Asymptotic Security Proof for Discrete-Modulated CVQKD without Cutoff Assumption

Twesh Upadhyaya | Thomas van Himbeeck | Jie Lin | Norbert Lütkenhaus

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

Introduction

Goal is to compute tight key rates for DMCVQKD with a small number of modulated states.

Want to harness the existing numerics framework for finite-dimensional optimizations [1][2].

Previous work assumes the state is finite-dimensional; the cutoff assumption. This gives numerically stable results but is not a rigorous security proof [3].

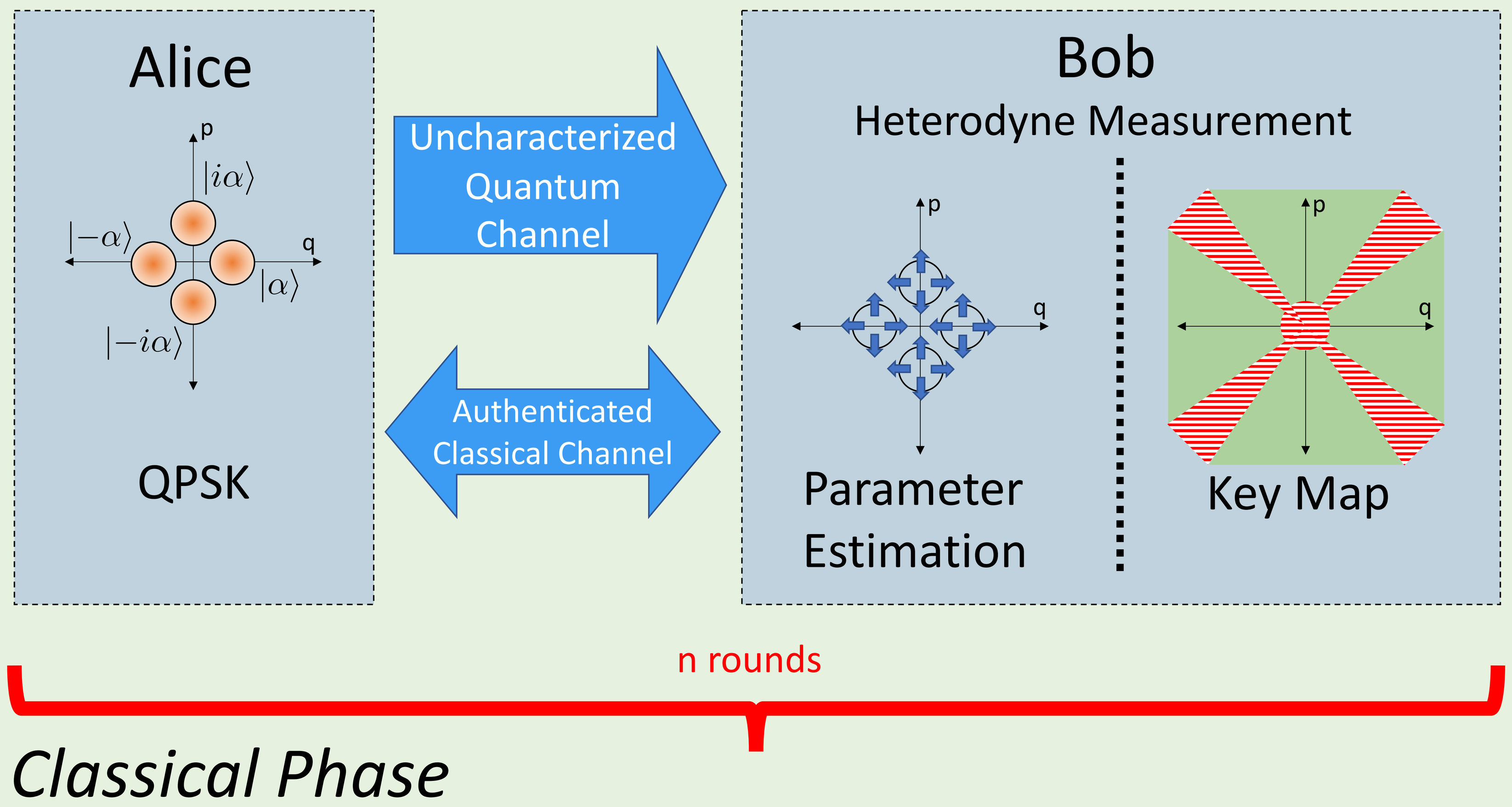
Our Contribution

We establish the asymptotic security of DMCVQKD with a small number of modulated states. In particular, we do not use a cutoff assumption.

We develop a framework to provide tight, reliable key rates for other protocols with infinite-dimensional Hilbert spaces.

Protocol

Quantum Phase



Classical Phase

- Error Correction: Reverse Reconciliation
- Privacy Amplification

Key Rate Formula

Under collective attacks in the asymptotic limit, given by Devetak-Winter formula [4]: $R = I(A:B) - \chi(X:E)$

Evaluated on worst-case state compatible with statistics from parameter estimation; after post-processing.

Reformulate: $R = \min_{\rho \in \mathbf{S}} [H(X|E)] - H(X) + I(A:B)$

If ϕ is the CPTNI map representing the post-processing performed by Alice and Bob, then: $f(\rho) = H(X|E)_{\phi(\rho)}$

Infinite and Finite Optimizations

$$
\begin{aligned}
f(\rho^\infty) = \min\ & f(\rho) \\
\text{subject to:}\ & \\
& \rho \in \mathrm{Pos}(\mathcal{H}_{AB}^\infty) \\
& \mathrm{Tr}(\rho) = 1 \\
& \rho \in \mathbf{S}_\infty
\end{aligned}
\qquad
\begin{aligned}
f(\rho^N) = \min\ & f(\rho) \\
\text{subject to:}\ & \\
& \rho \in \mathrm{Pos}(\mathcal{H}_{AB}^N) \\
& 1 - W \leq \mathrm{Tr}(\rho) \leq 1 \\
& \rho \in \mathbf{S}_N
\end{aligned}
$$

Require: $\Pi_N \mathbf{S}_\infty \Pi_N \subseteq \mathbf{S}_N$

$$W \geq 1 - \mathrm{Tr}(\rho \Pi_N) \quad \forall \rho \in \mathbf{S}_\infty$$

Uniform Continuity of Conditional Entropy

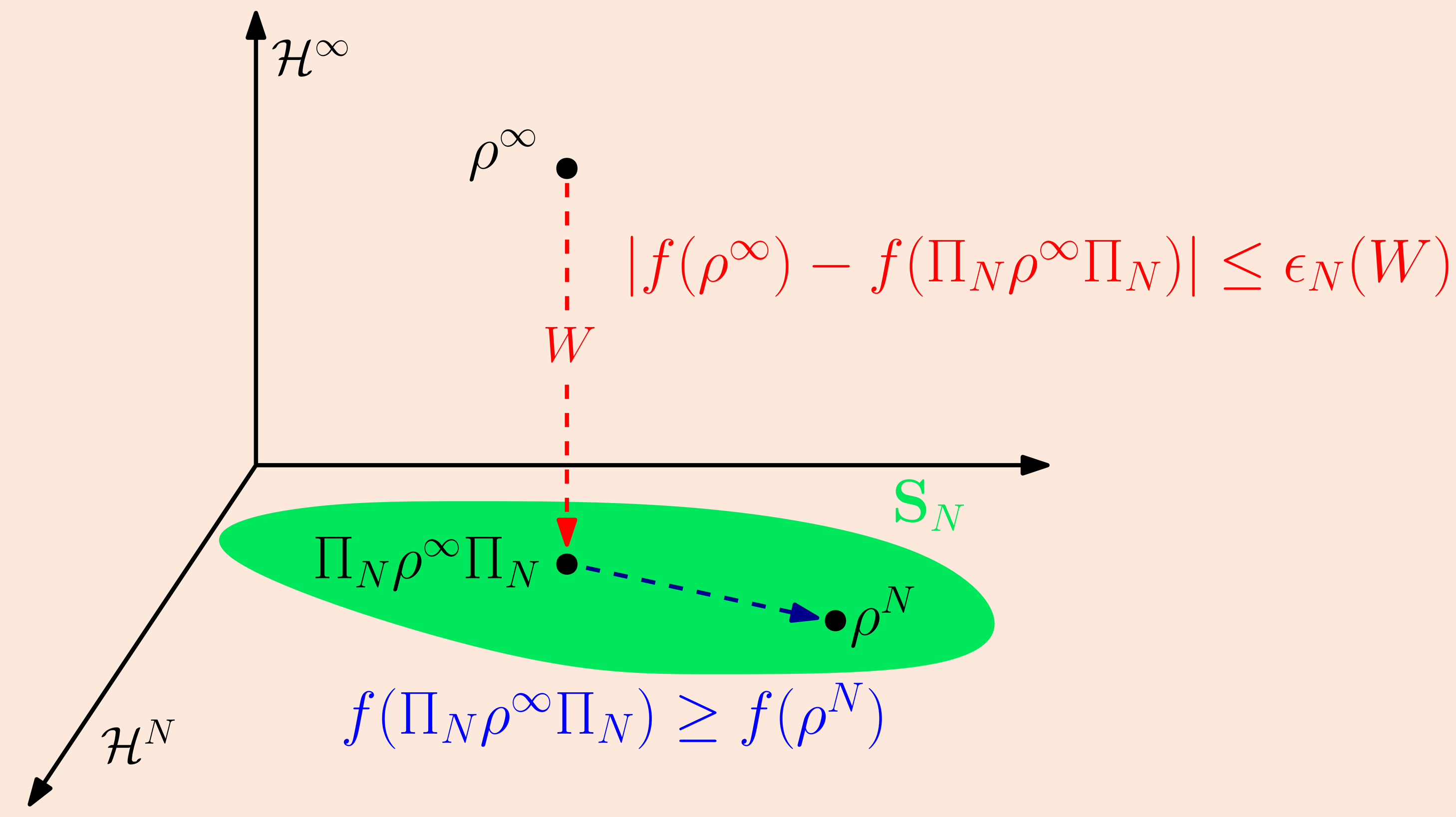
W can be determined from parameter estimation, and characterizes the weight outside the finite subspace.

Using a generalization of the result in [5], we have:

$$1 - W \leq F(P,Q) \implies |f(P) - f(Q)| \leq \epsilon_N(W)$$

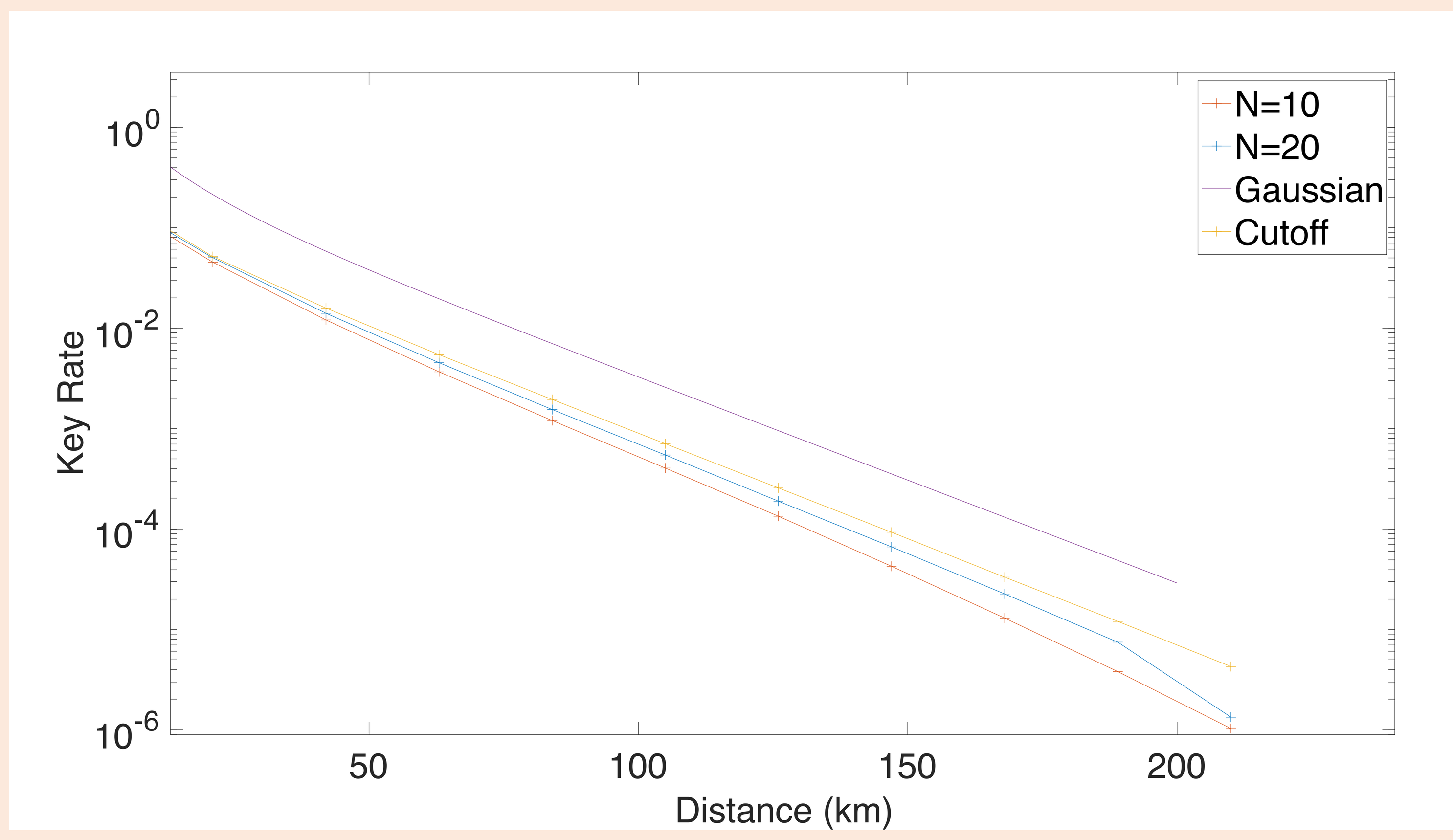
Main Theorem

$$f(\rho^N) - \epsilon_N(W) \leq f(\rho^\infty)$$



Results for Typical Simulation

Lossy and noisy channel, with ξ=0.01



Future Work

Extend to finite-size numerical framework for CVQKD.

Encapsulate different models of imperfect detectors, e.g. trusted noise.

References

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[3] J. Lin, T. Upadhyaya, and N. Lütkenhaus, Phys. Rev. X **9**, 041064 (2019).
[4] I. Devetak and A. Winter, Proc. R. Soc. A **461**, 207 (2005).
[5] A. Winter. (2016) , Communications in Mathematical Physics, **347**(1):291–313.

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