Quantum Attacks on Classical Proof Systems

The Hardness of Quantum Rewinding

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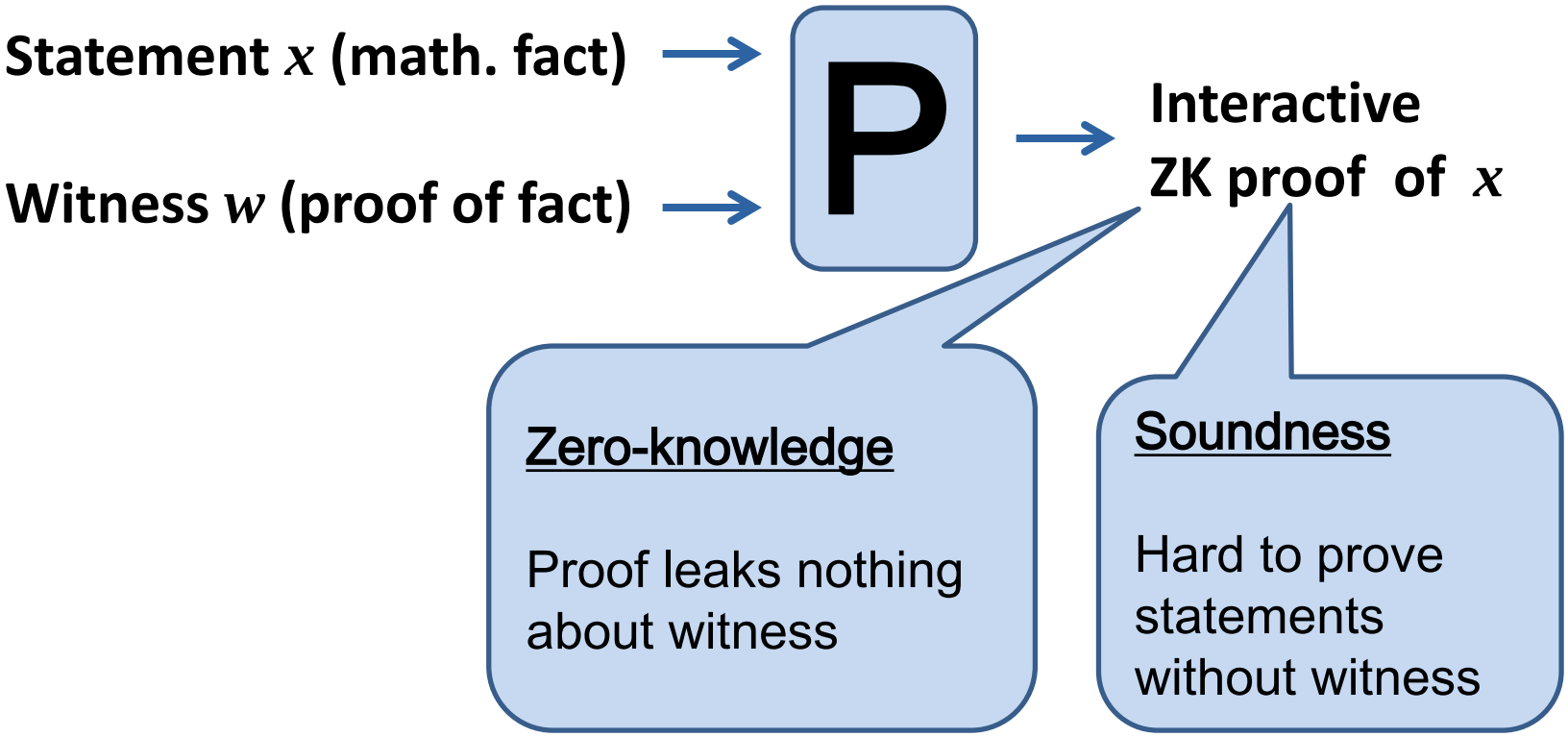
Classical Crypto

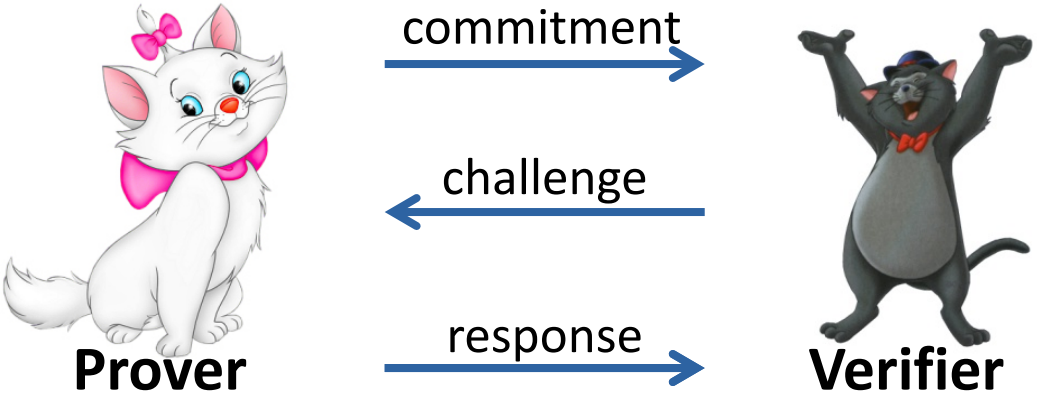
(Quick intro.)

Zero-knowledge proofs (of knowledge)



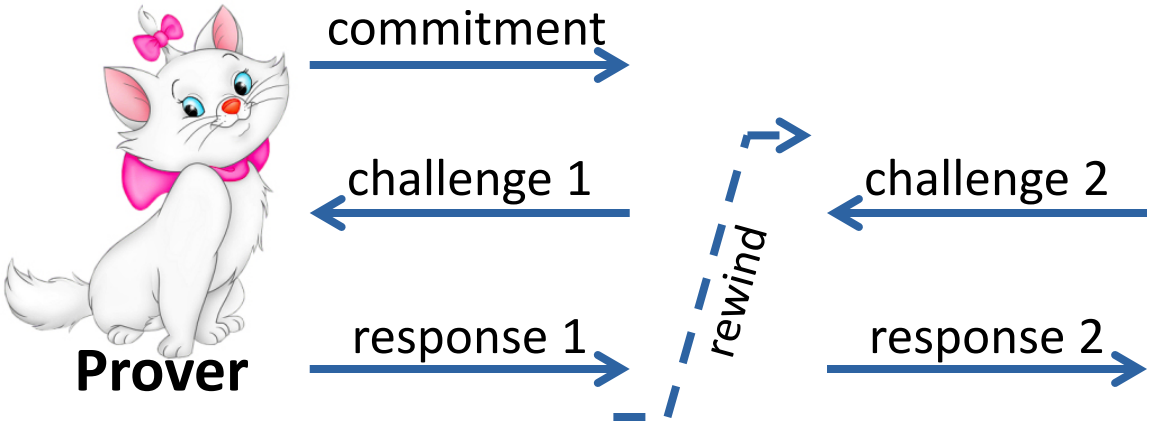
Uses: Proving honest behavior, drosophilia of crypto, …

Towards efficient ZK: Sigma protocols



"Special soundness": Two different responses allow to compute witness

Proving soundness



Special soundness ➔ We extract the witness

➔ Correct proof implies knowledge of witness

Classical security easy.

Quantum!

But if adversary has a quantum computer?

Impossibility result

There is a sigma-protocol
- with special soundness
- that is not sound

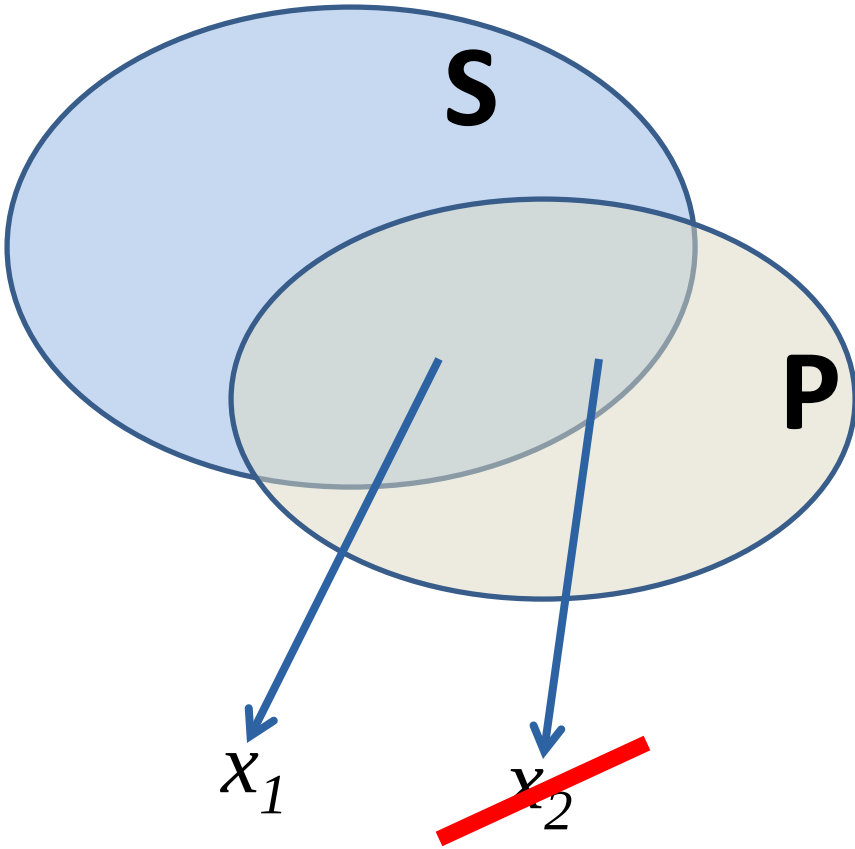
(Relative to some oracle.)

Consequence:
A classically secure sigma-protocol may be quantum insecure*

* See terms and conditions for oracle-separations

The "pick-one trick" (simplified)

- Given a set S
- can encode it as a quantum state $|\Psi\rangle$
- s.t. for any set P
- you find one $x_1 \in S \cap P$
- but not two $x_1, x_2 \in S$



[Up to some constraints]

Pick-one trick: Finding $x_1 \in S \cap P$

Grover's algorithm

- Create
$$|\Psi\rangle := \sum_x |x\rangle$$
- Repeatedly apply:
$$I - 2|\Psi\rangle\langle\Psi|$$
and stuff.
- Get: $x \in P$

Picking x_1

- Create
$$|\Psi\rangle := \sum_{x \in S} |x\rangle$$
- Repeatedly apply:
$$I - 2|\Psi\rangle\langle\Psi|$$
and stuff.
- Get: $x \in P \cap S$

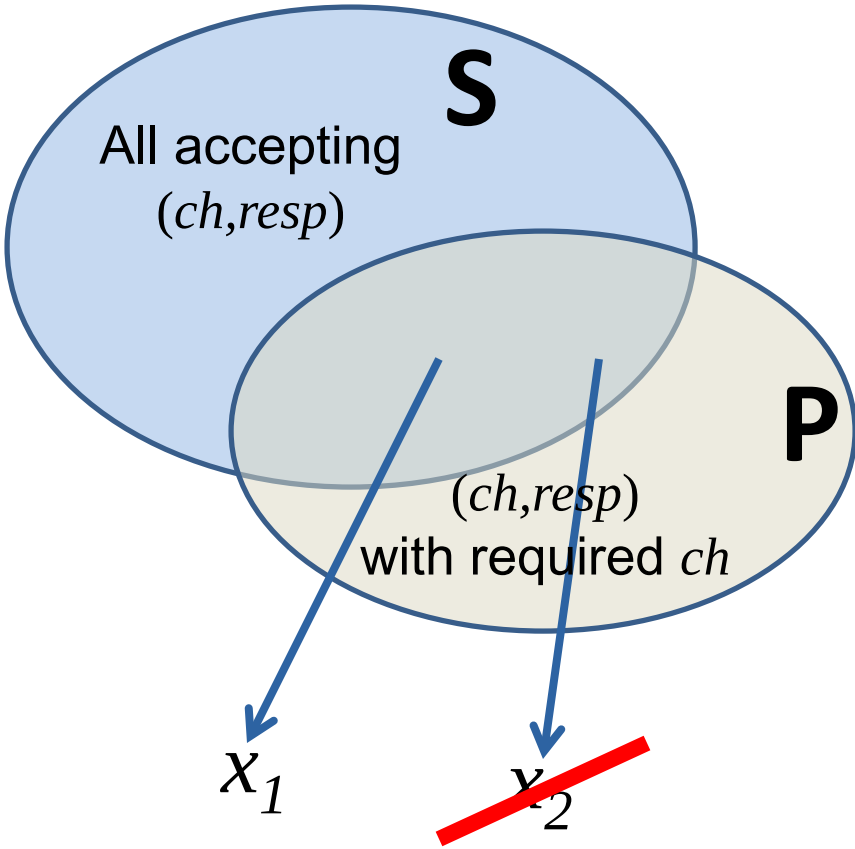
Pick-one trick: Not finding $x_1, x_2 \in S$

- $x_1, x_2 \in S$ hard to find.
- Even with oracle for $I - 2|\Psi\rangle\langle\Psi|$.
- Assuming *S* is a random set.

- Query complexity problem.
- Proved using Ambainis' "adversary method"

Breaking sigma-protocols

- Given a set S
- can encode it as a quantum state $|\Psi\rangle$
- s.t. for any set P
- you find one $x_1 \in S \cap P$
- but not two $x_1, x_2 \in S$



No quantum secure sigma protocols?

- No: under extra conditions, they are secure [Watrous 2006, Unruh 2012]

- But general security unlikely under same assumptions as classical

Other results

Same technique (pick-one trick) gives impossibilities for:

- Computationally-sound proofs
- Fiat-Shamir's NIZK proofs/signatures
- Fischlin's NIZK proofs
- Commitments

Open problems

- Can we do it without oracles? [Aaronson, Christiano 2012]?
- Under what conditions are sigma-protocols et al. secure?
- Alternative constructions that are secure?

I thank for your attention

Logo soup

DoRa

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NIZK with random oracles

Fiat-Shamir

com →

← chal

H(com)

resp →

Prover

- NIZK consists of com,chal,resp
- Prover can't cheat: H is like a verifier
- Security-proof: Rewinding

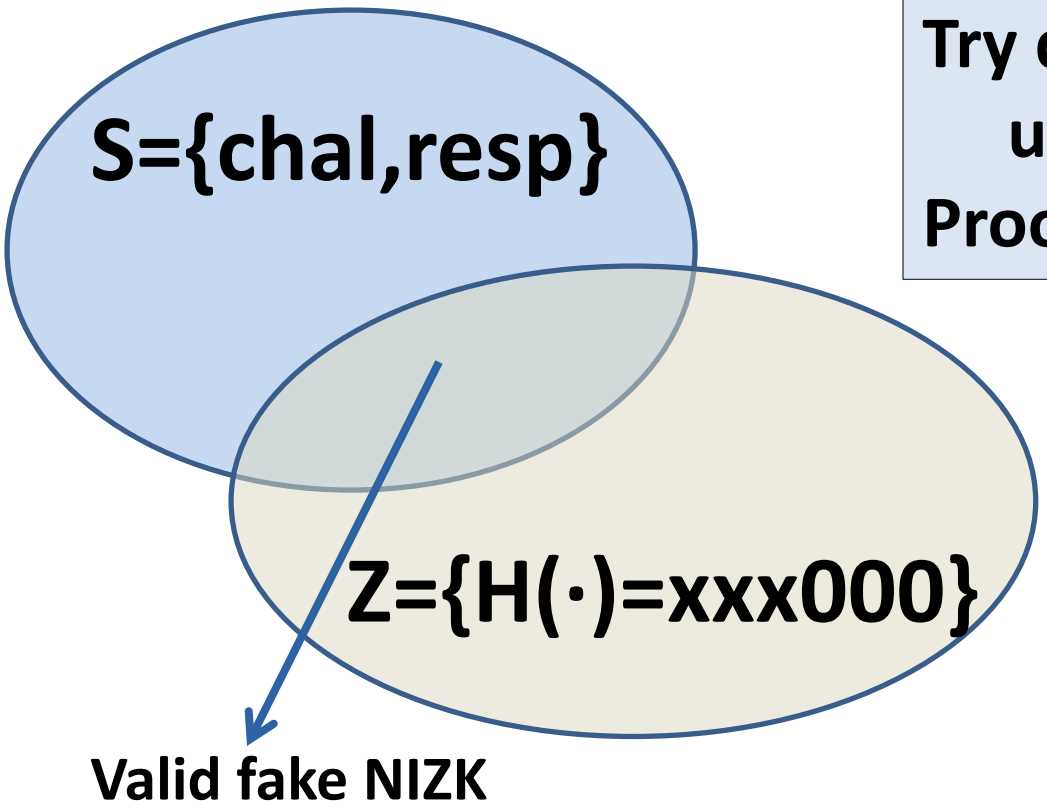
Fischlin

Fix com
Try different chal, resp
until H(chal,resp)=xxx000
Proof := com,chal,resp

- Need to query several chal,resp
- Implies existence of witness

Attacking Fischlin

Fix com
Try different chal, resp
until H(chal,resp)=xxx000
Proof = com,chal,resp



Without knowing witness!

(Because we have only one S-element)

[Fiat-Shamir attacked similarly]

How does "one-pick trick" work?

- Grover: Quantum algorithm for searching
- Observation:
  - First step of Grover produces a state encoding the search space
- This state (plus modified Grover) implements "one-pick trick"
- Hard part: Prove "can't find two $x_1, x_2 \in S$"