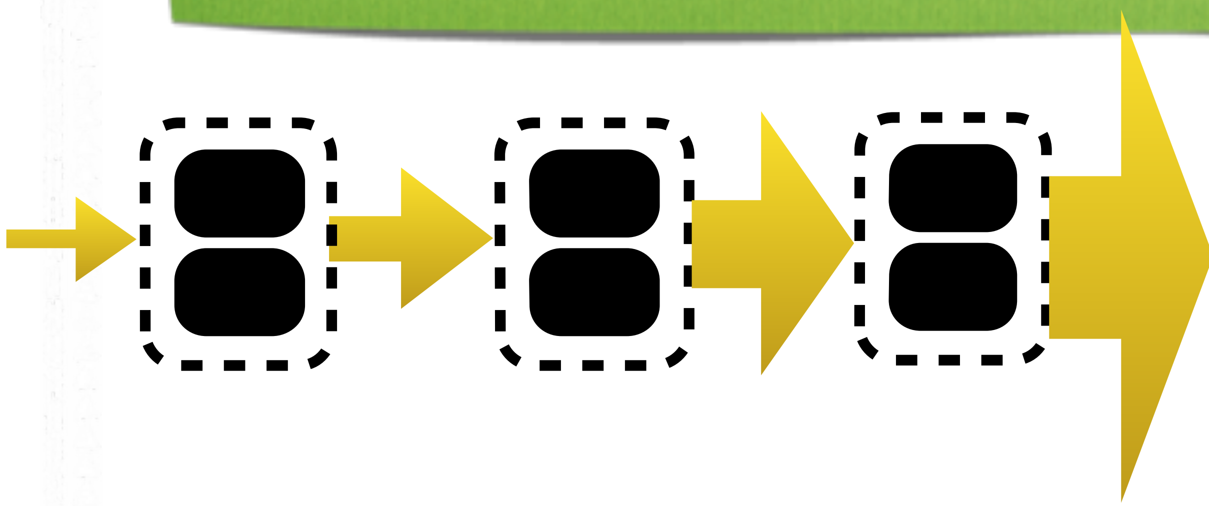

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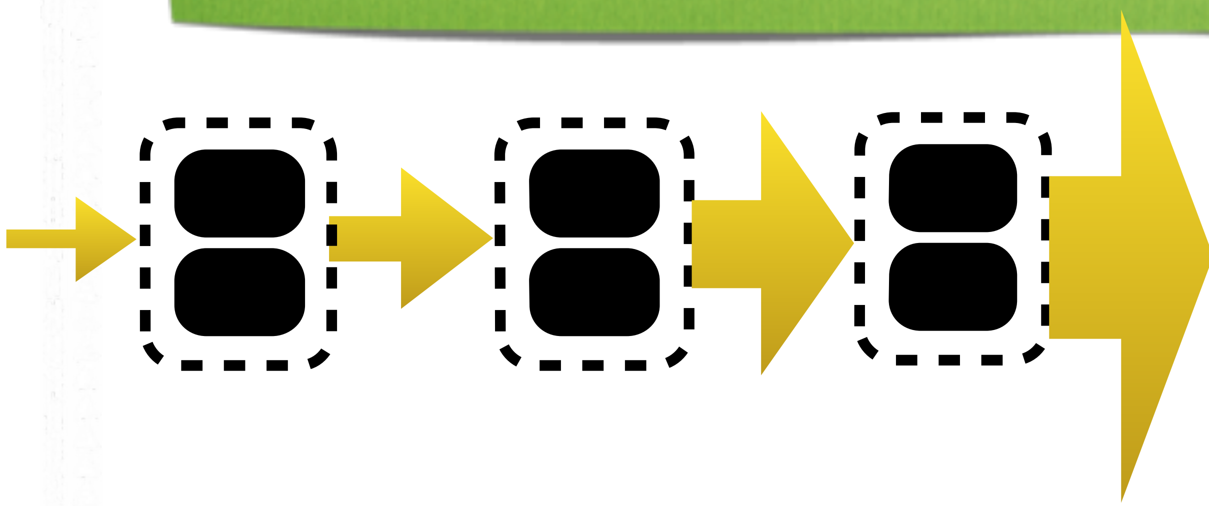

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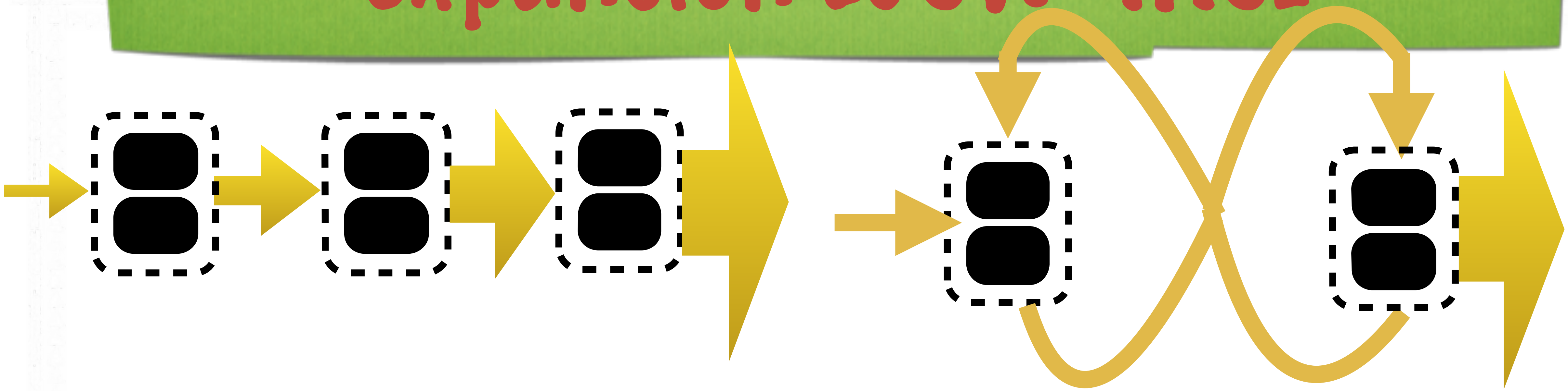

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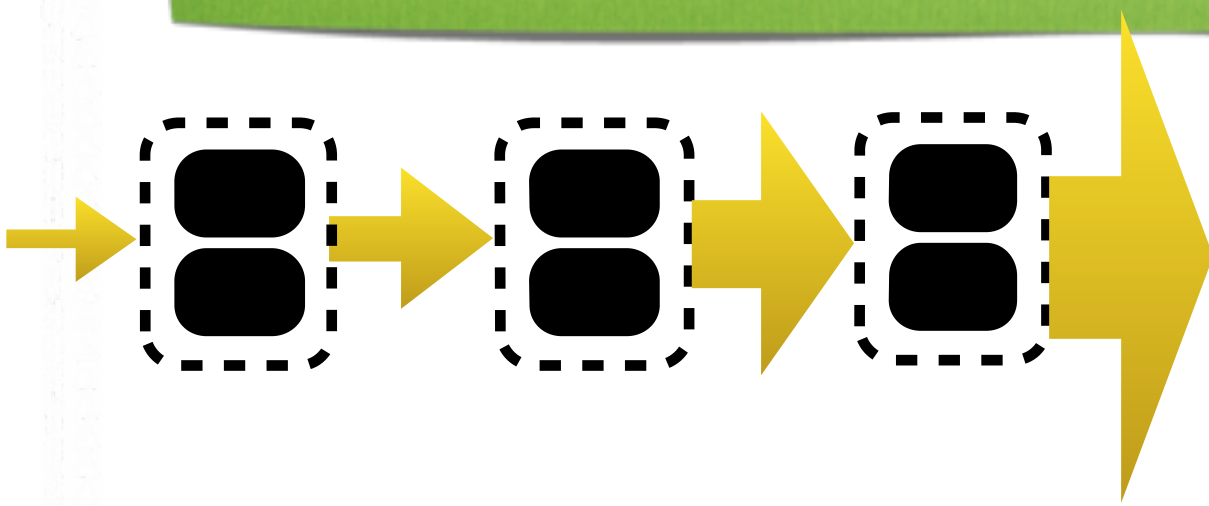

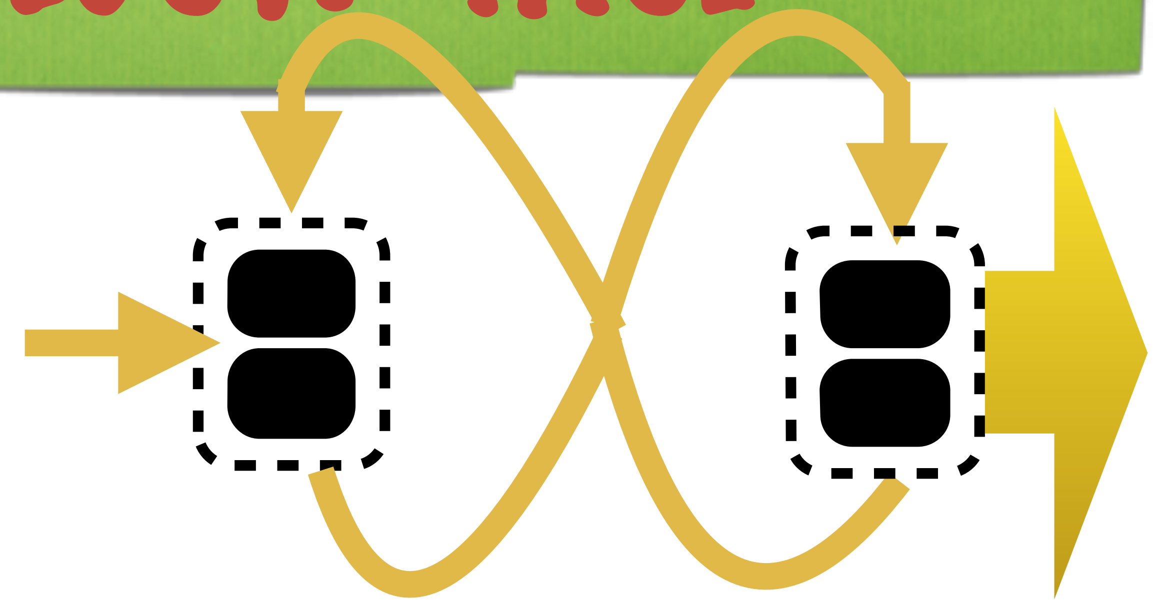

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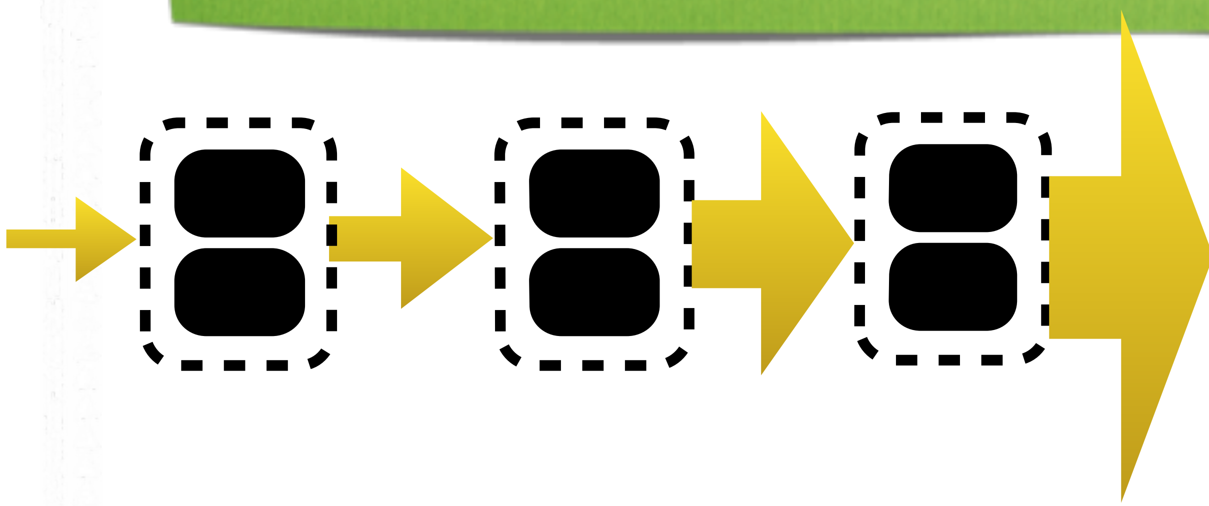

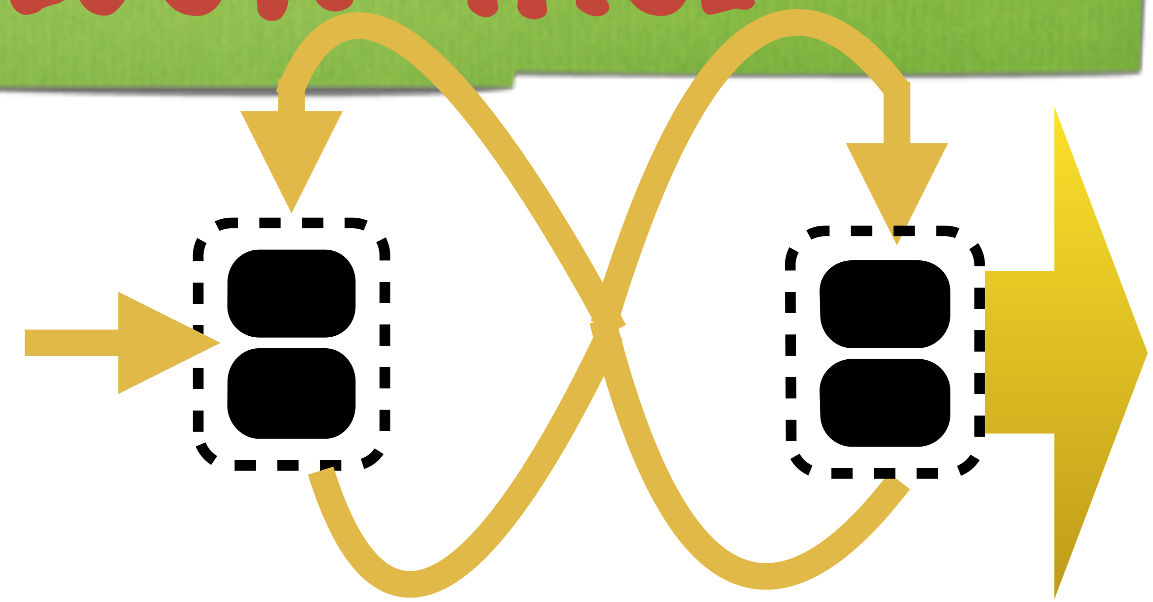

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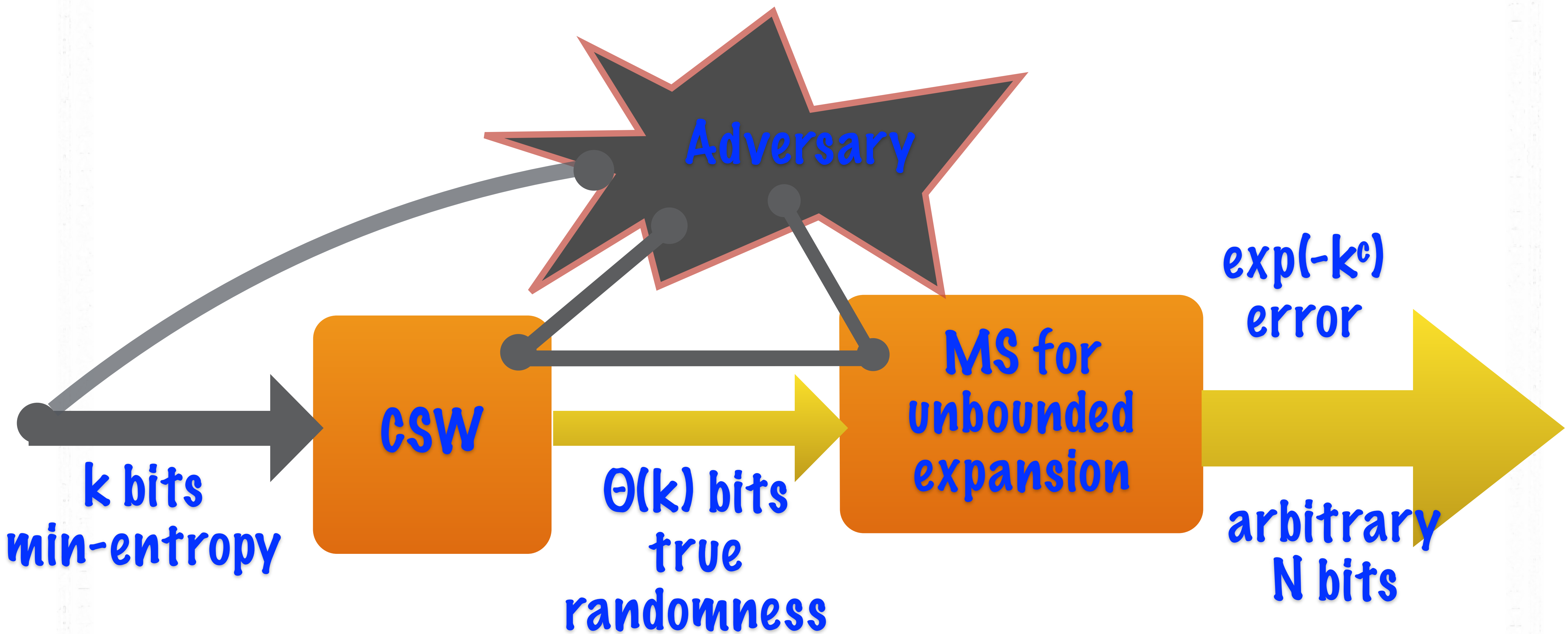
- Yes: [Coudron-Yuen'13] based on VV'12+Reichardt-Unger-Vazirani'13: 8 devices, non-robust
- CSW+MS: any expanding protocol safe for cross-feeding with doubled number of devices and about the same error
  - Using Miller-Shi: robust, 4-devices

CSW+MS: robust unbounded expansion from just one and arbitrary min-entropy source and almost optimal error



Application: expanding key distribution [Miller-Shi]

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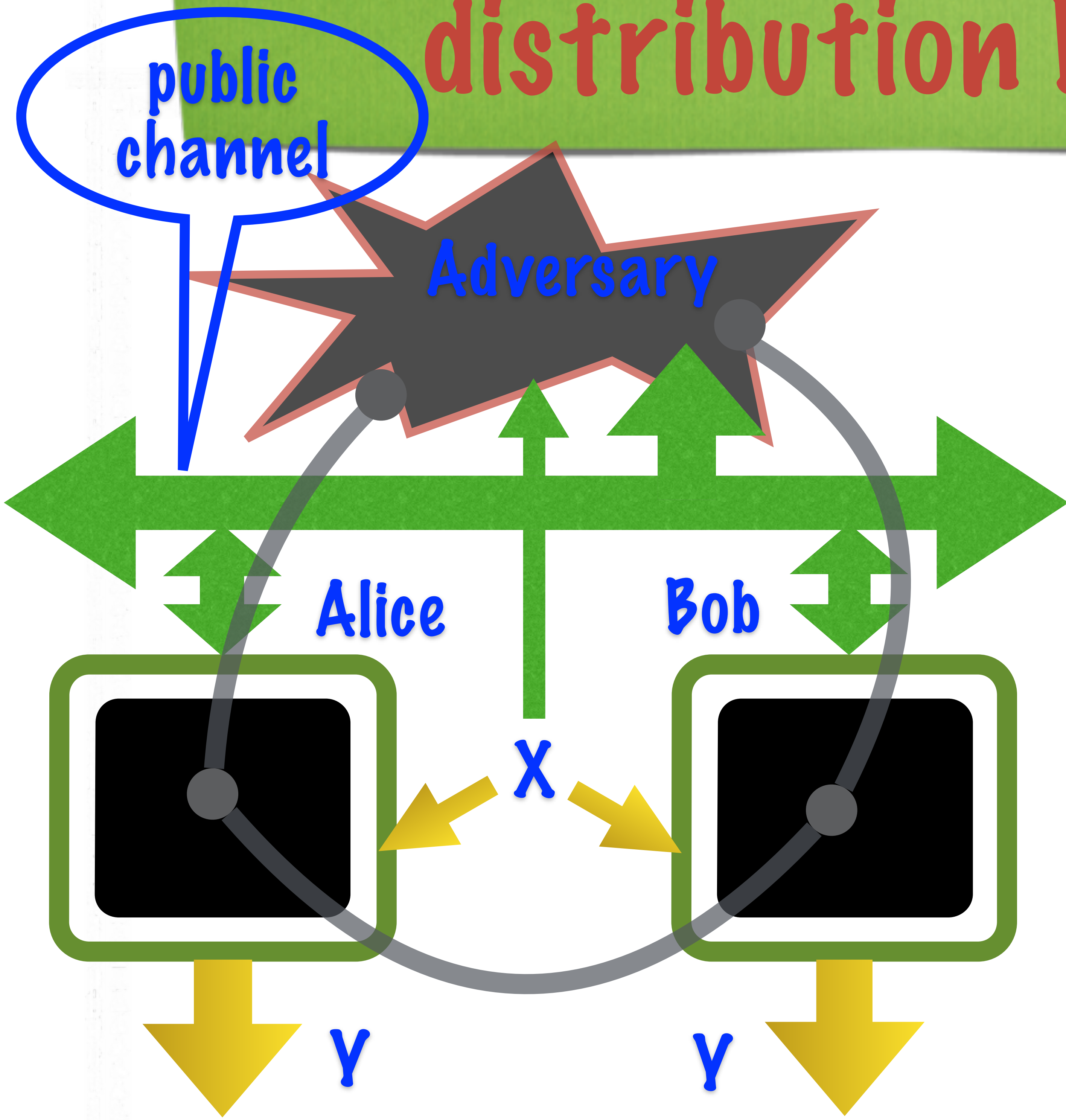

- Robust untrusted device qkd first proved by Vazirani-Vidick'13

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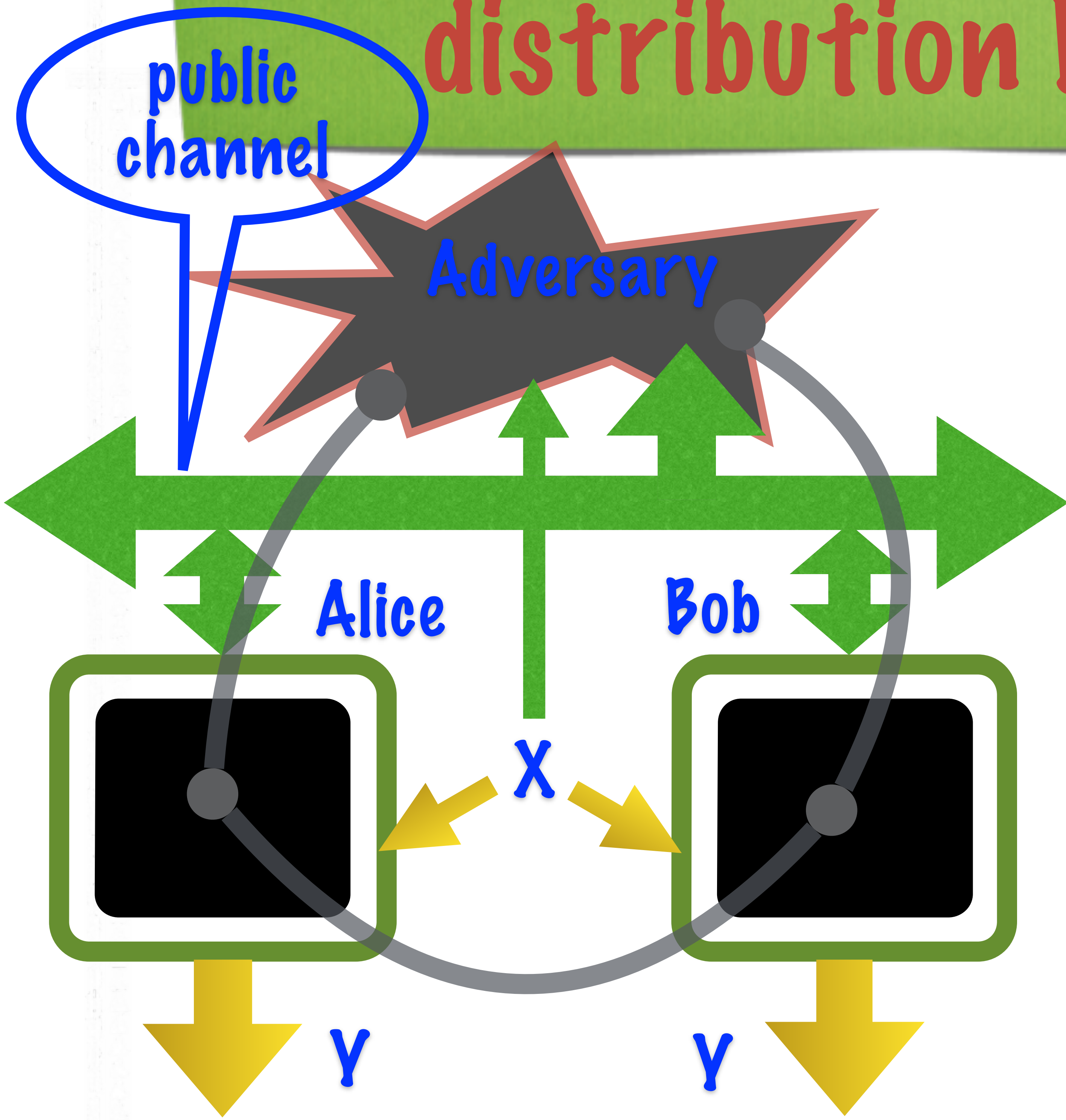
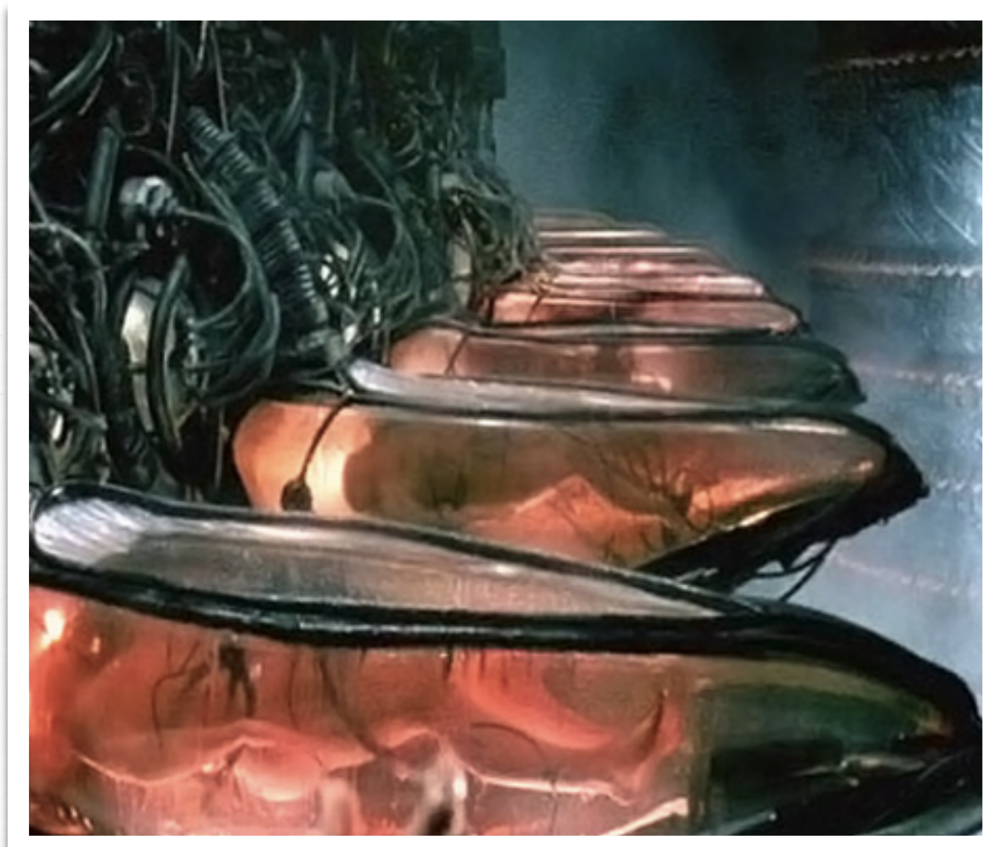


- Robust untrusted device qkd first proved by Vazirani-Vidick'13
- New in the adapted Miller-Shi: exponentially expanding key with 2 devices (unbounded with 4)

Dichotomy between deterministic and fundamentally random world

[Colbeck-Renner'12, Gallego+'13, CSW'14]

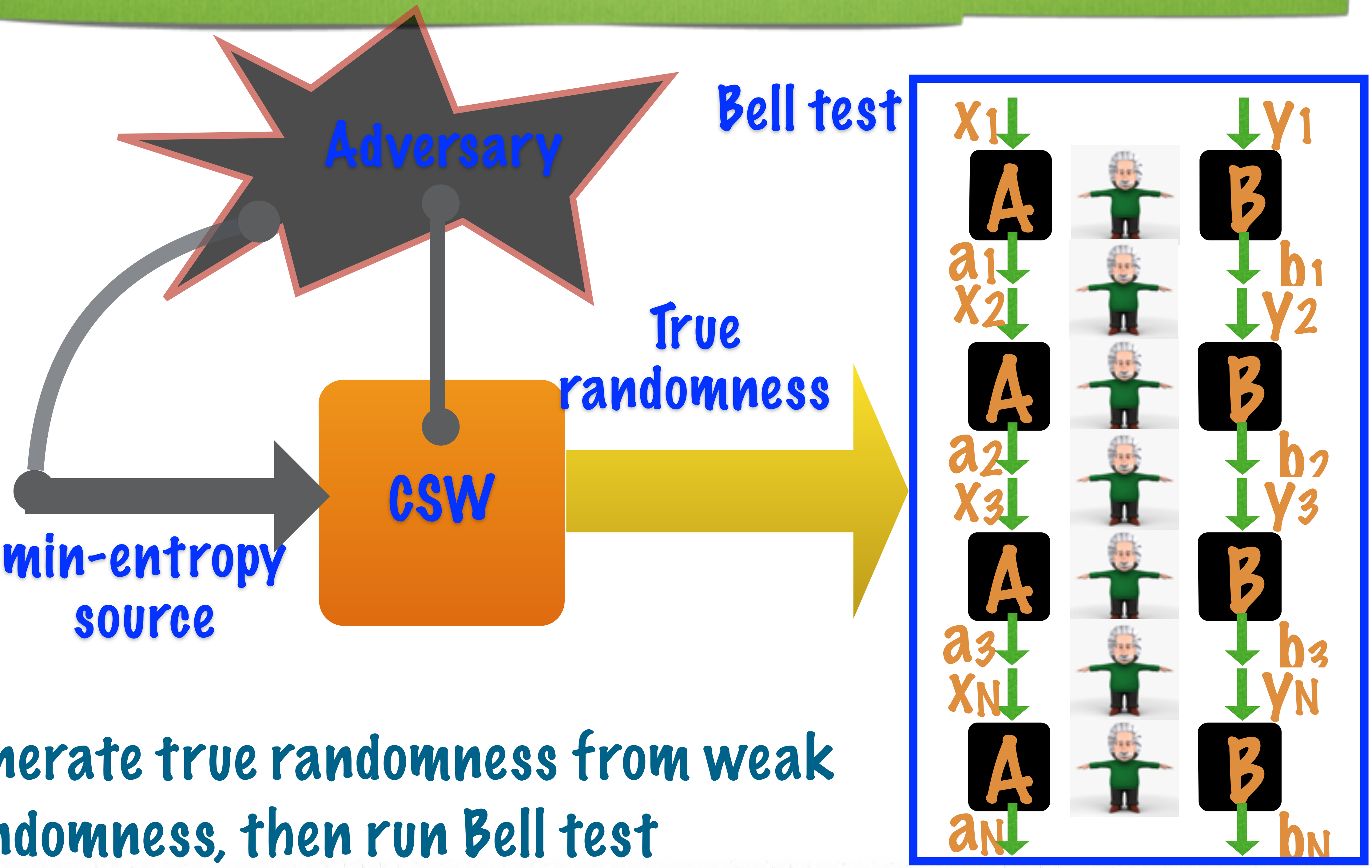


V.S.

True randomness in Nature either does not exist or exist in almost perfect quality and unbounded quantity

Untrusted-device protocols

Mitigating freedom-of-choice loophole



Generate true randomness from weak randomness, then run Bell test

3. Protocols and Proof Method

Foundation: Classical Test of a Quantum Duck

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- Test if the devices behave like the ideal devices

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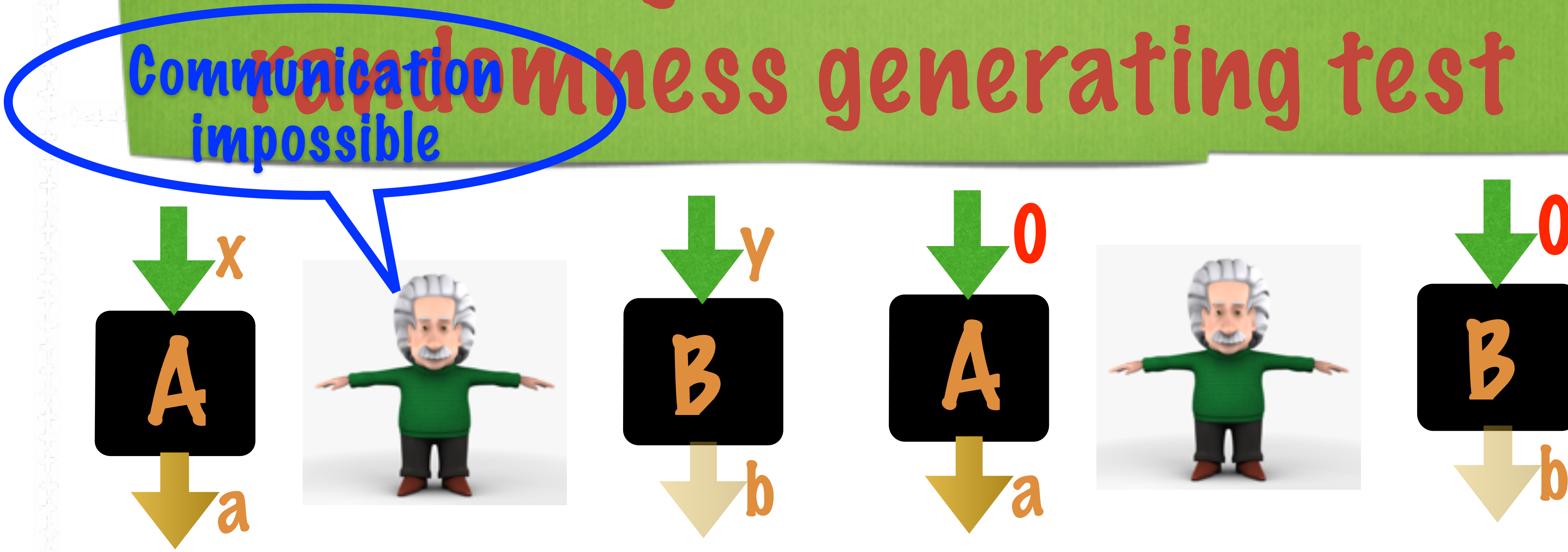
- Test if the devices behave like the ideal devices
- Idea devices generate randomness

CHSH game: a robust & randomness generating test



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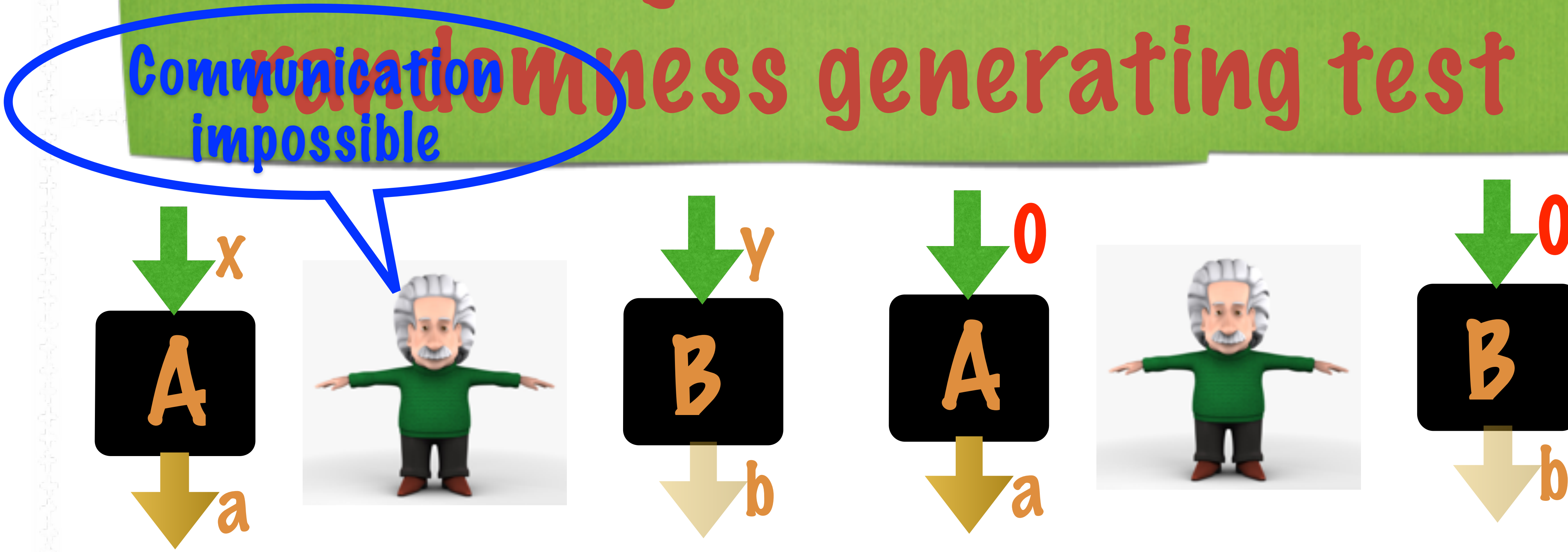

- No communication allowed during game
- Each device receives a bit, outputs a bit

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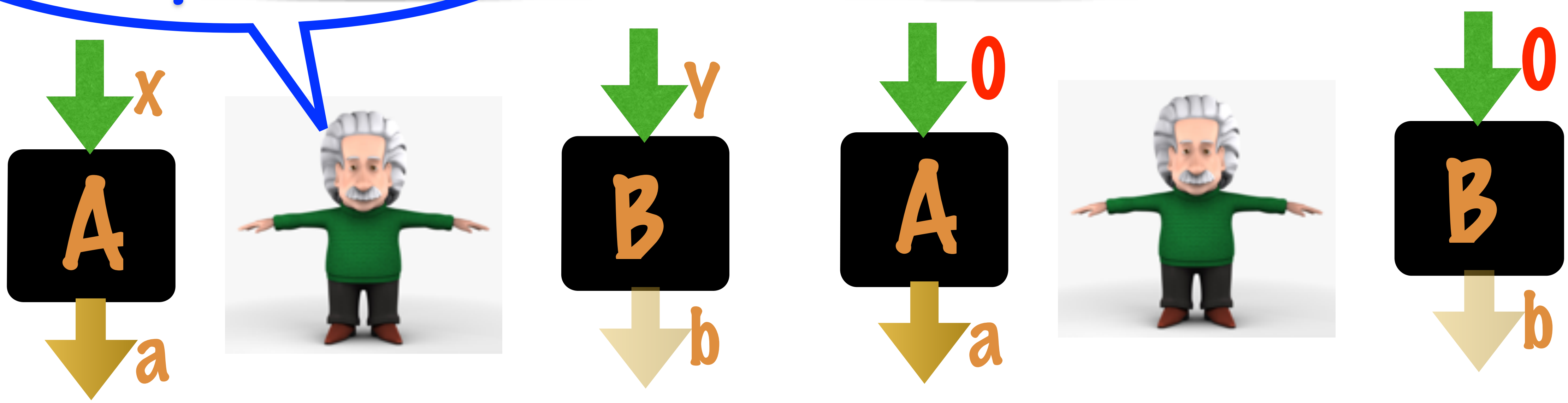

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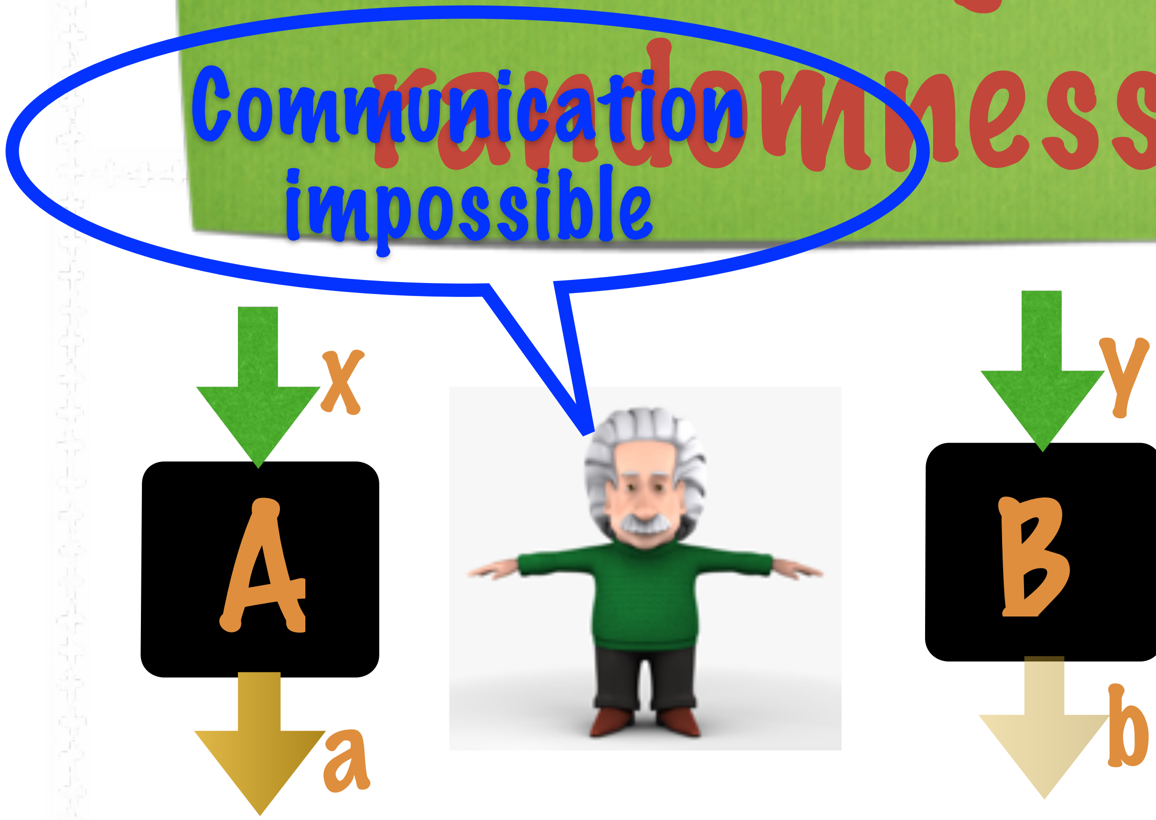

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- On uniformly random input, OPT quantum wins prob = .85 > OPT classical = .75

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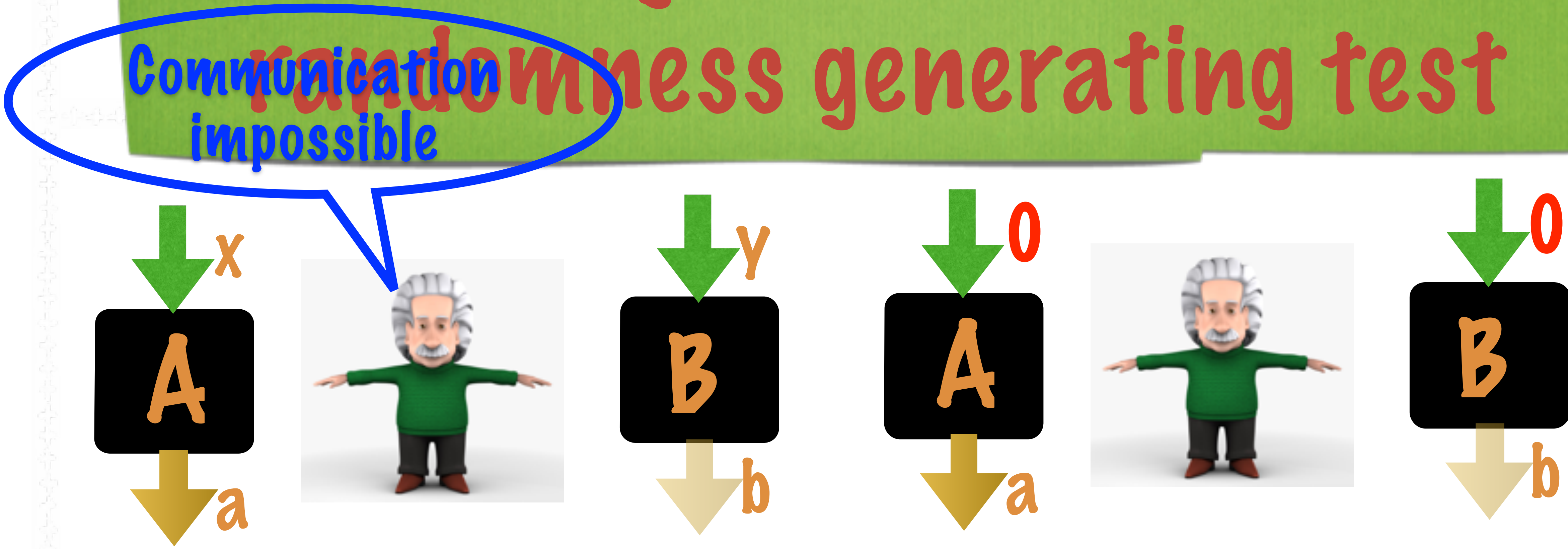

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- Each device receives a bit, outputs a bit
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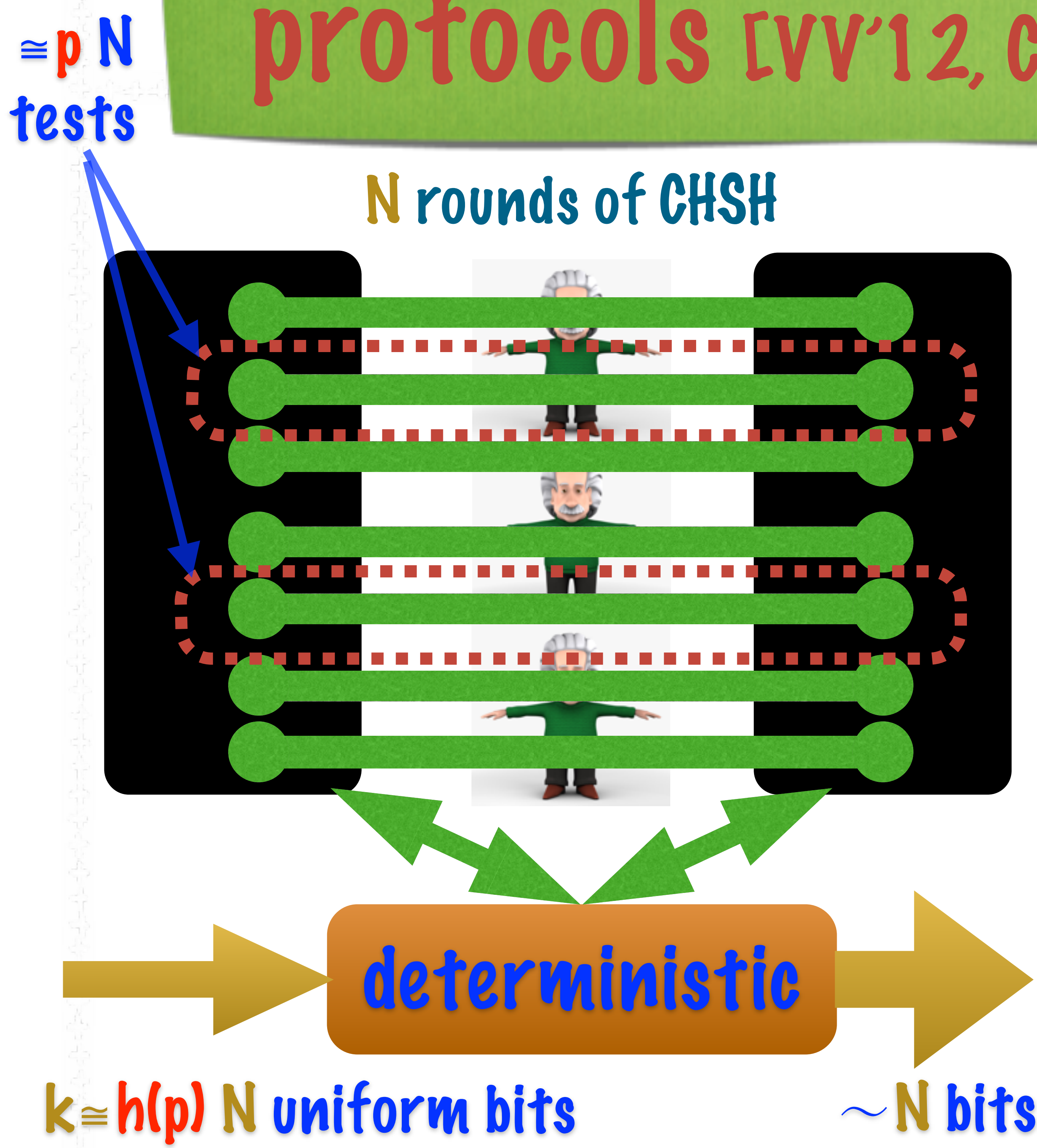
- On uniformly random input, OPT quantum wins prob = .85 > OPT classical = .75
- Bit a in OPT quantum uniform to the input+Adversary, on all inputs, including (0,0)

Seeded extraction: Spot-checking protocols [VV'12, Coudron-Yuen-Vidick'13]



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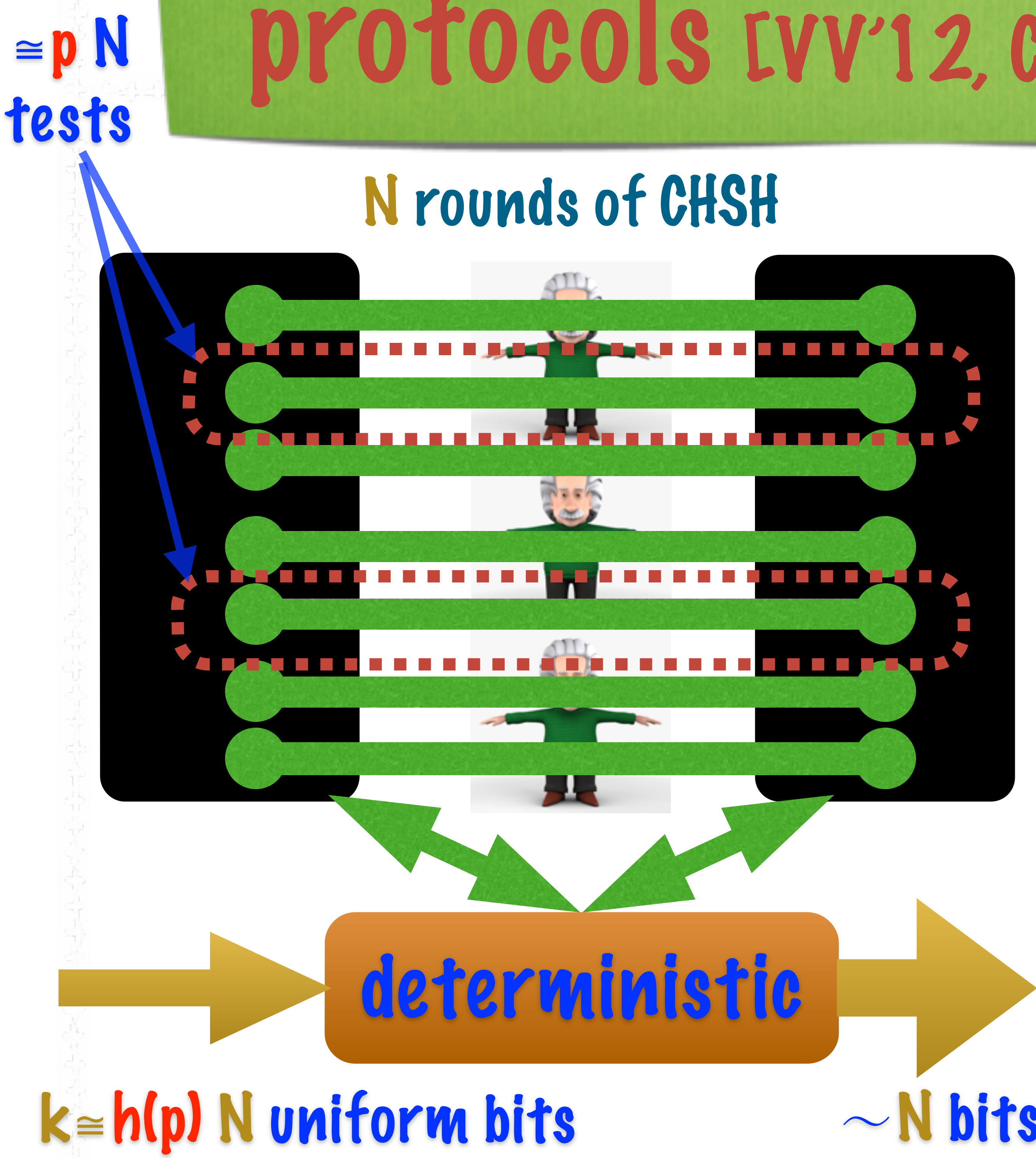

- Play N sequential CHSH

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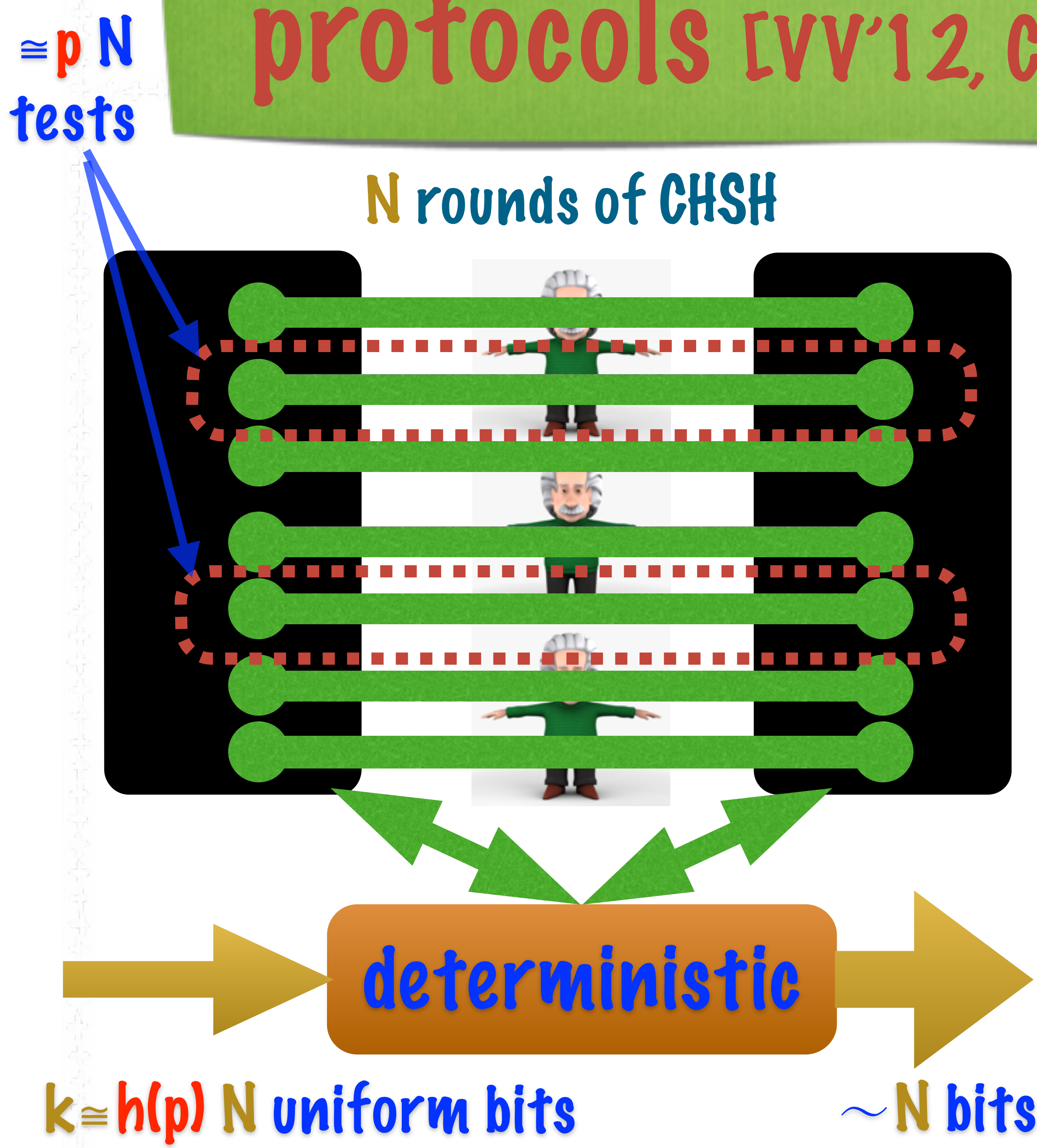

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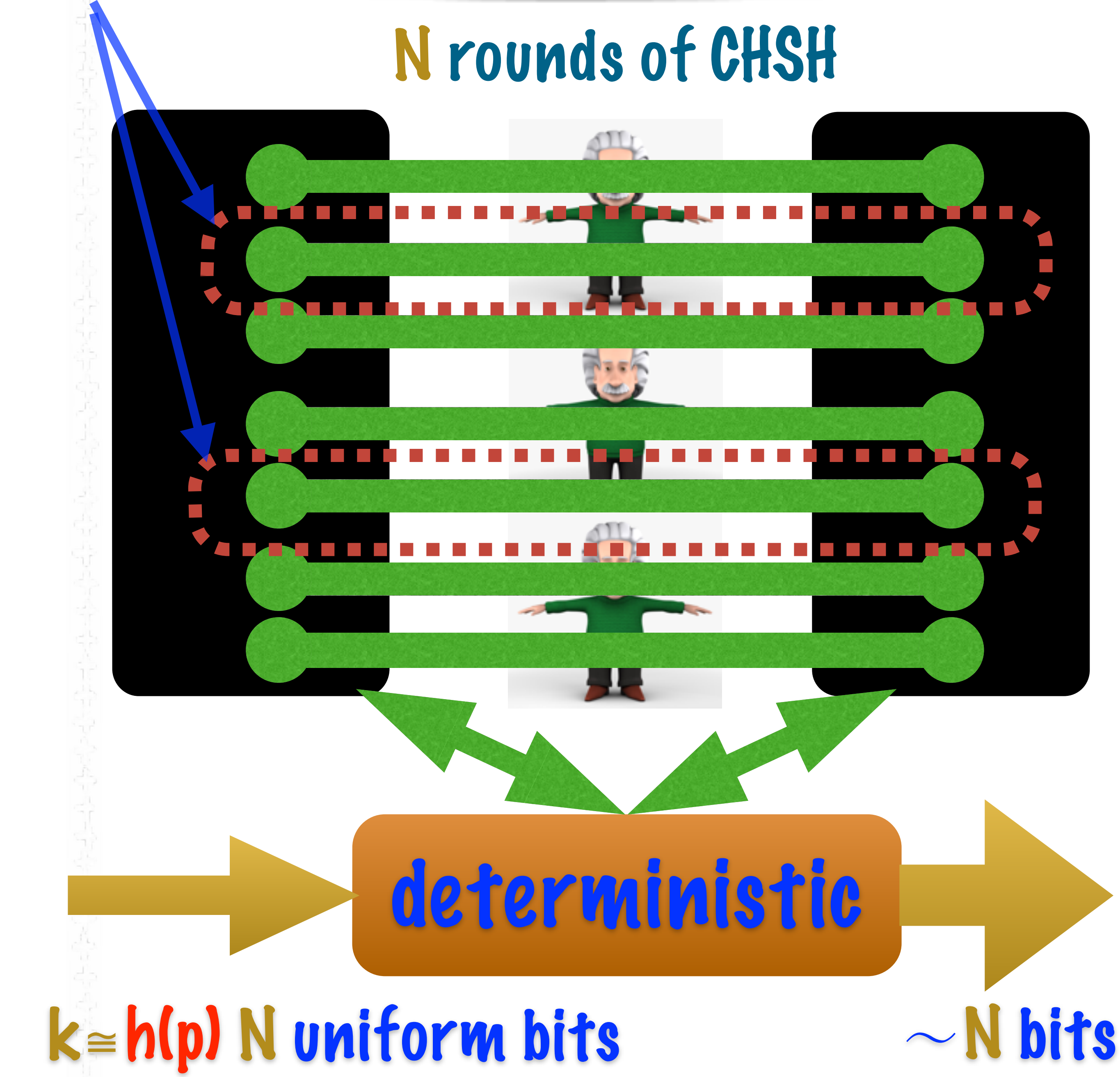
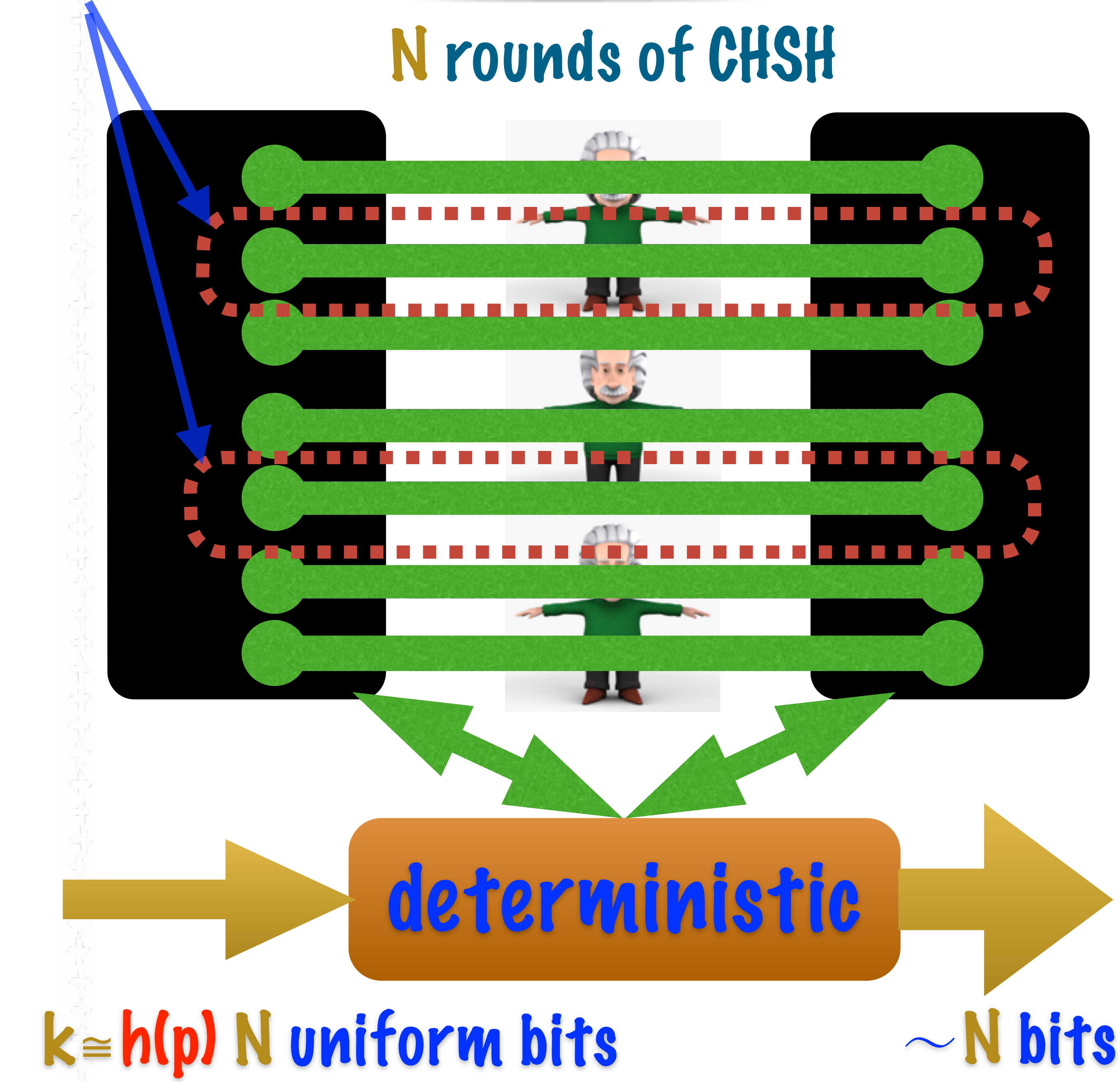


- Play N sequential CHSH
- Choose a small number of games for testing; others for randomness generation
    - CYV'13: test independently with a small probability p

Seeded extraction: Spot-checking protocols [VV'12, Coudron-Yuen-Vidick'13]



- Play N sequential CHSH
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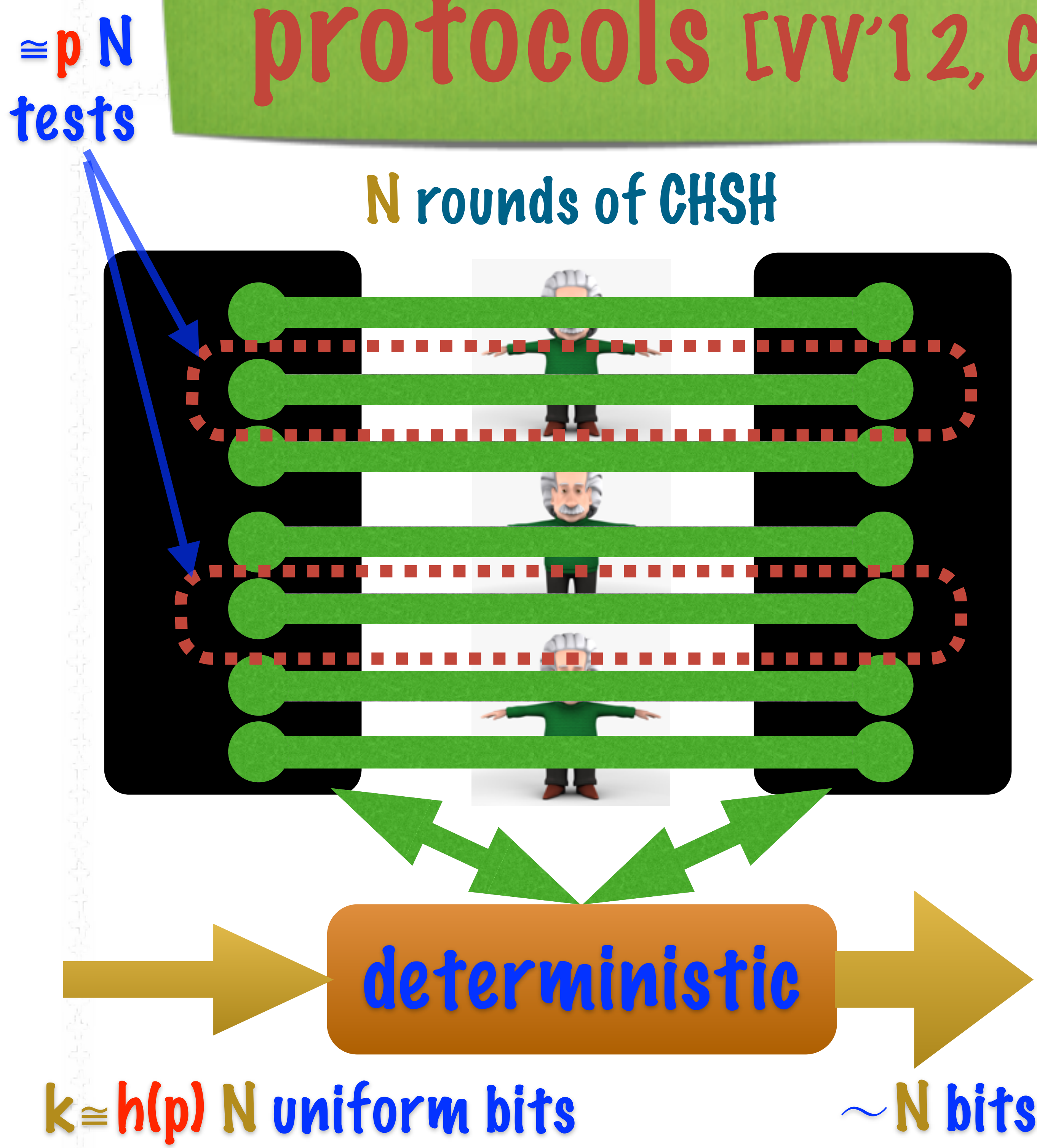

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Seedless extraction: Master protocol [Chung-Shi-Wu]

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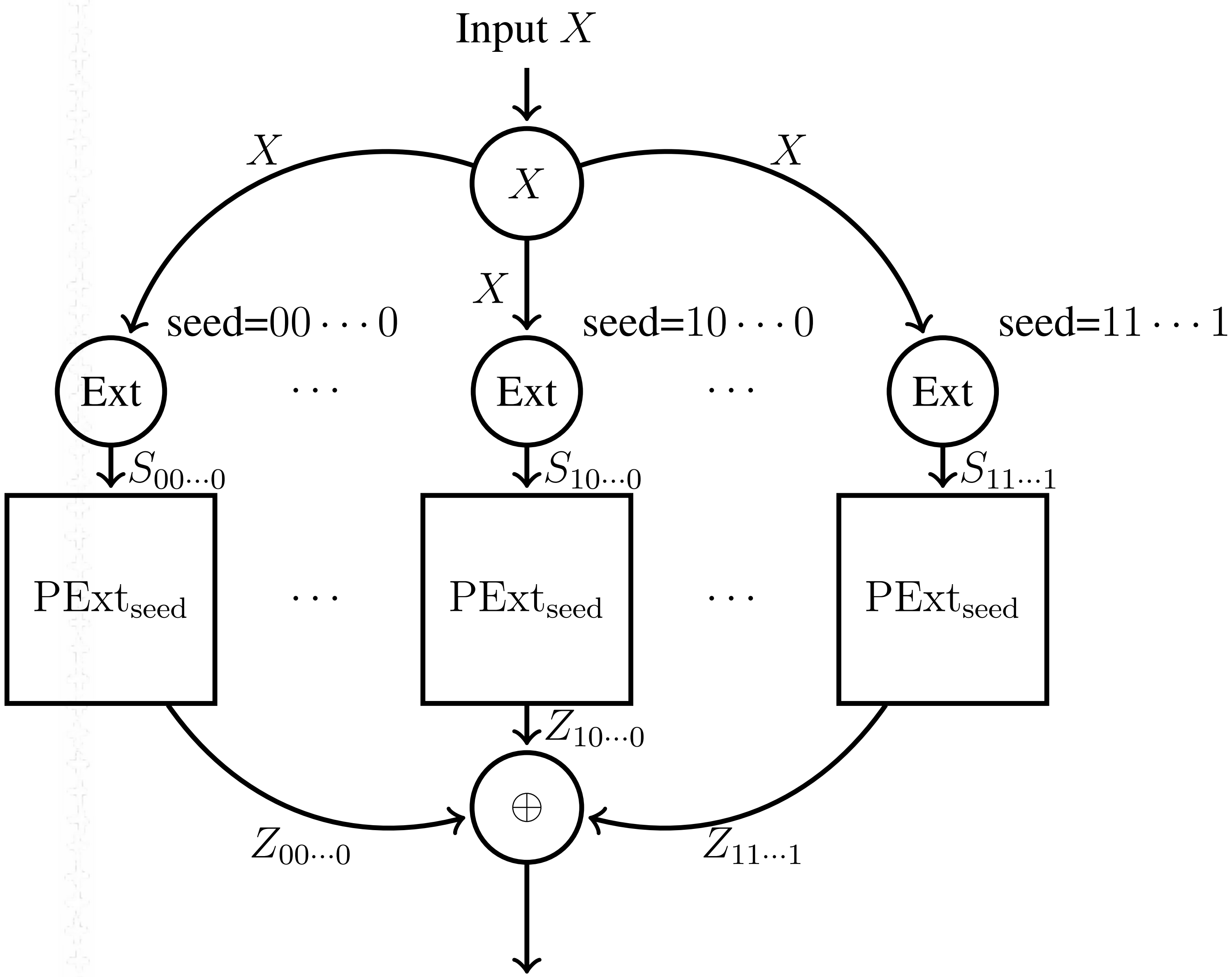

Output Z if no more than η fraction of $\mathrm{PExt}_{\mathrm{seed}}$ reject.

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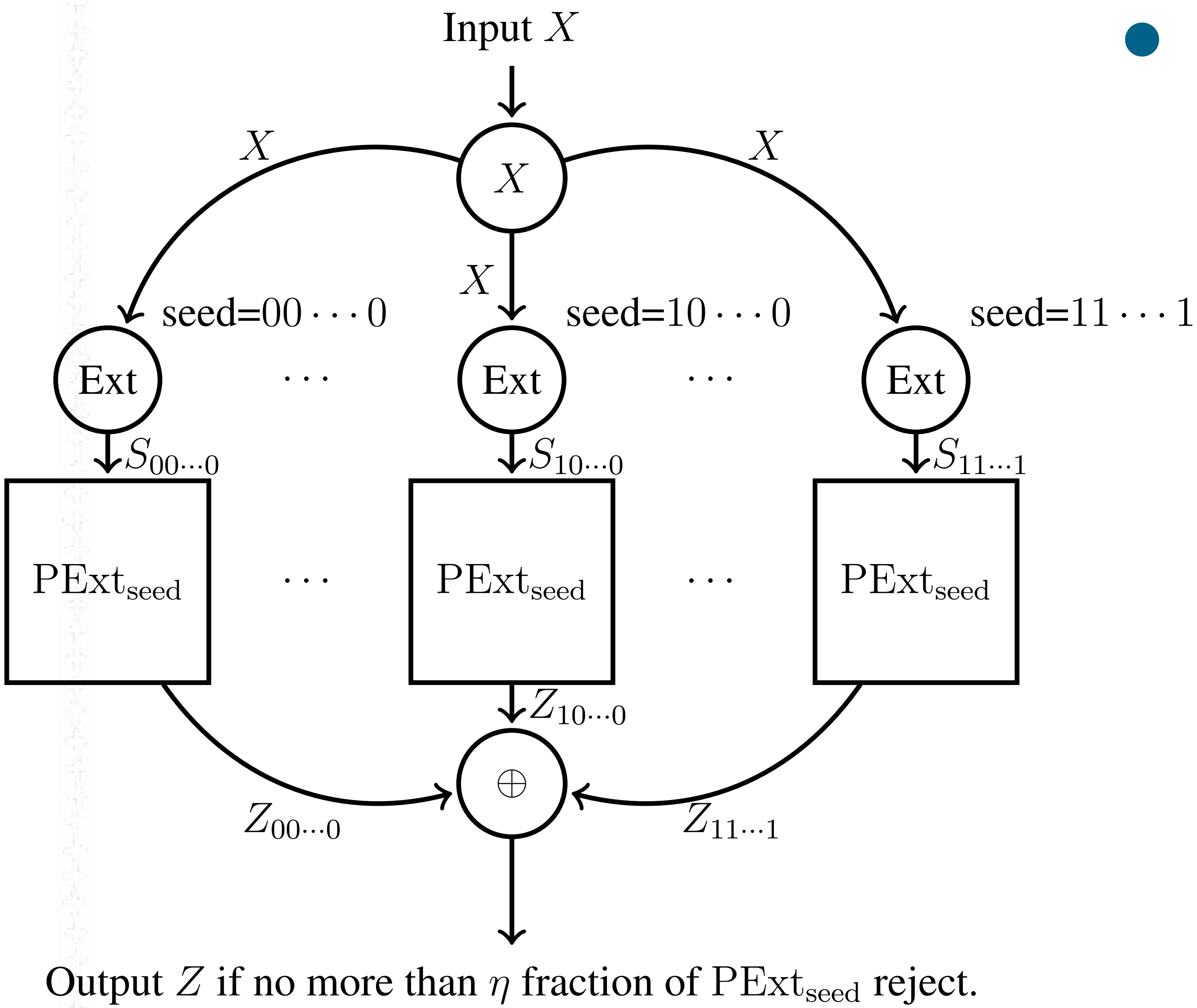

- Create "quantum somewhere randomness",

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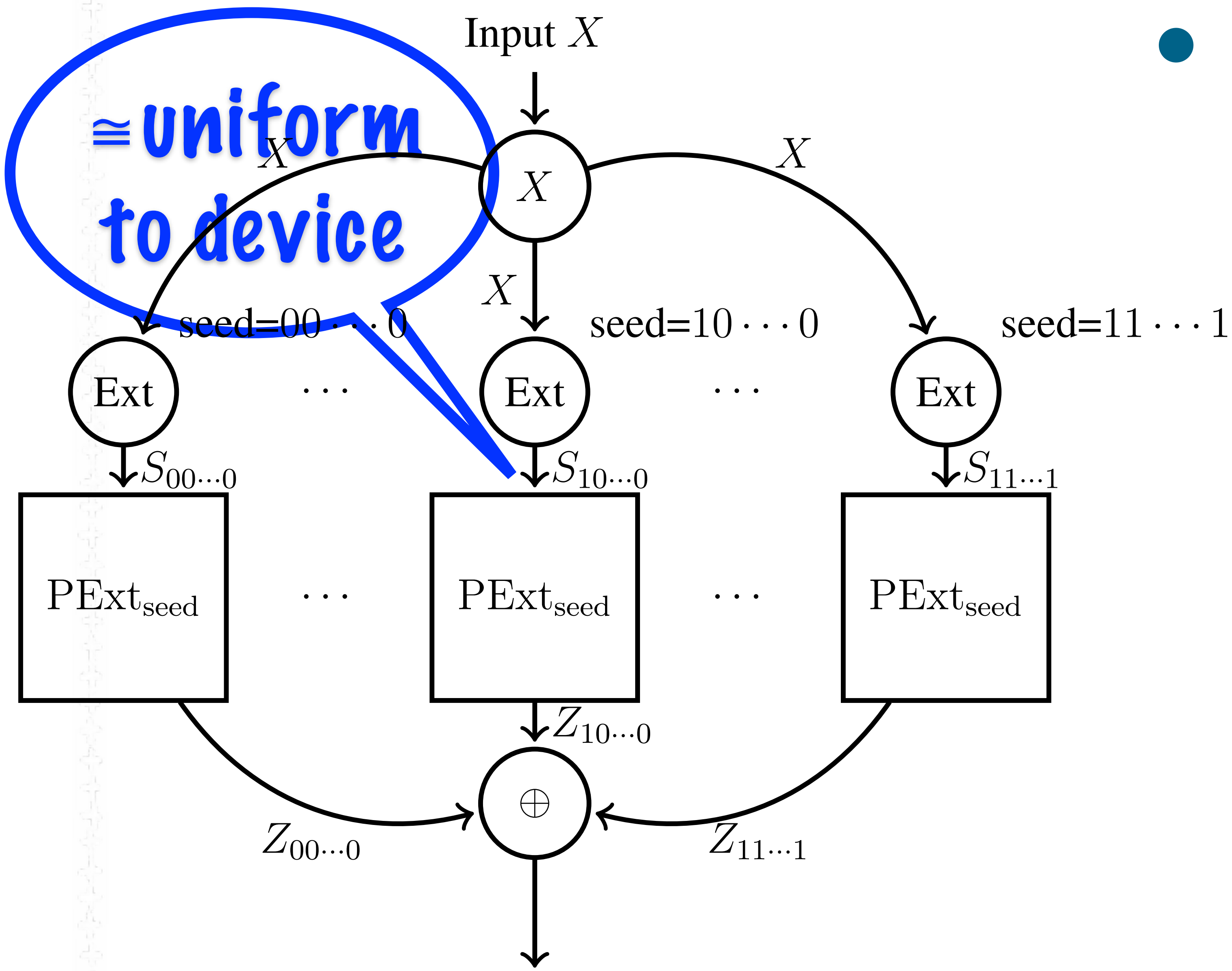

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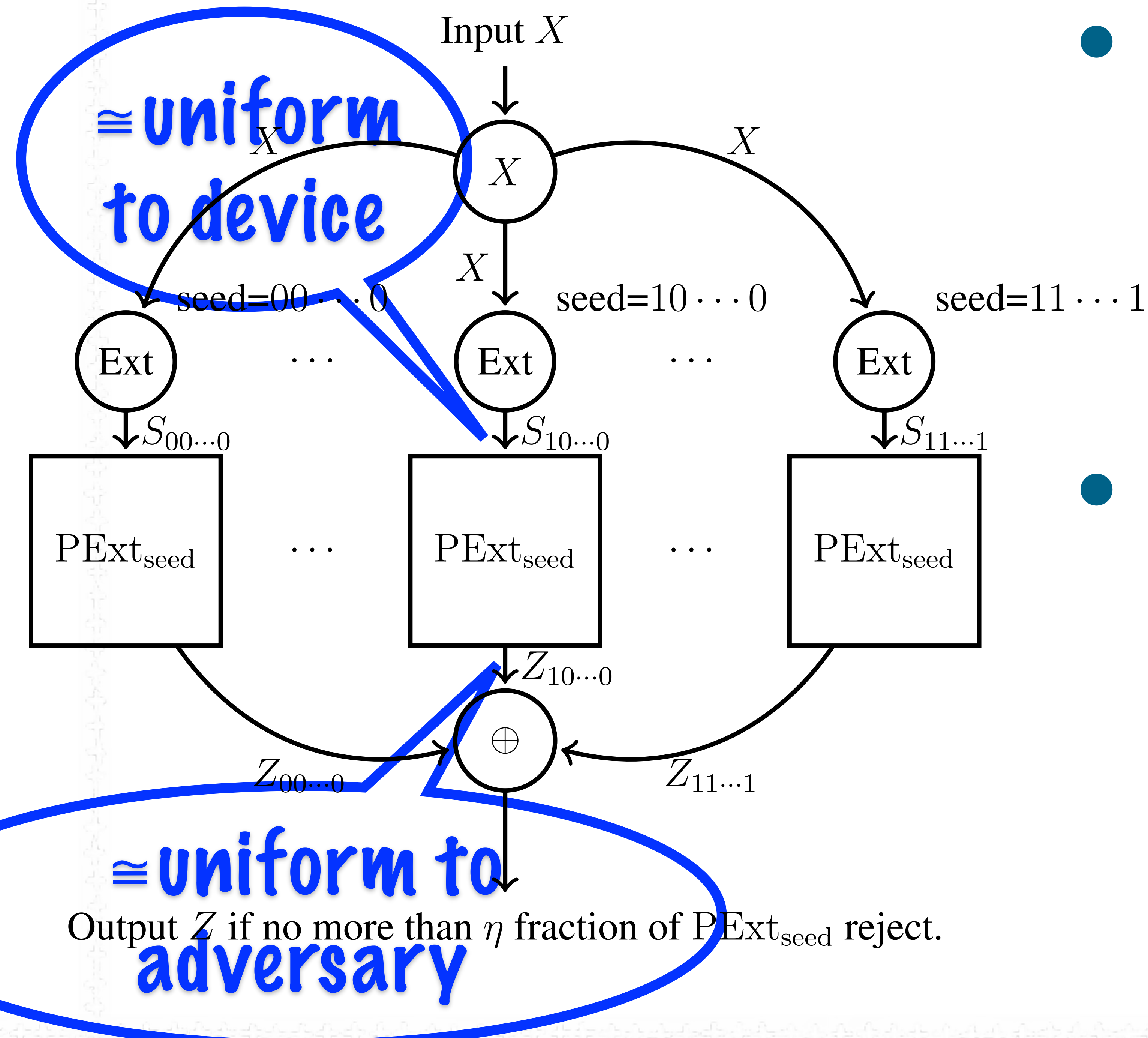

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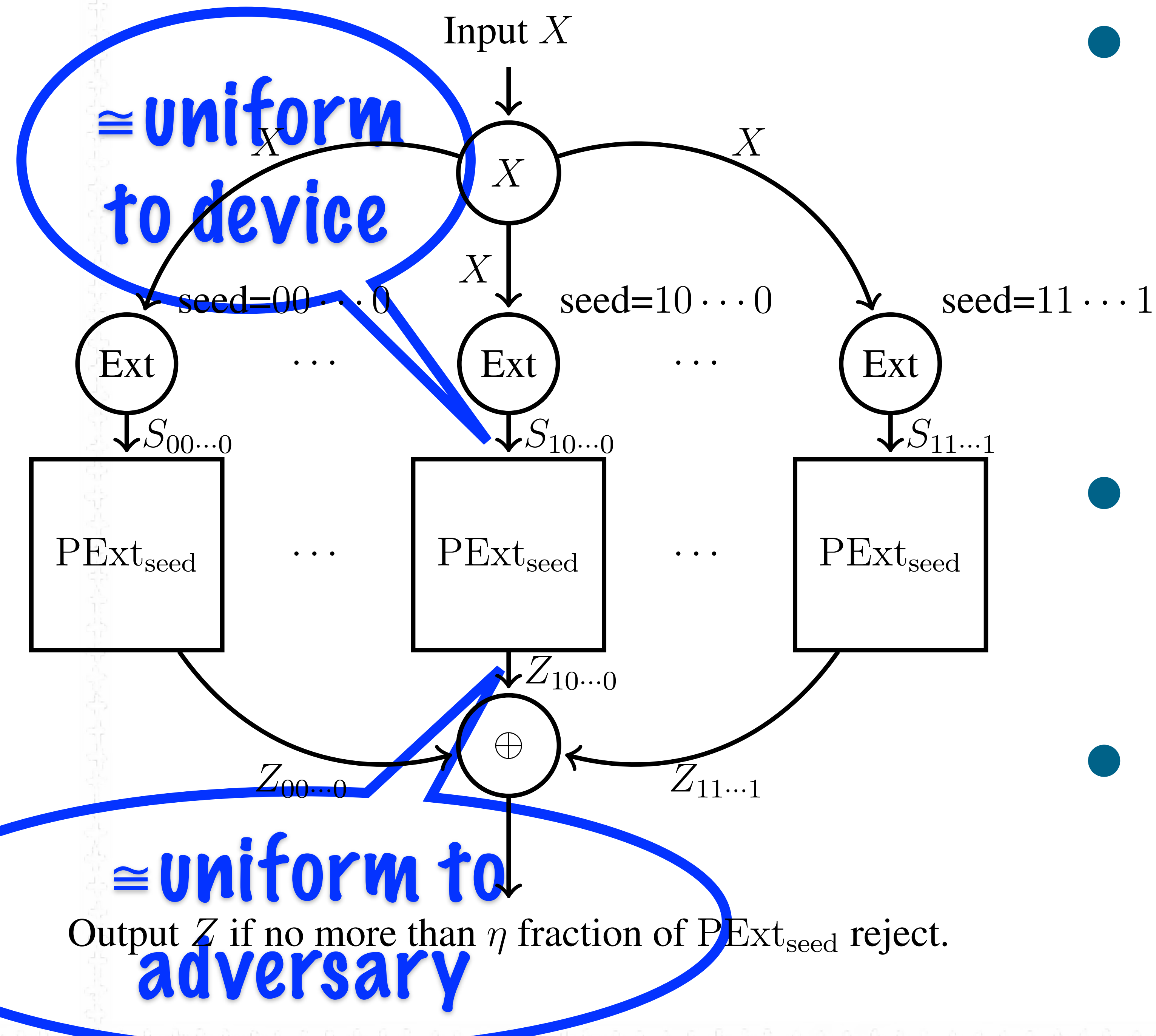

- Create "quantum somewhere randomness"
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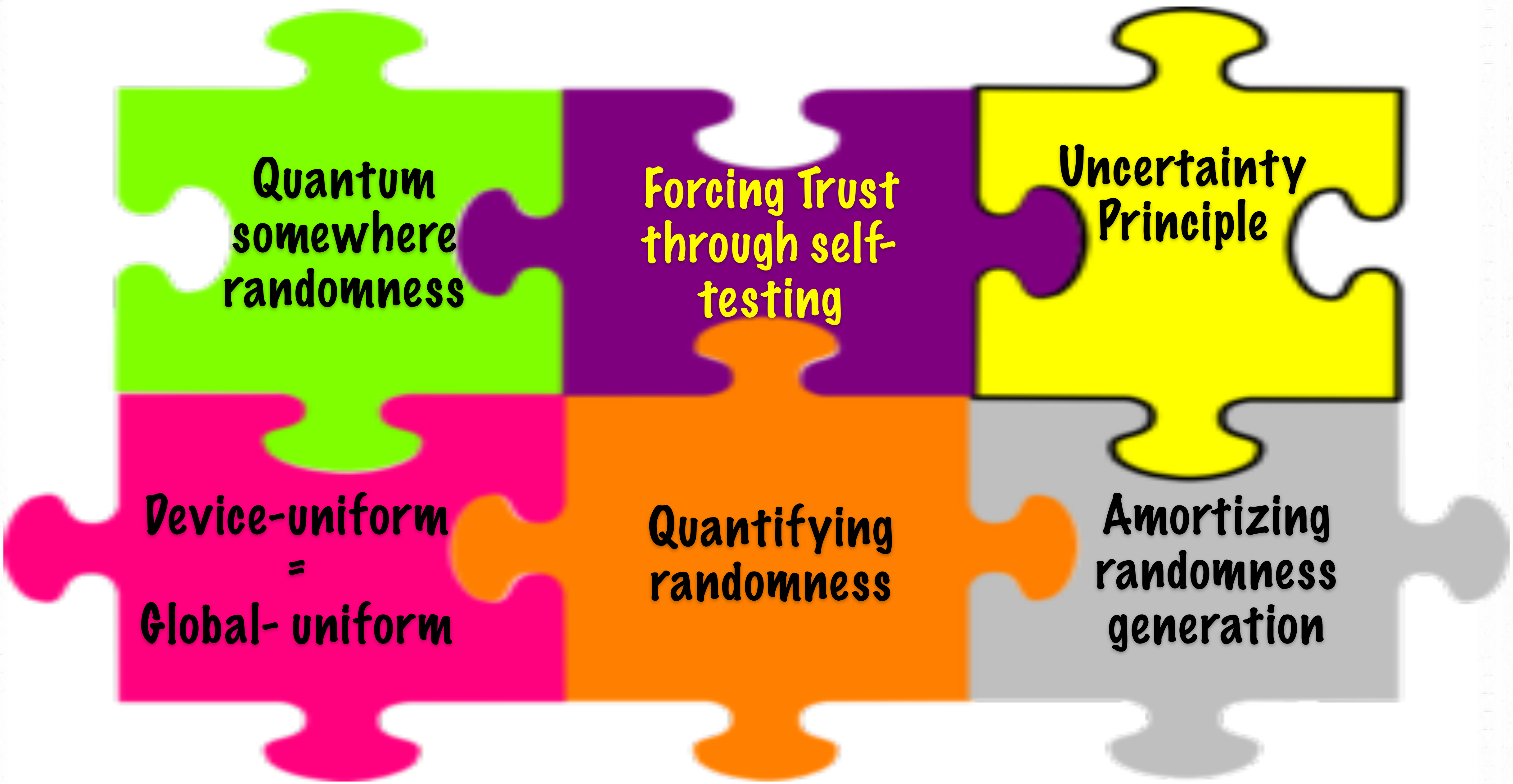
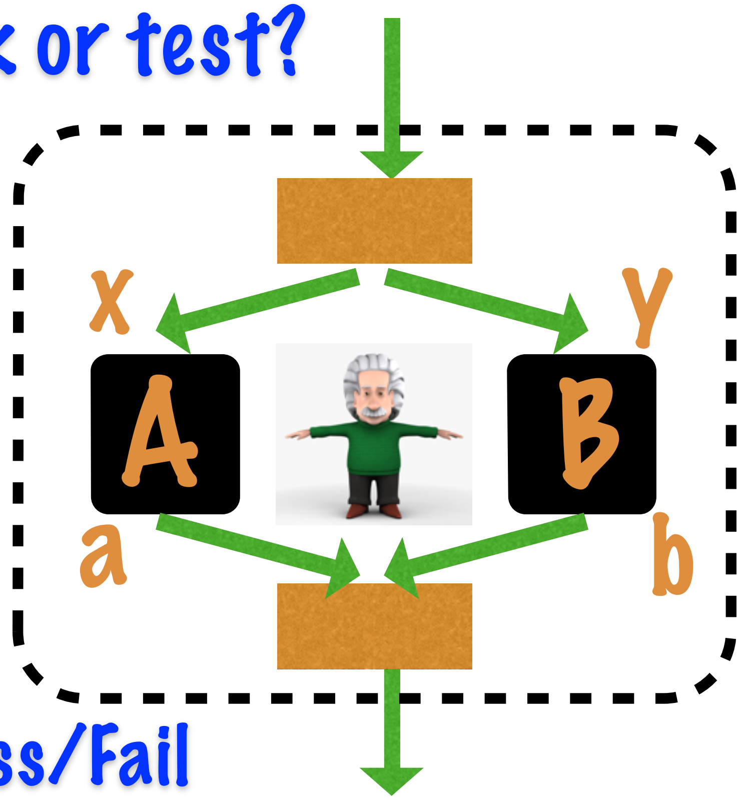
- Accept if the number of acceptance exceeds a threshold. XOR accepted outputs close to adversary random

Our techniques: many pieces to the puzzle...



Forcing trust on adversarial devices

work or test?



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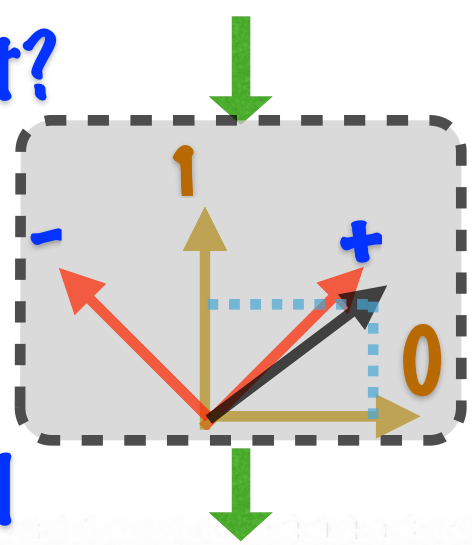

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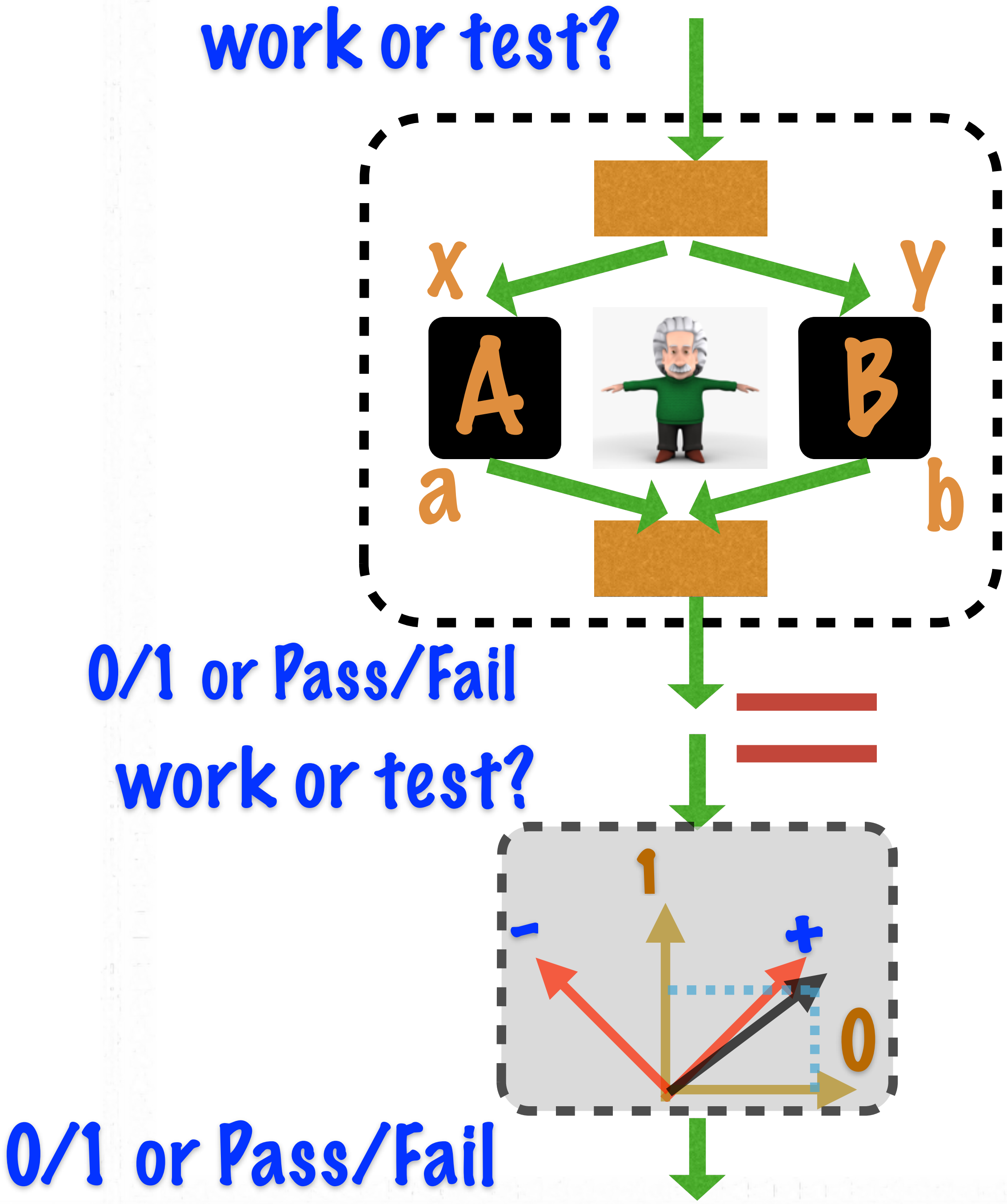

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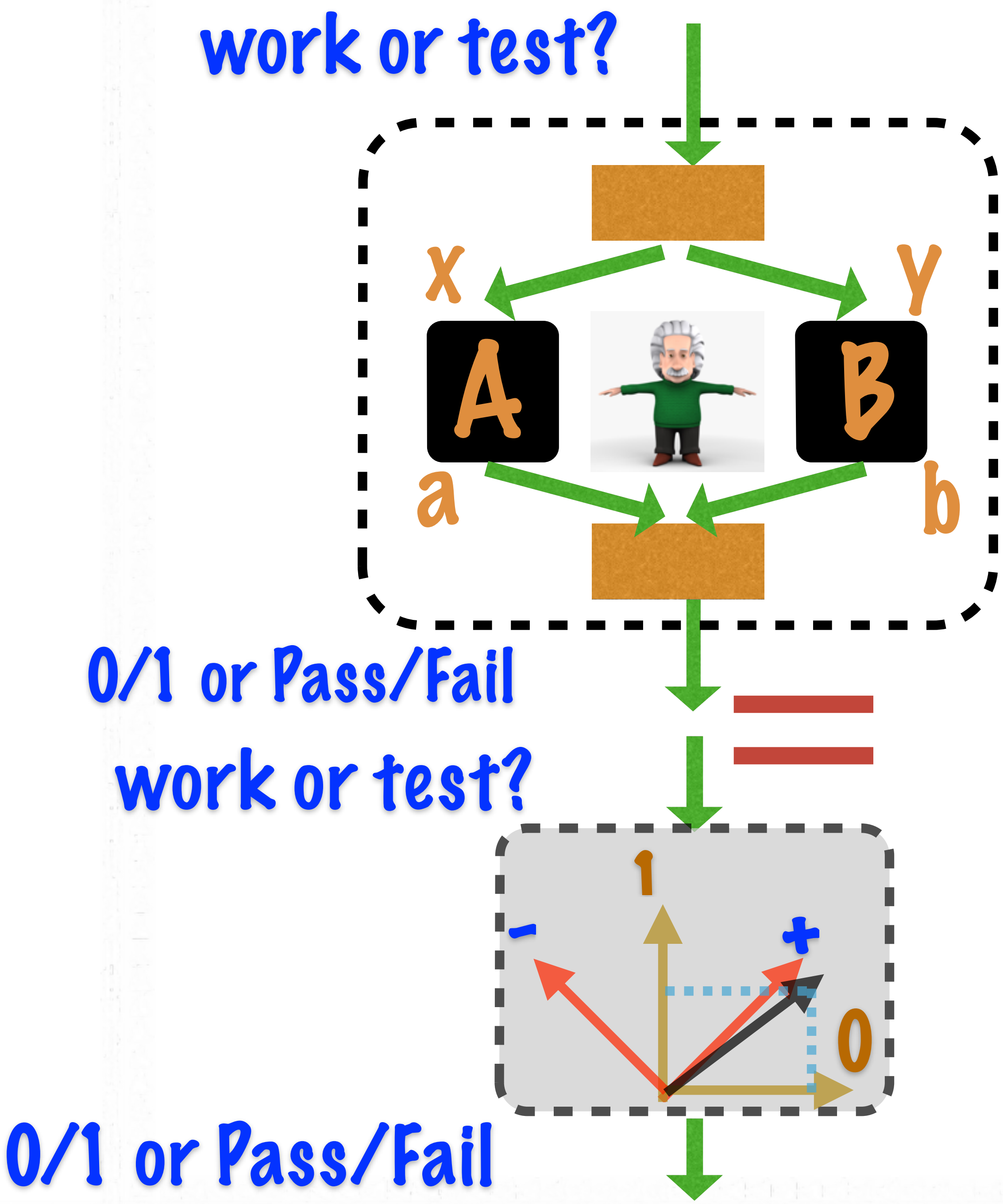

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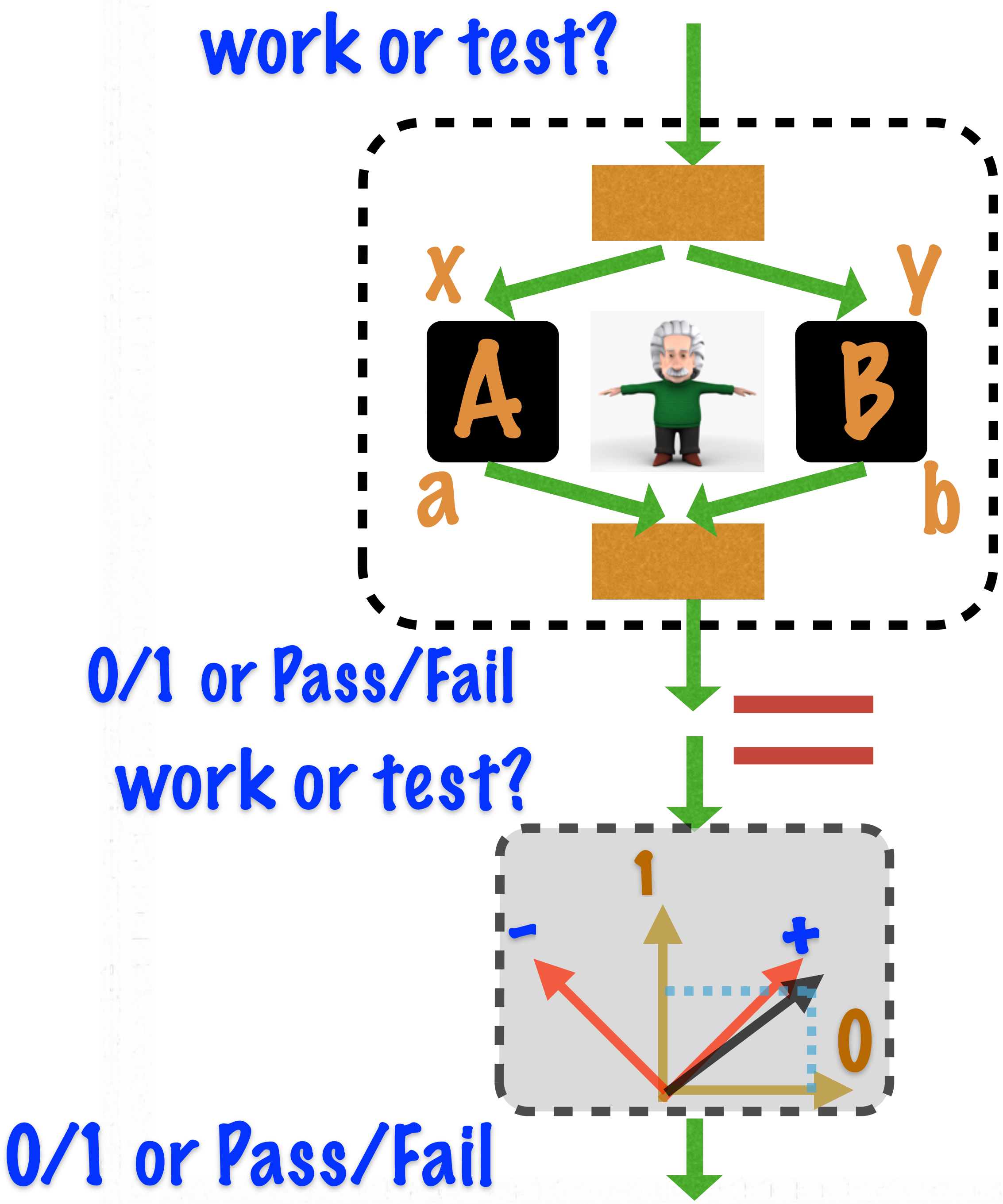

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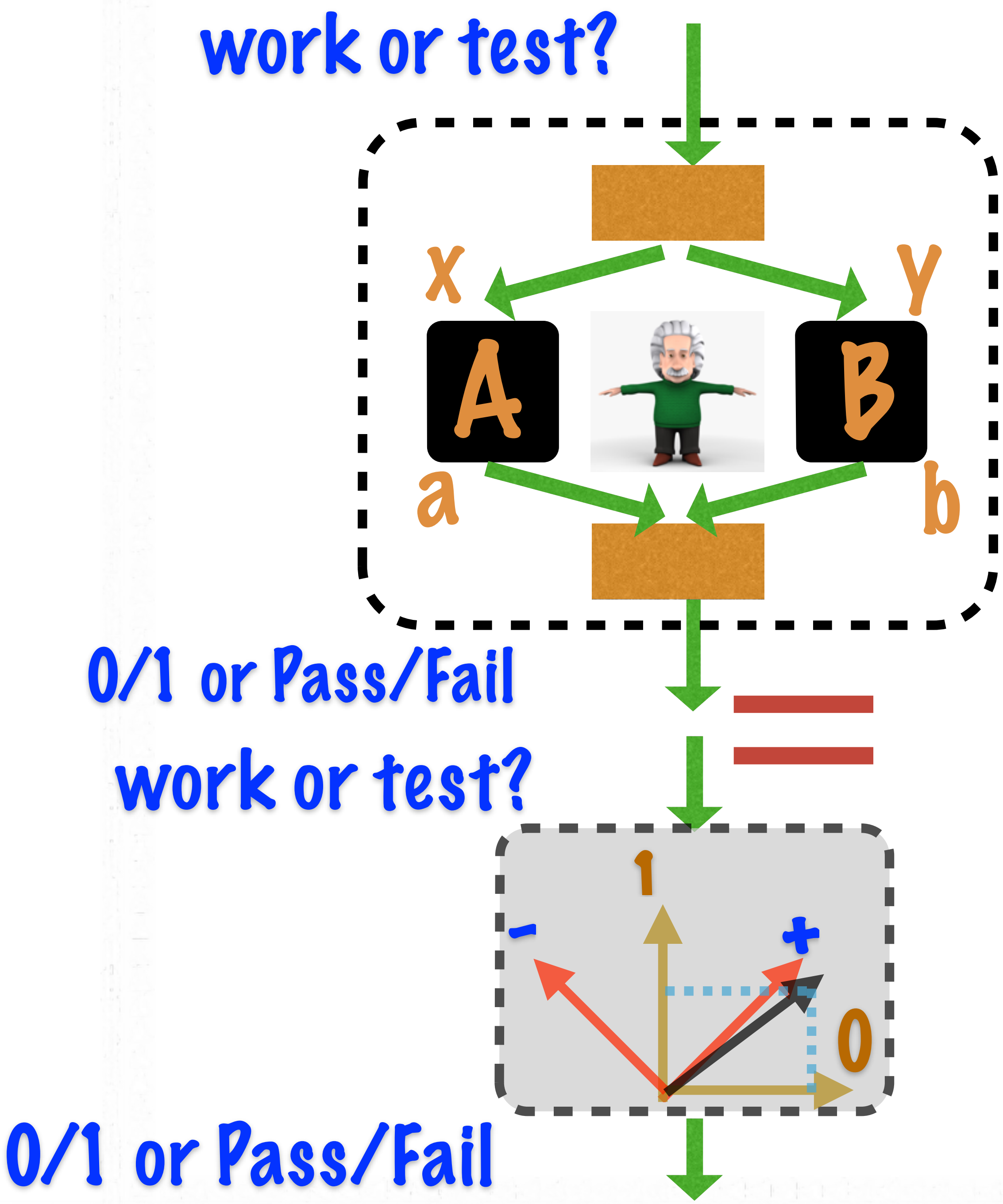

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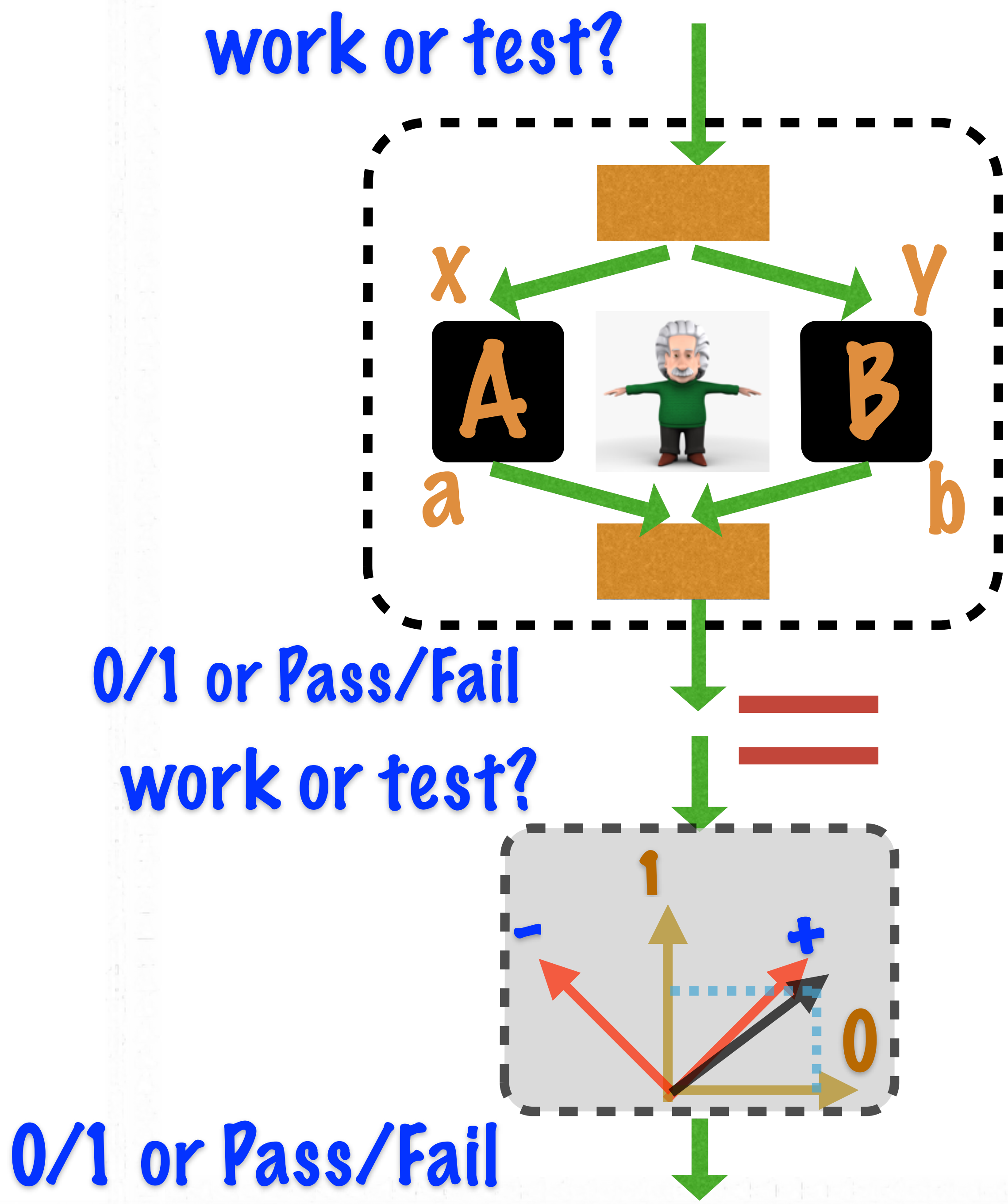

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- Creating uniform input for seeded extractor: somewhere randomness from quantum-proof extractor
- Security in composing seeded protocols: Equivalence Lemma

Method::seeded extraction

4. Open Problems

“Randomness capacity” of untrusted-devices

- What quantifies the maximum amount of extractible randomness from (non-communicating) untrusted devices?
  - All published proofs require linear amount of entanglement
  - Is entanglement really needed?!

Maximum noise tolerable: the boundaries between quantum-classical-no security

- What is the maximum level of imperfection allowed for ensuring quantum security?
  - Trivial upper bound: quantum-classical gap
  - Another trivial but better (?) bound: quantum - OPT when output is deterministic
- Is there a range of noise values that provide classical security but not quantum security?

Maximum output bit rate under noise

- A more quantitative version of the previous question; important for practical use
- Two ways to improve the rate under noise based on Miller-Shi
  - Improve the trust coefficient
    - Method for computing the optimal trust coefficient?
  - Improve the Schatten norm uncertainty principle

Open problems

What are the most general class of games allowed?

- Anything having a quantum-classical gap?
  - Kochen-Specker games?

Minimum device number for unbounded expansion

- What is the minimum number of devices required for unbounded expansion?
  - MS+CSW: <= 4
  - 3?
  - 2?
  - 3 for constant noise, 2 for almost perfect devices?

Minimum device number for seedless extraction

- What is the minimum number of devices that can be used to extract from all (n, k) sources with a desired ϵ error?
  - CSW's upper bound >= poly(n/ϵ)
  - Could it be polylog(n/ϵ) or even constant?
  - Possibly no...
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Optimal quantum-proof classical extractors

- What is the shortest seed length allowed for a quantum-proof classical extractor?
  - As a function of the source, output, and error parameters
  - Trevisan's extractor [De et. al.'12]: $\log^2(n/\epsilon)\log(m)$
  - Just $O(\log(n/\epsilon))$?

Open problems

Perfect Physical Extractor?

- A perfect physical extractor?
  - Optimizing all parameters simultaneously or necessary tradeoffs?

A general principle translating classical security to quantum security?

Open problems

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Thanks!

Open problems

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Questions?

Forcing trust on adversarial devices

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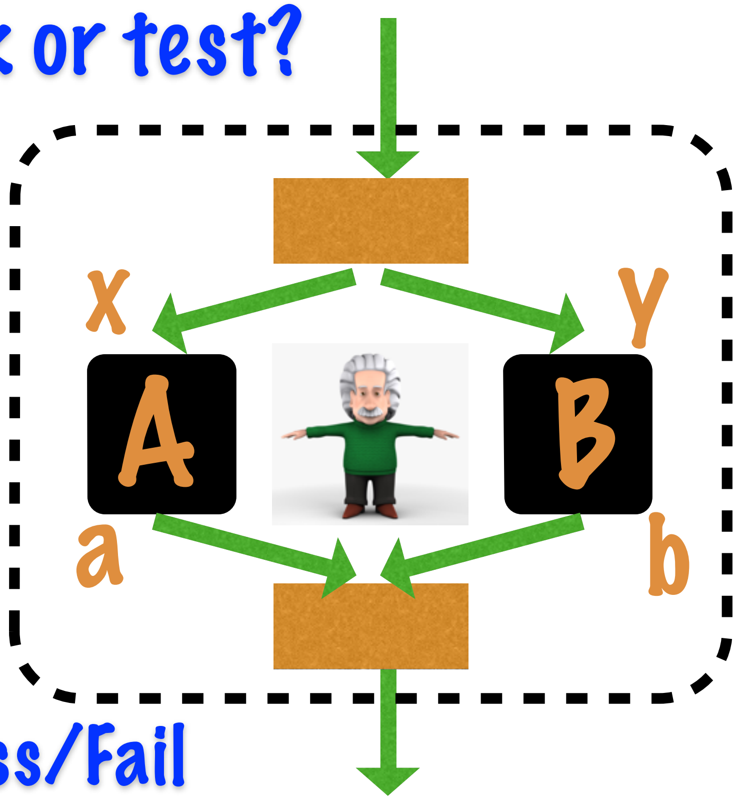

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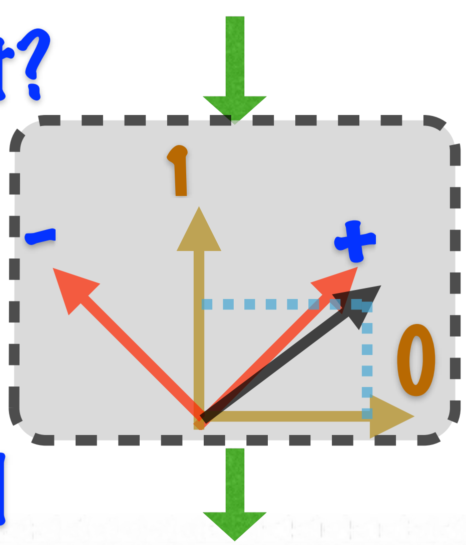

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Method::seeded extraction

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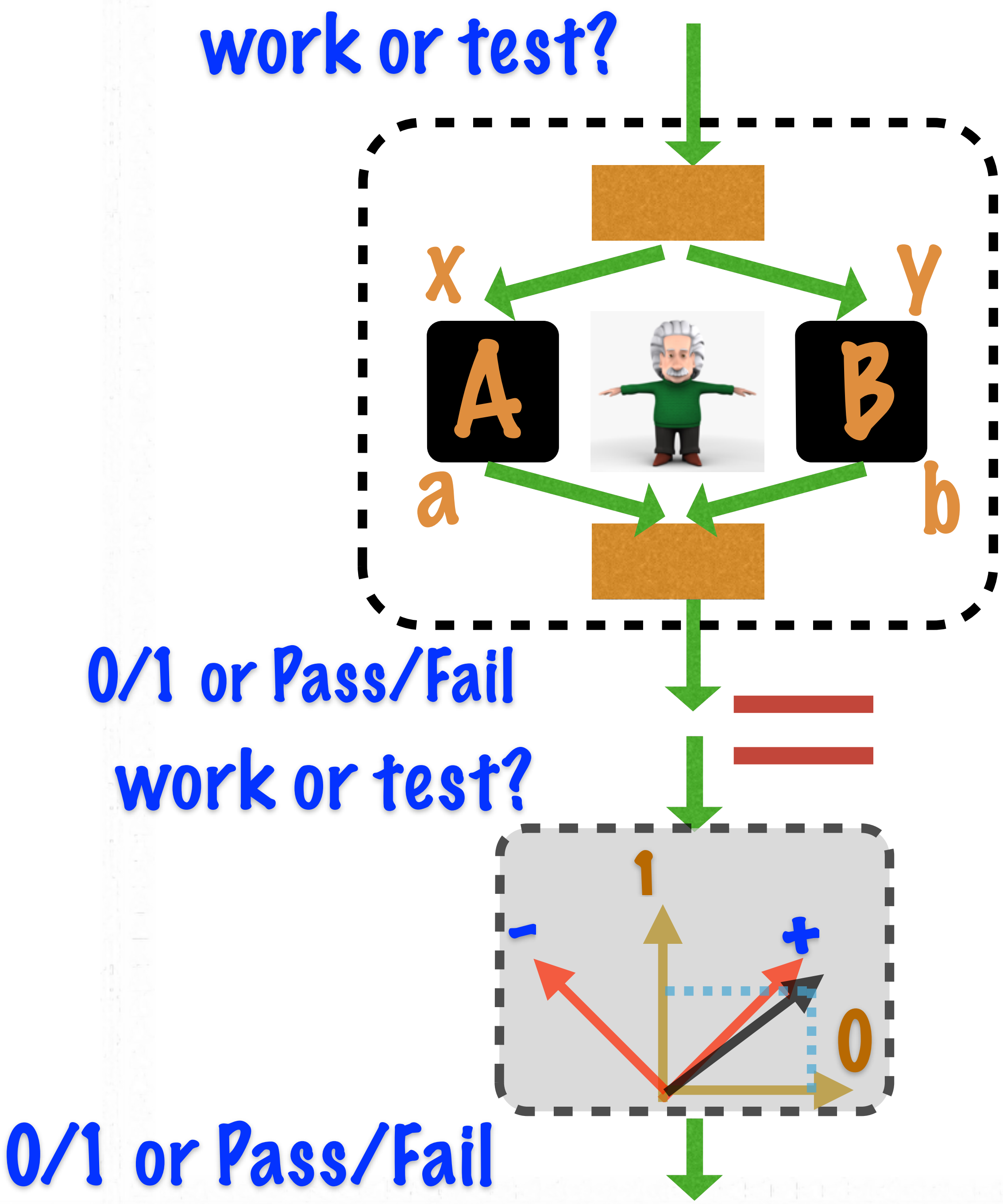

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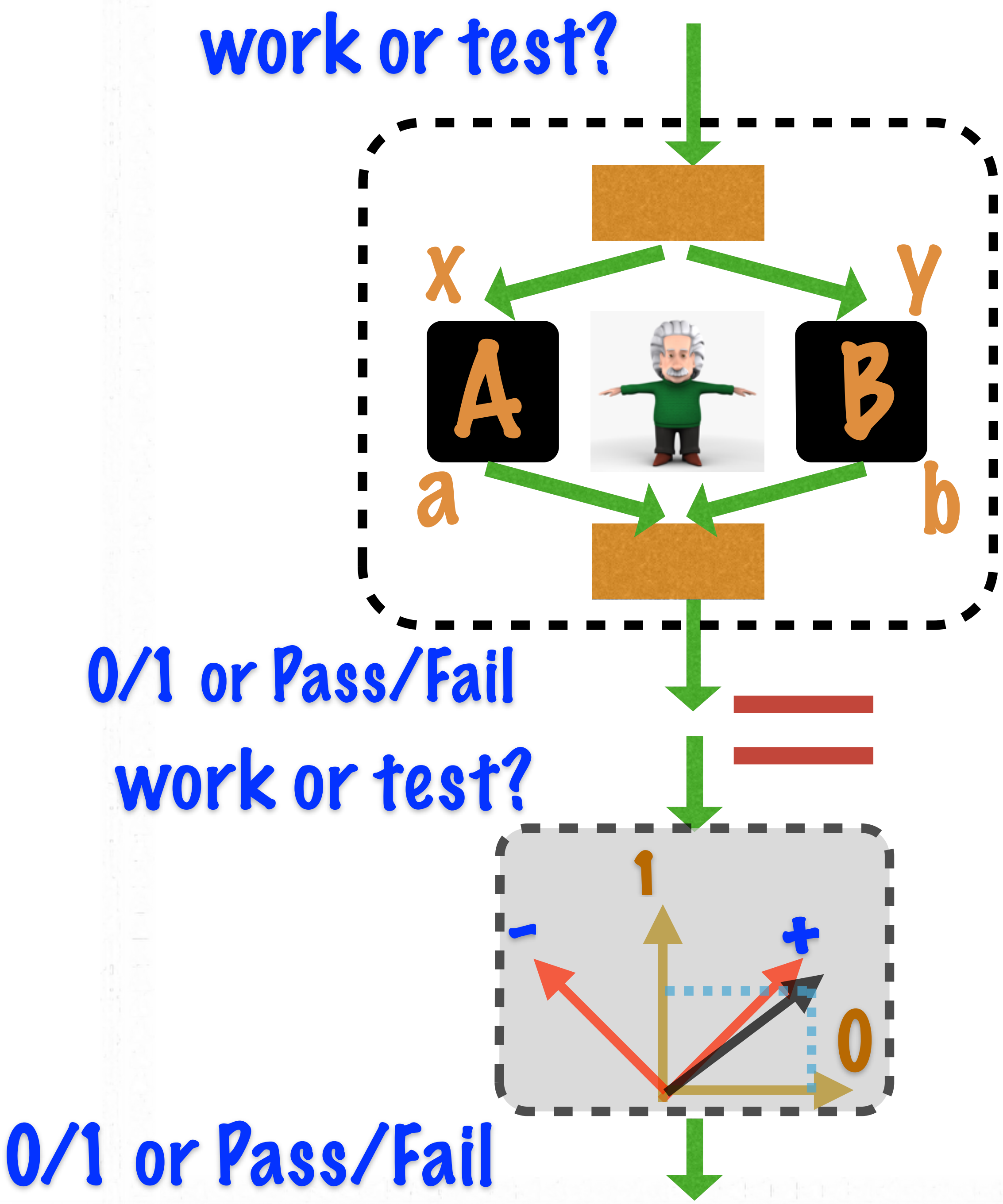

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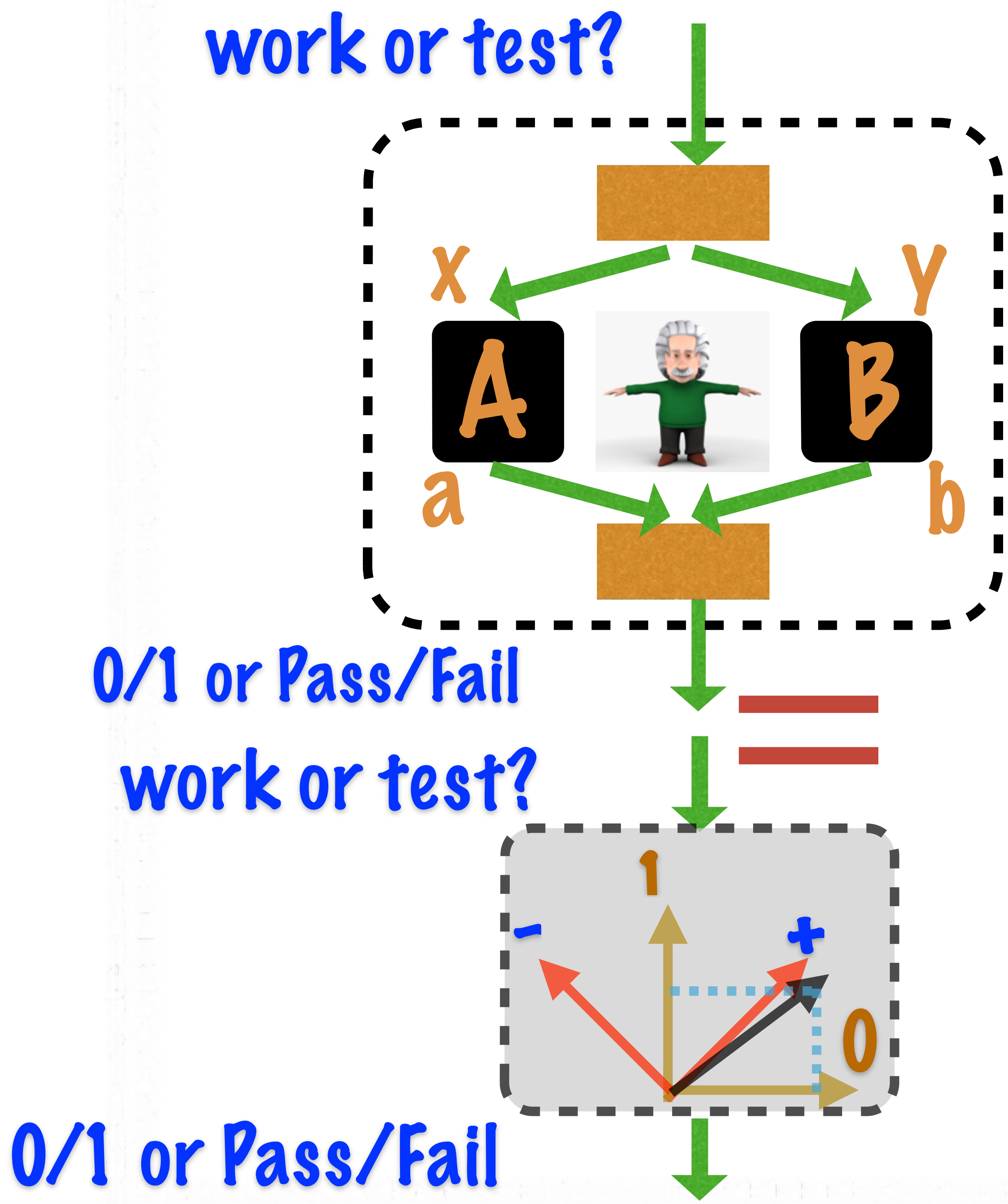

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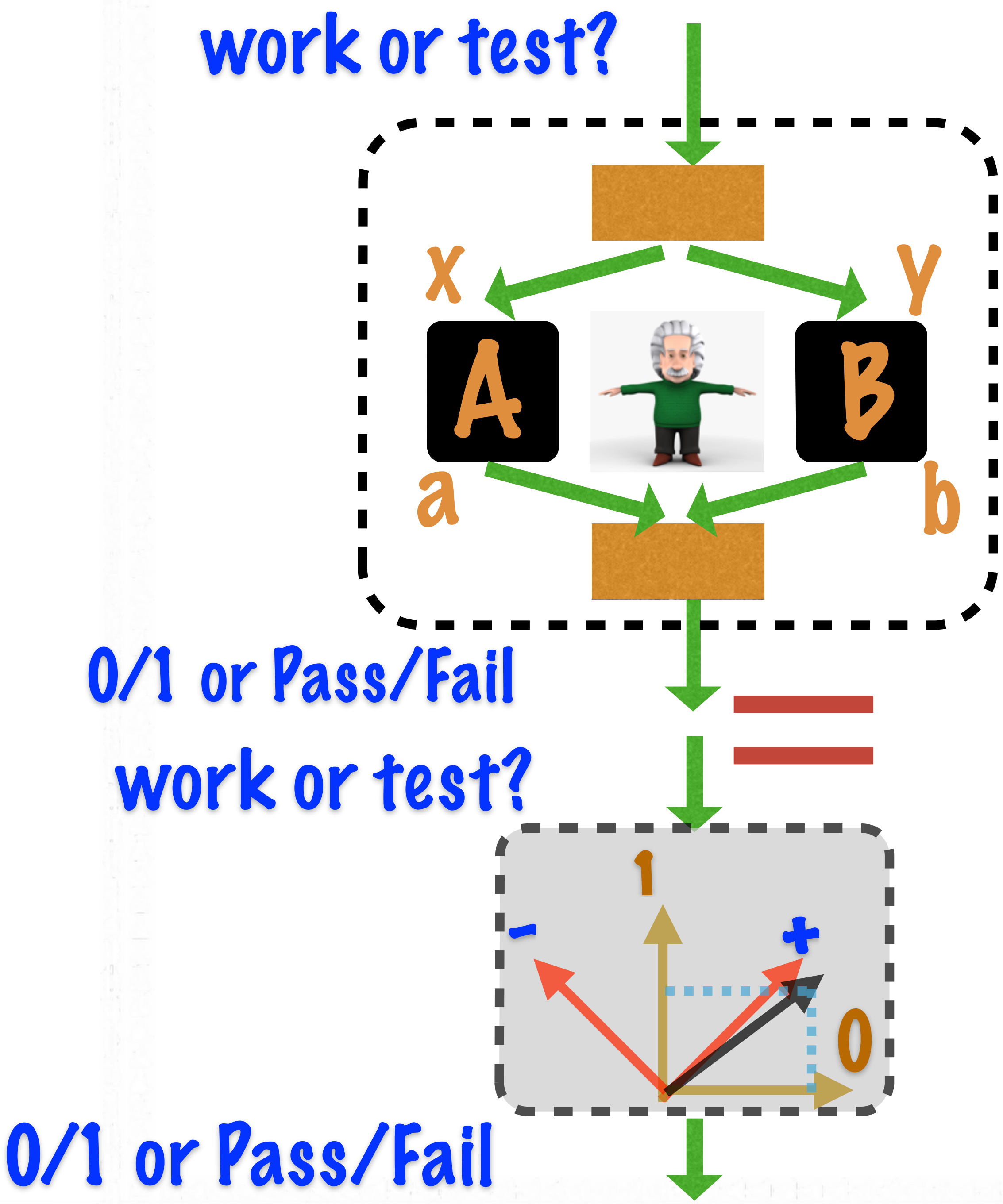

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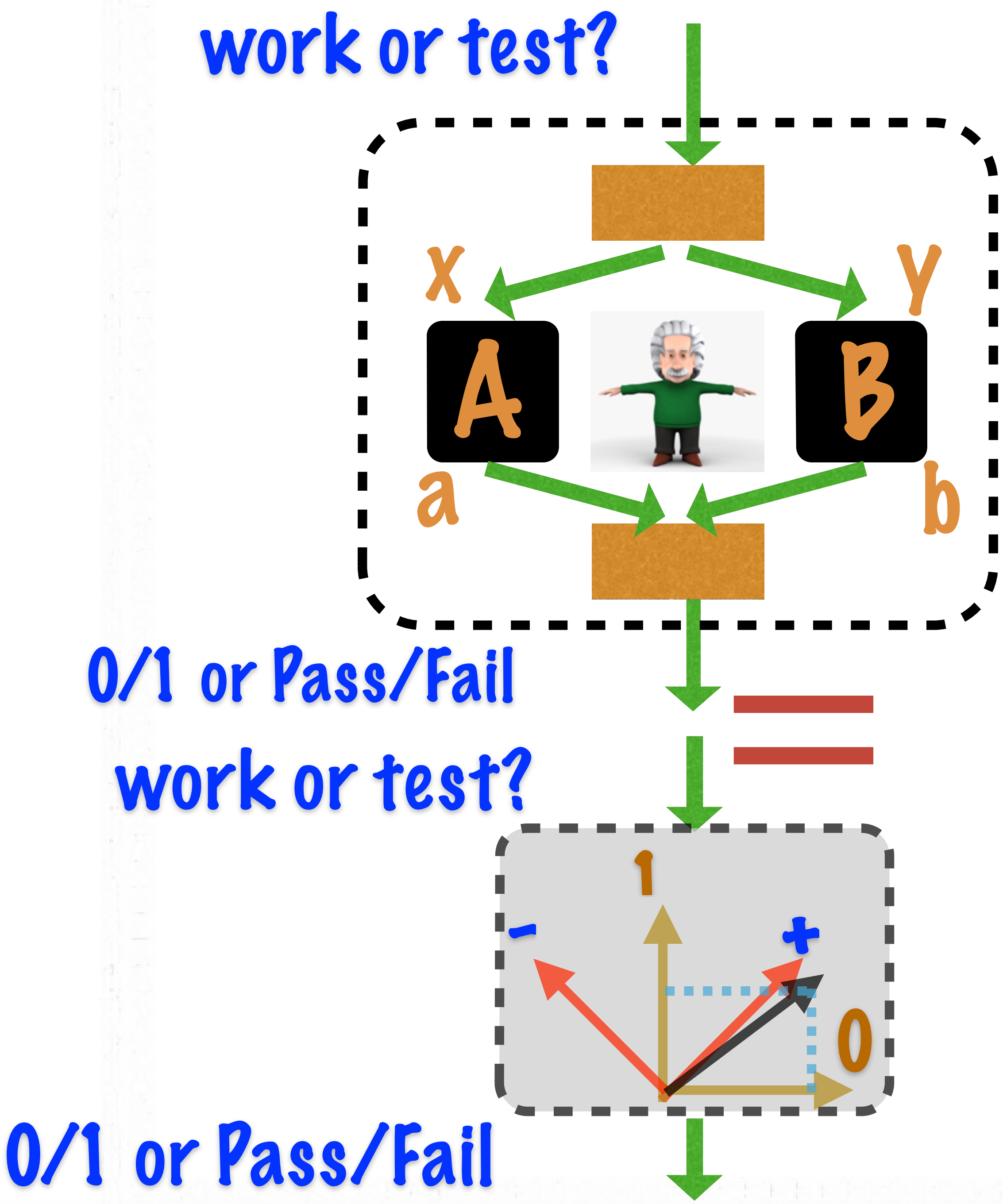

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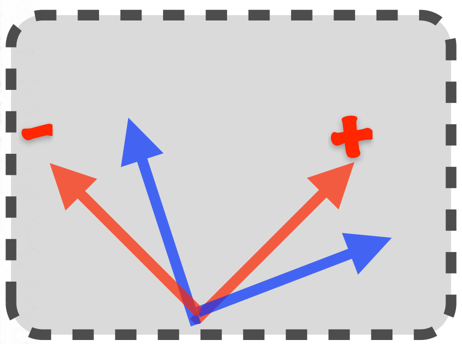
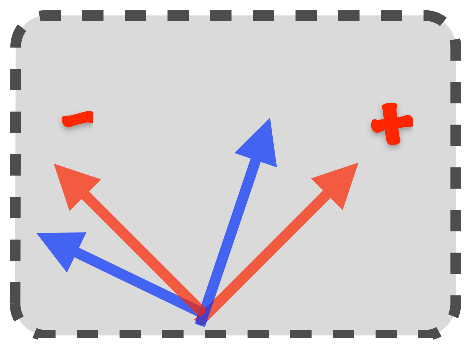
Forcing trust: the case of CSHS

- On 0 measure M, on 1 measure N
- Simplify and normalize





$$M_A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad M_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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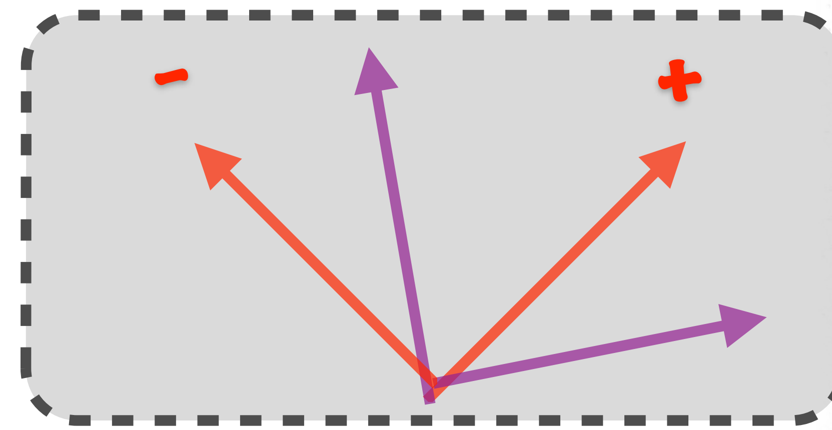

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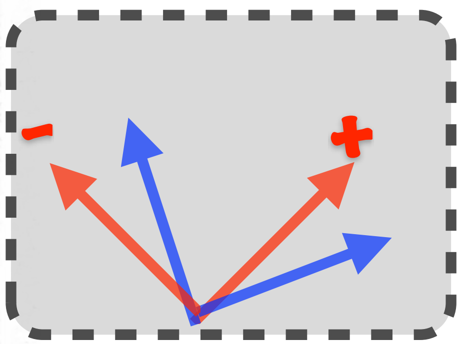

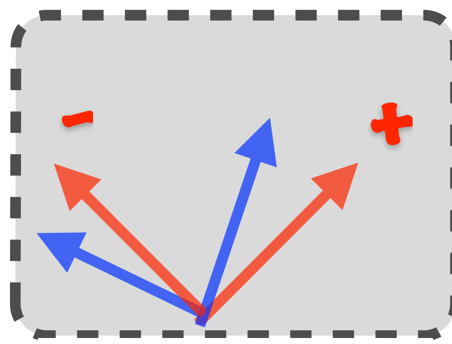

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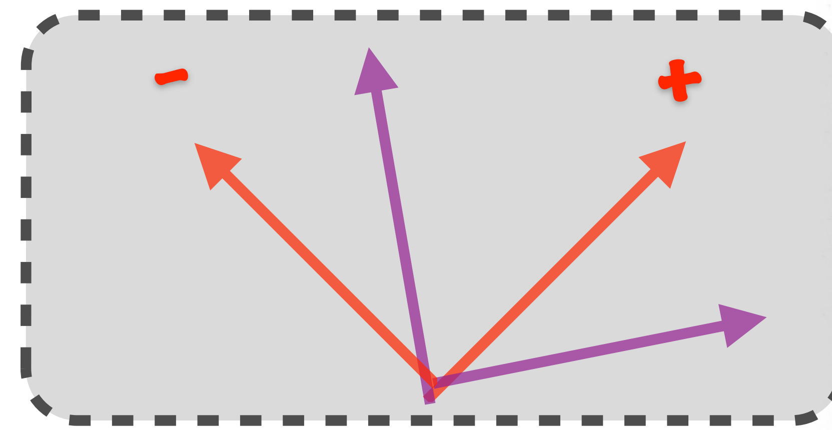

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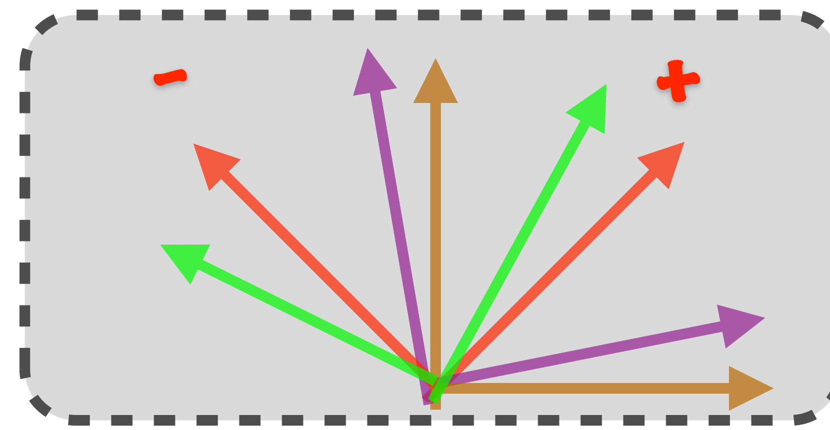
- **Proposition.** There exists a constant v, $0<v<1/\sqrt{2}$, s.t. for any N_{AB}, there exist T, N,
  - $N_{AB} = v\,T + (1/\sqrt{2}-v)\,N'$
  - $TM_A+M_AT=0$, $\|N'\|$, $\|T\|\leqq 1$ and
- Largest v: trust coefficient
  - $v \geqq .15$
- $1-1/\sqrt{2}$: coefficient for random coin flipping



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Method::seeded extraction

Quantifying randomness

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- Schatten norm $\mathrm{Tr}(\rho^{1+q})$: turns out to be appropriate

  $G_\epsilon(\rho_{YE})^q \leqq (2/\epsilon^2)\,\mathrm{Tr}[\rho'^{1+q}]$

Method::seeded extraction

Why anti-commuting measurements are good? —A Schatten norm Uncertainty Principle

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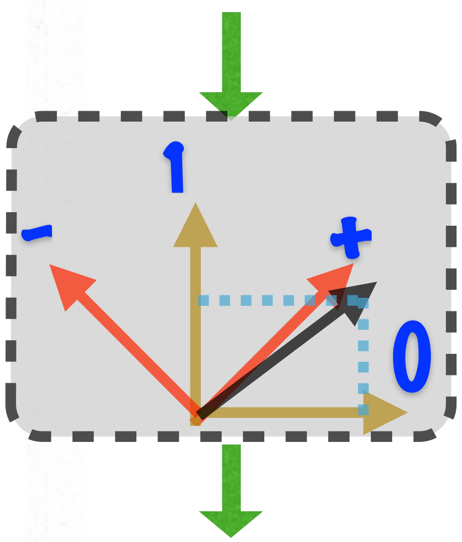

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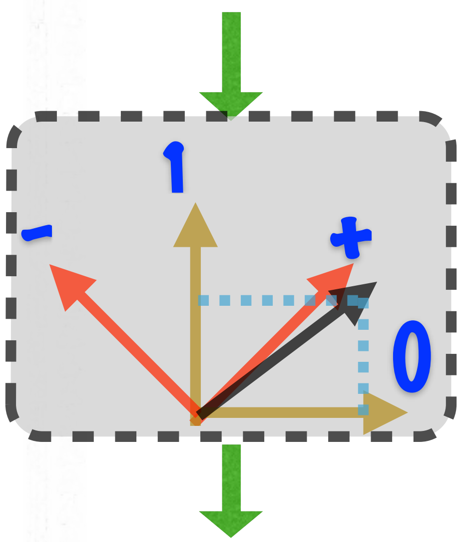

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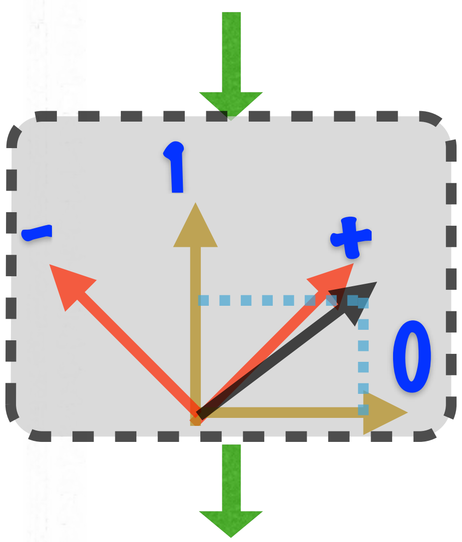

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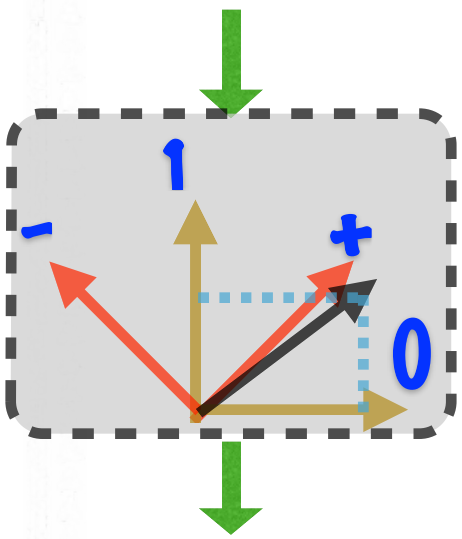

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Theorem. Let ρ_0, ρ_1, ρ_P, ρ_F, be the adversary's "states" from measuring 0/1 and +/-, respectively. For sufficiently small δ and q,

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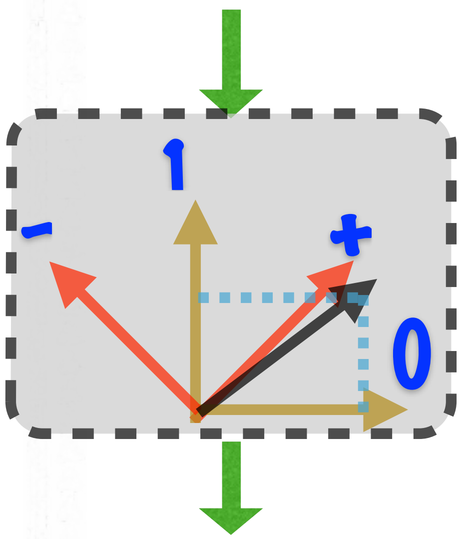

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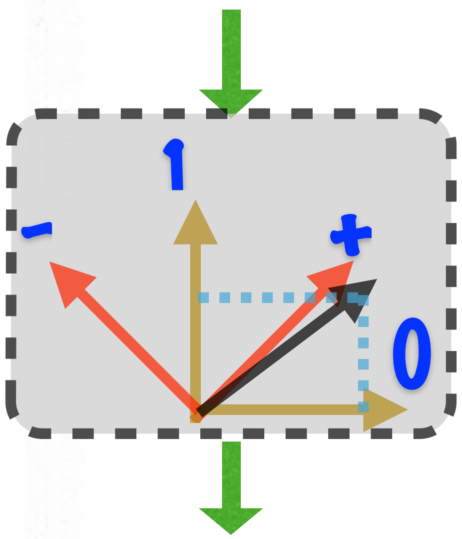
Theorem. Let ρ_0, ρ_1, ρ_P, ρ_F, be the adversary's "states" from measuring 0/1 and +/-, respectively. For sufficiently small δ and q,

$$\text{if } \mathrm{Tr}(\rho_F^{1+q}) \leq \delta\, \mathrm{Tr}(\rho^{1+q}), \qquad \mathrm{Tr}(\rho_0^{1+q}) + \mathrm{Tr}(\rho_1^{1+q}) \leq (1/2)^{q\pi(q,\delta)}\, \mathrm{Tr}(\rho^{1+q})$$

Method::seeded extraction

Why anti-commuting measurements are good? —A Schatten norm Uncertainty Principle

work or test?



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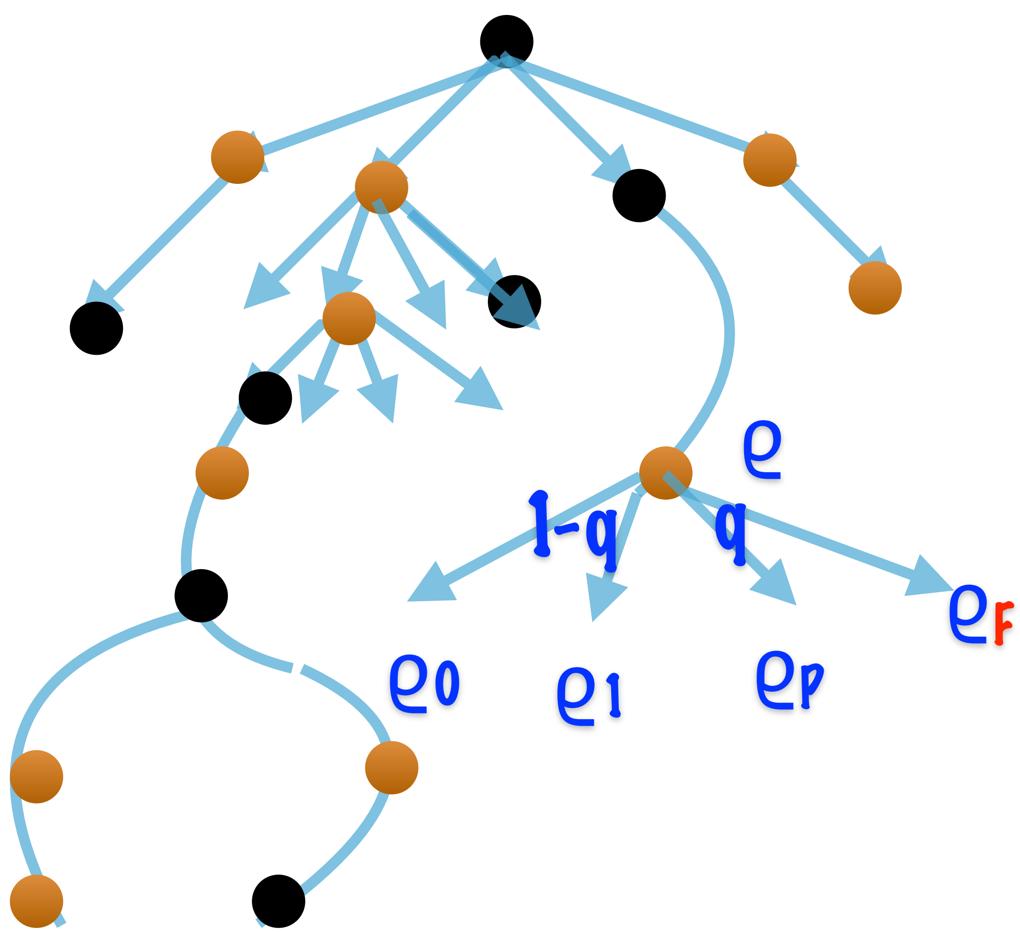
$\pi(q,\delta) \to$ 1, when q, $\delta \to 0$

Method::seeded extraction

Amortized randomness generation



Method::seeded extraction

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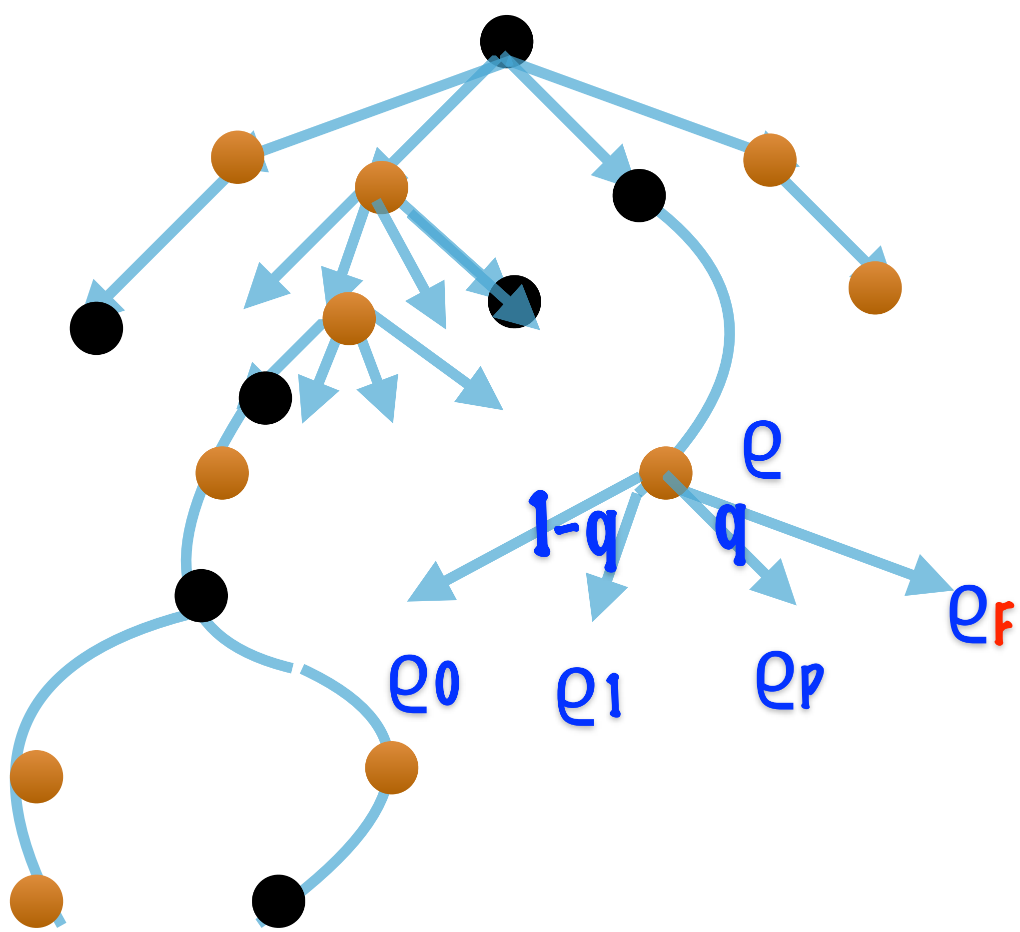

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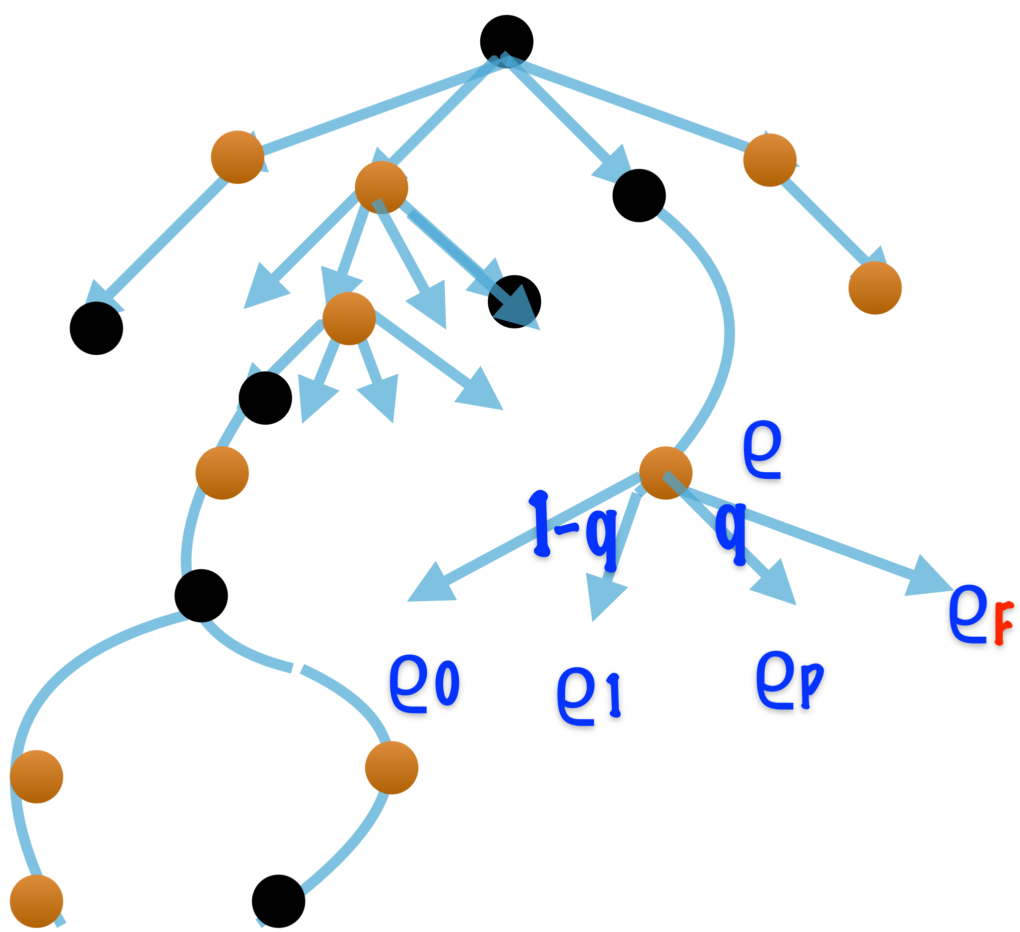
Method::seeded extraction

Amortized randomness generation



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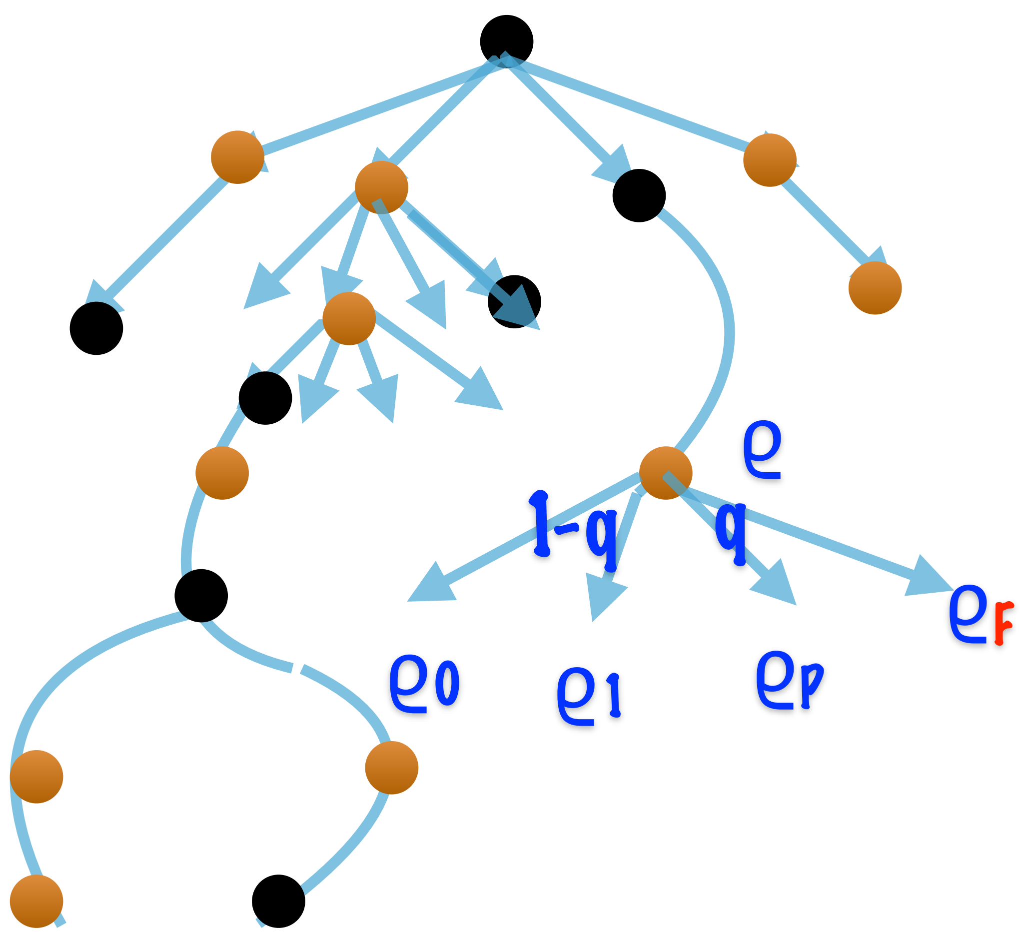

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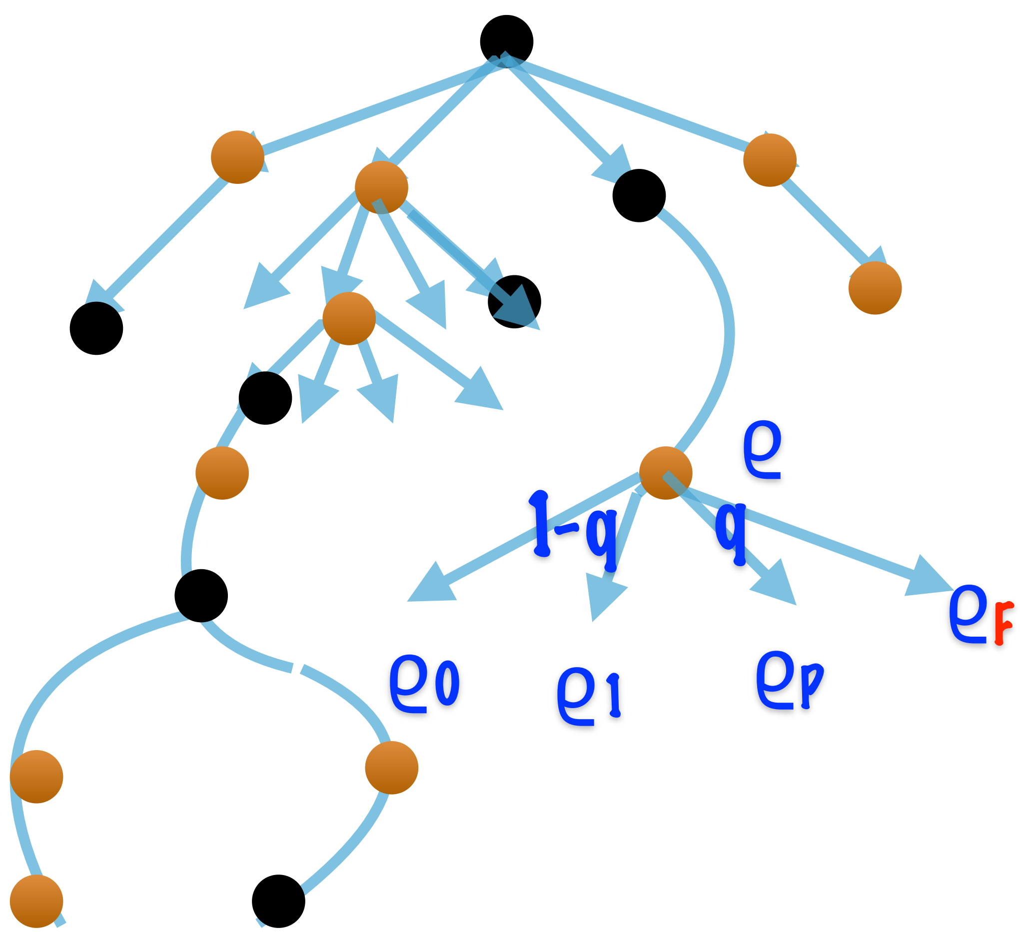

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$$\mathrm{Tr}[(1-q)\rho_0^{1+q} + (1-q)\rho_1^{1+q} + q\rho_P^{1+q} + (1/2)q\rho_F^{1+q}]$$

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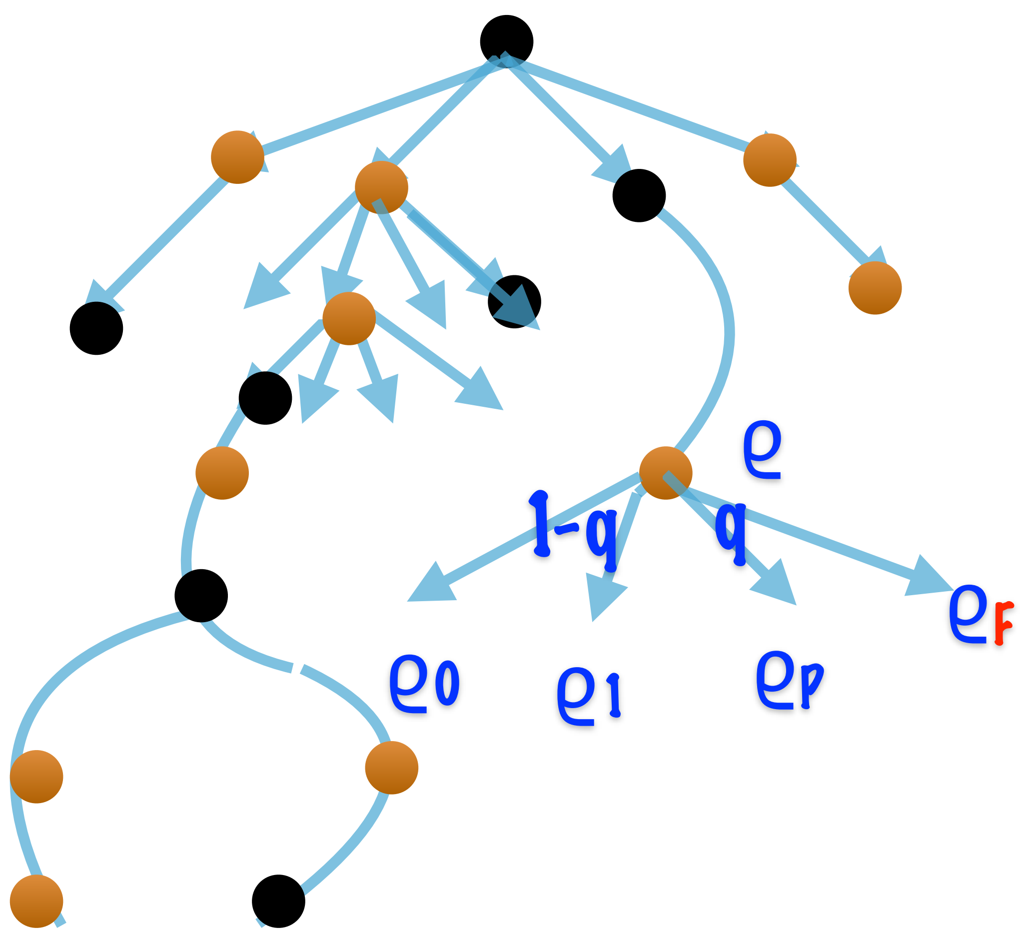

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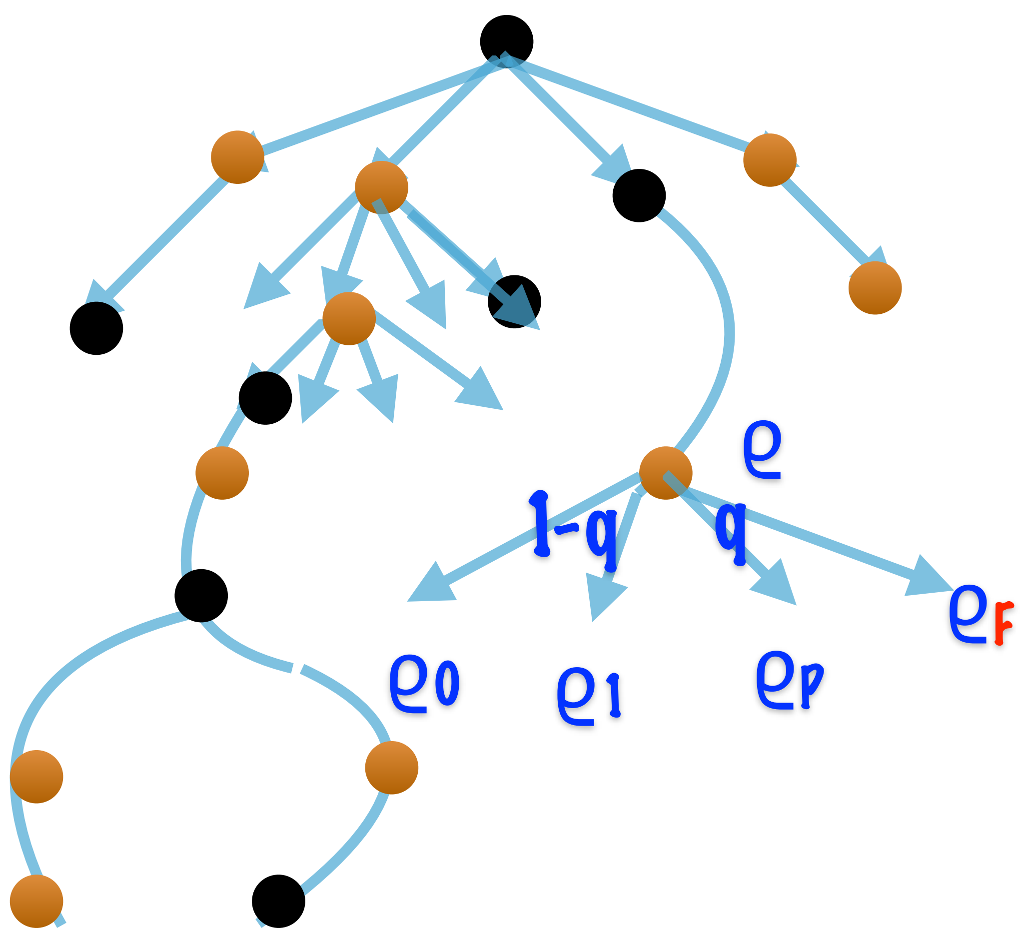
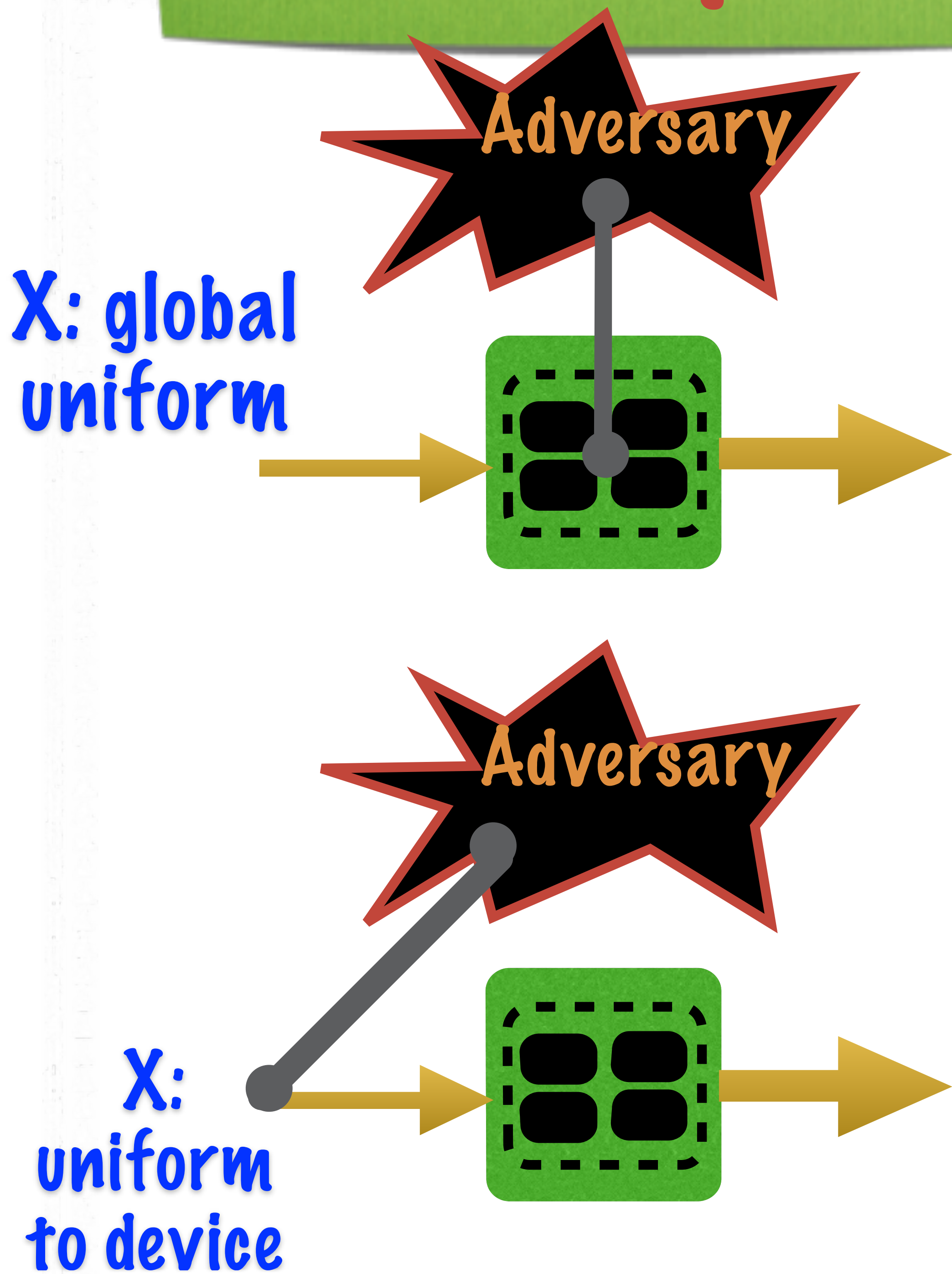


- The behavior at each round depends on the history
- If failing test, "toss a coin" and "loan" some randomness to the protocol
    - Ensuring each step increase randomness
- Total amount of loans is small (few testing rounds)
    - Fresh randomness still a lot

Geometrically decreasing:

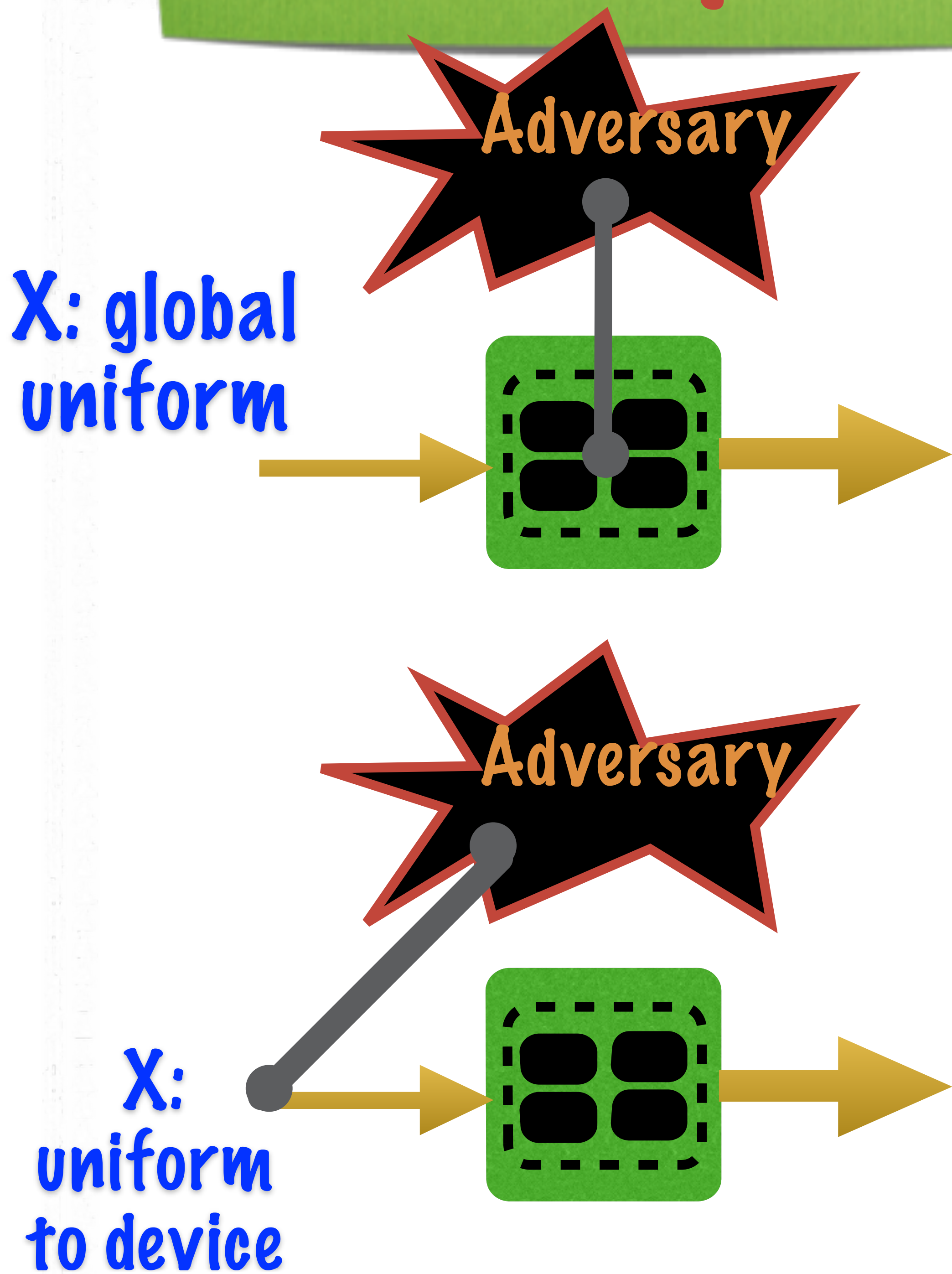
$$\mathrm{Tr}[(1-q)\rho_0^{1+q} + (1-q)\rho_1^{1+q} + q\rho_P^{1+q} + (1/2)q\rho_F^{1+q}]$$

Security under composition: Equivalence Lemma



Method::seedless extraction

Security under composition: Equivalence Lemma



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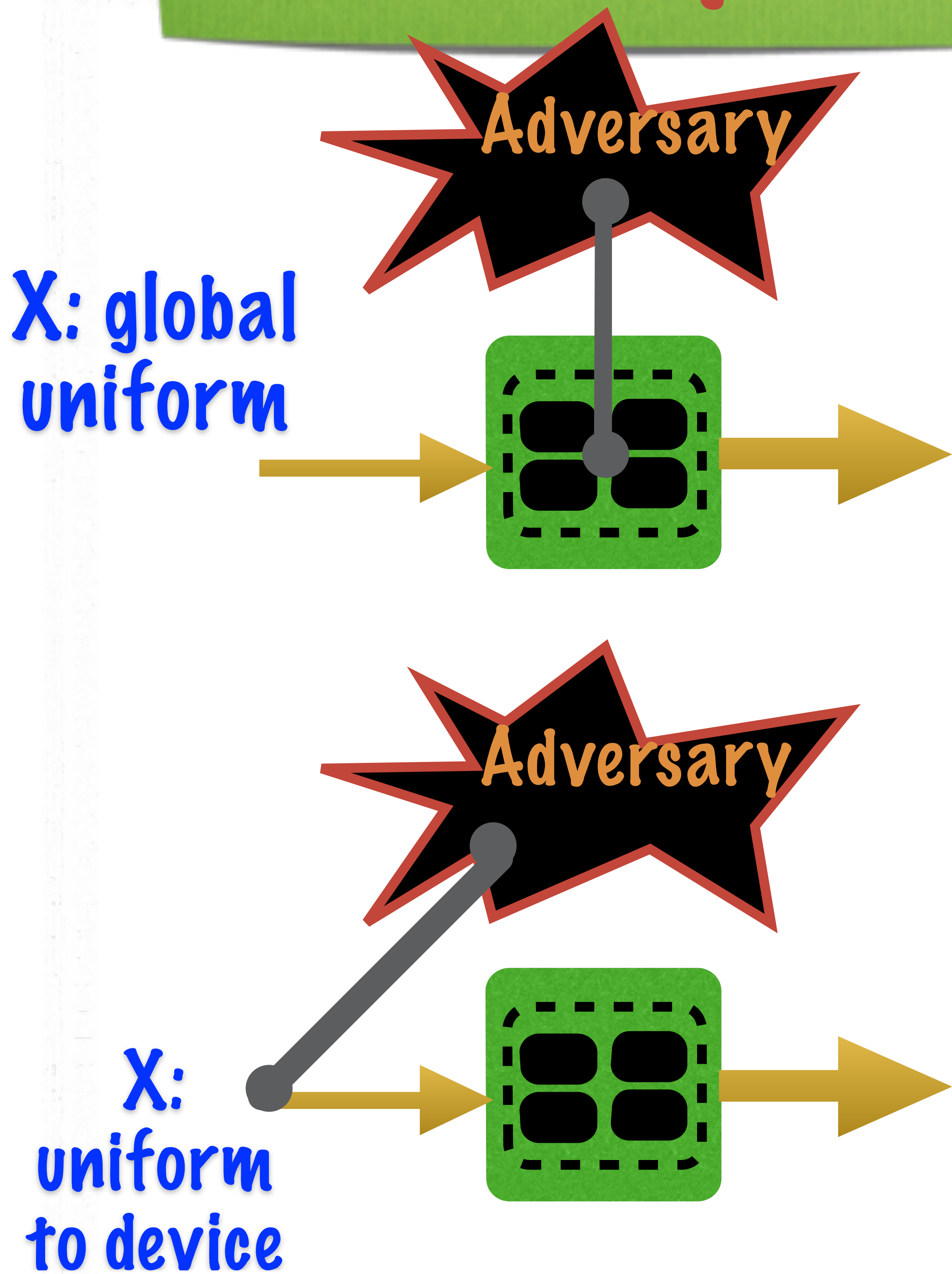
Security under composition: Equivalence Lemma



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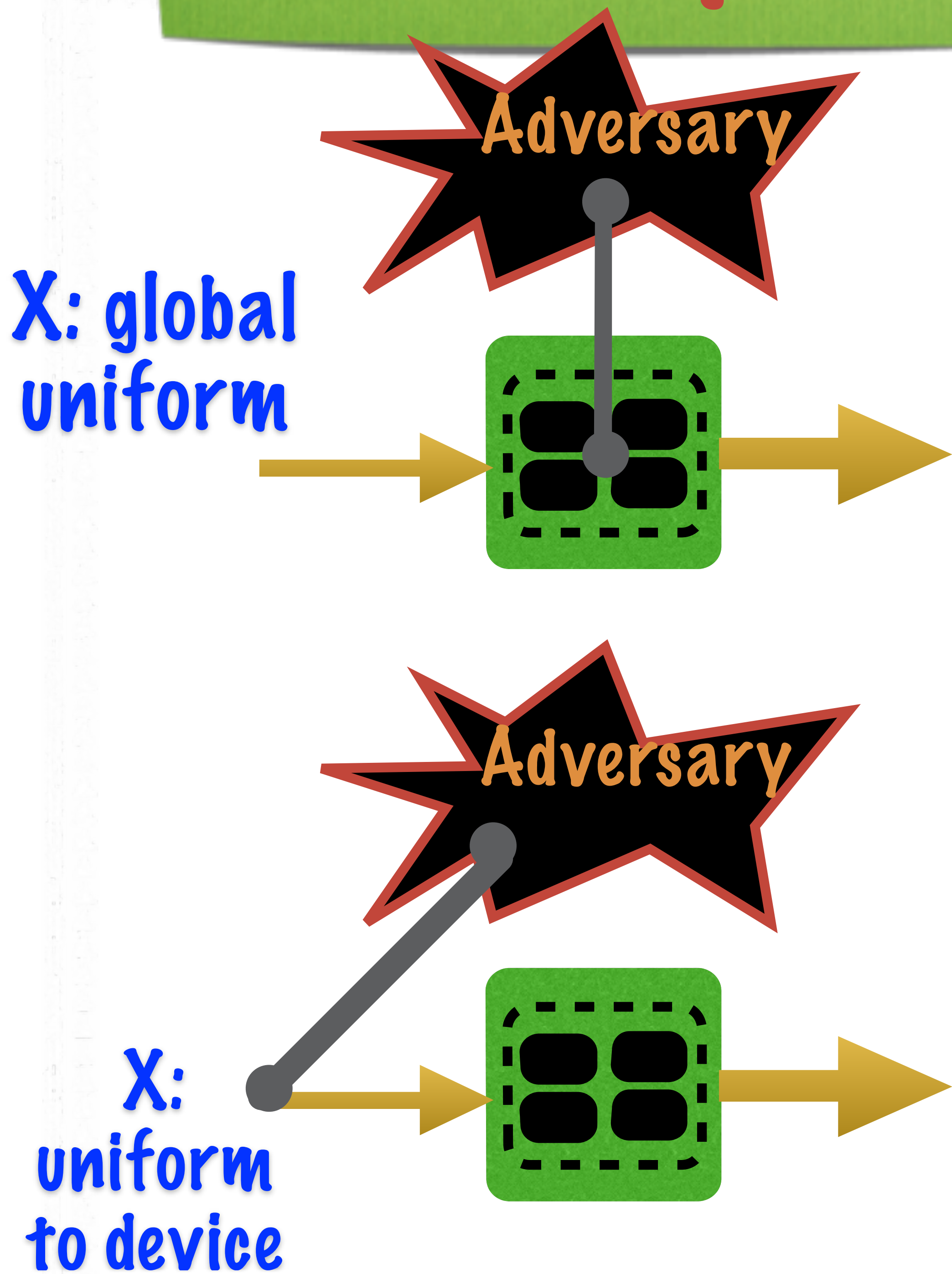

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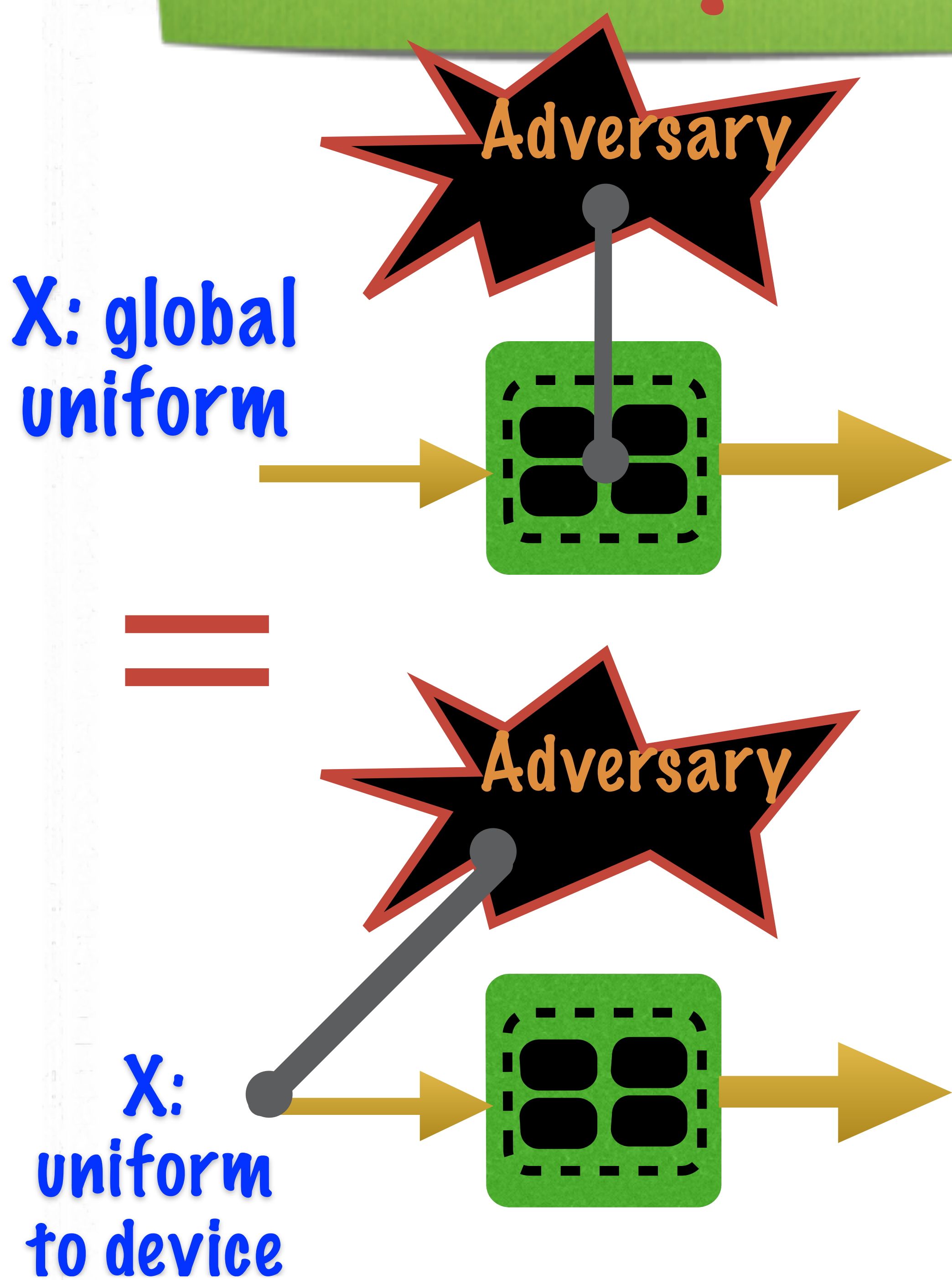

- All known expansion protocols were proved assuming globally uniform input
- Equivalence Lemma: same performance using uniform-to-device input
  - All correlations with Adversary can be produced from global uniform input by an operator OP commuting with protocol
  - OP does not change performance

Method::seedless extraction

E.L. implies unbounded expansion from cross-feeding any protocol

Invariant: each device's output is (close to) uniform to the other device

Method::Equivalence Lemma

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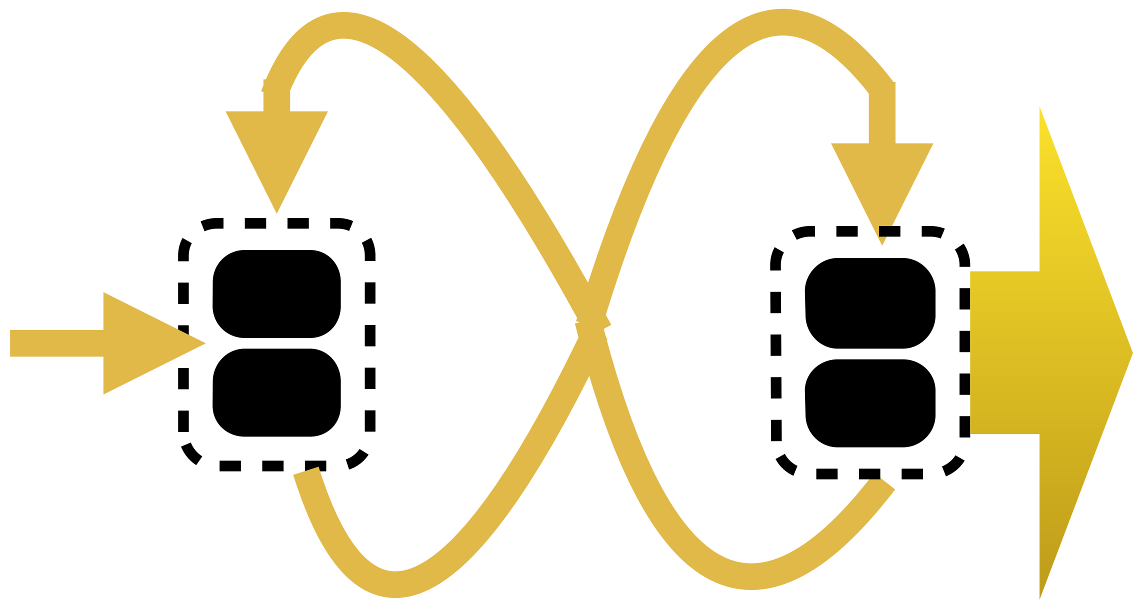

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Method::Equivalence Lemma