Improved analytical bounds on delivery times of long-distance entanglement

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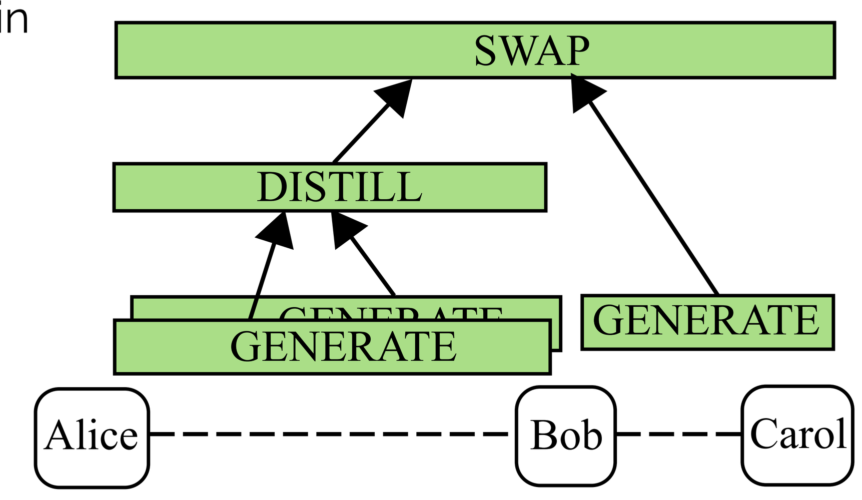
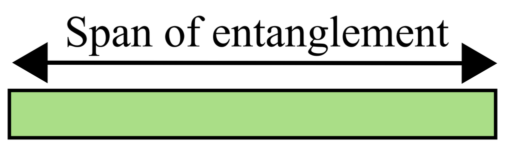
The fundamental distance limit for quantum key distribution due to photon loss can be overcome by intermediate nodes called quantum repeaters. We provide analytical bounds on the mean and quantiles of the entanglement delivery time for a very general class of repeater schemes, which significantly improve upon existing work. Our bounds enable the analytical assessment of repeater in the presence of time-dependent noise, such as imperfect memories, and are useful for the design and analysis of network sizes beyond the reach of numerics.

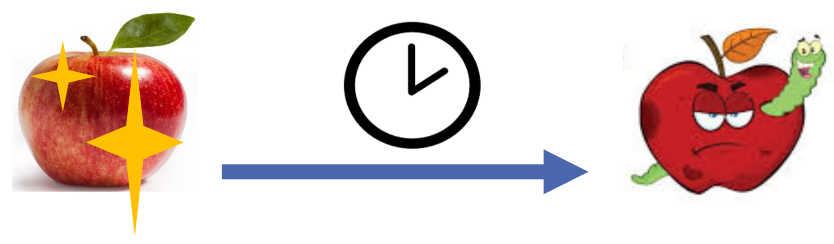
Context

- Example quantum repeater chain scheme on 3 nodes:



- Realizations of quantum repeaters will suffer from **time-dependent noise**, such as memory decoherence. Specifically: many times, an entangled pair is generated which needs to wait for another pair, and decoheres during this waiting:





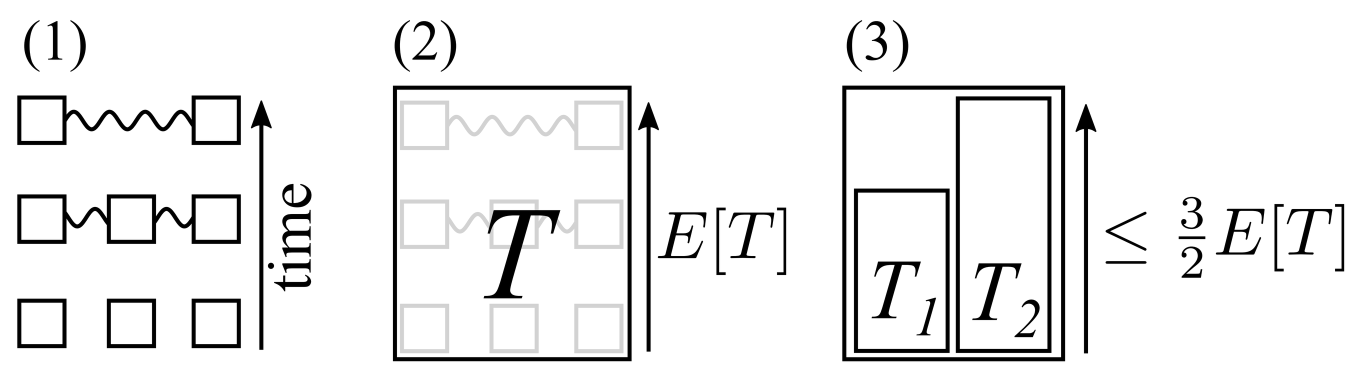
- To characterize time-dependent noise, we need to know:

> **Problem statement:** Given a repeater chain scheme, find the probability distribution of the time until entanglement delivery by the repeaters' end nodes

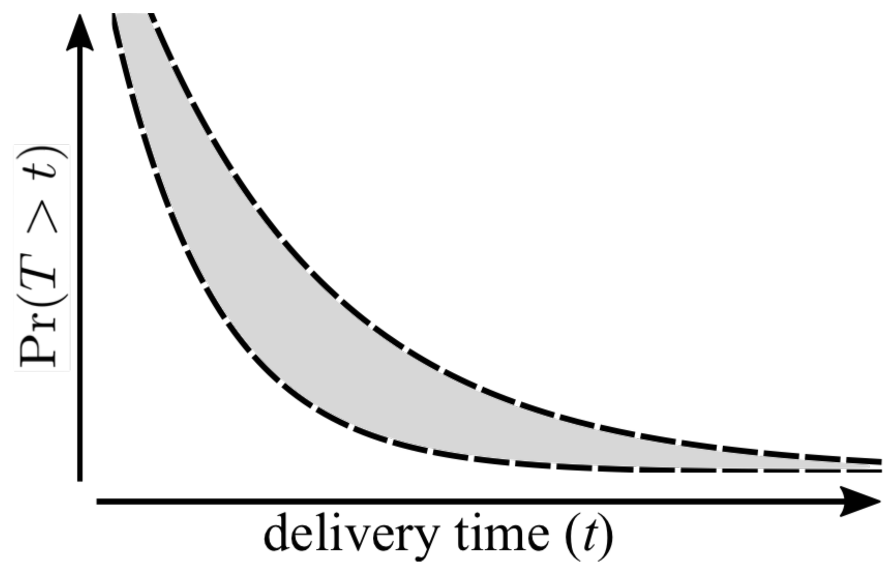
- Existing literature: often even mean time not known!
- We consider hierarchical repeater chain schemes (based on the BDCZ scheme [1]), which are:
  - composed of probabilistic components (GENeration of fresh entanglement, DISTillation, SWAPping) with Pr(success)≥const
  - No two components wait for the same entangled pair before proceeding, i.e. the entanglement dependency graph is a tree

Results: analytical bounds on the entanglement delivery time

(1) Bounds on mean delivery time. For two input entangled pairs:



(2) Exponentially-fast decaying bounds on Pr(it takes longer than time t to deliver entanglement)



Tool: reliability theory. A random variable T is **new-better-than-used** (NBU) if:

$$\forall x, y \geq 0: \Pr(T > x + y) \leq \Pr(T > x) \cdot \Pr(T > y)$$

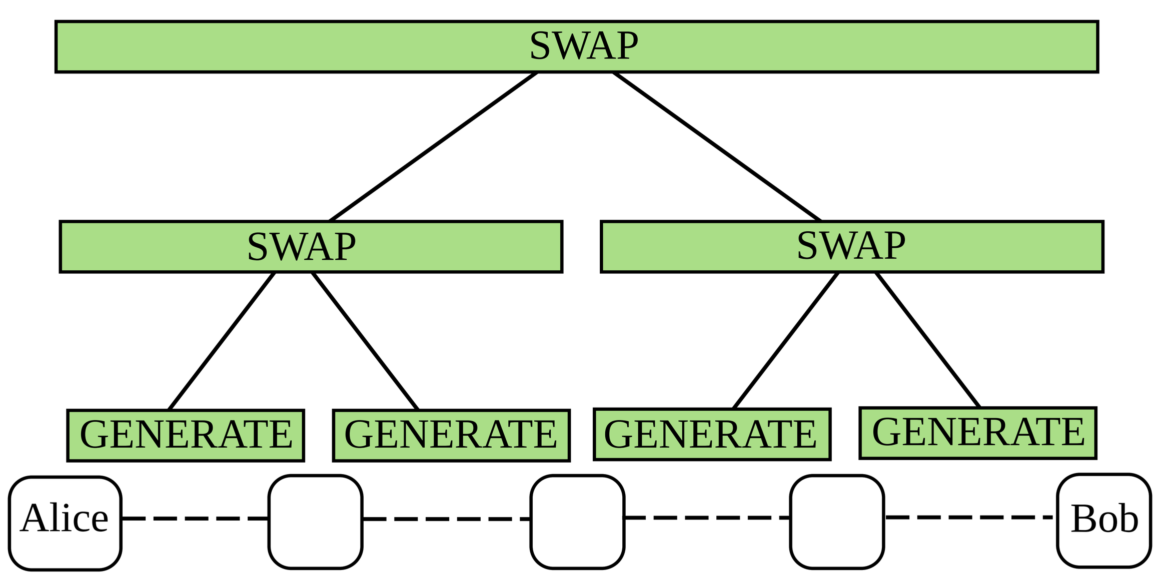
Formally, we show:

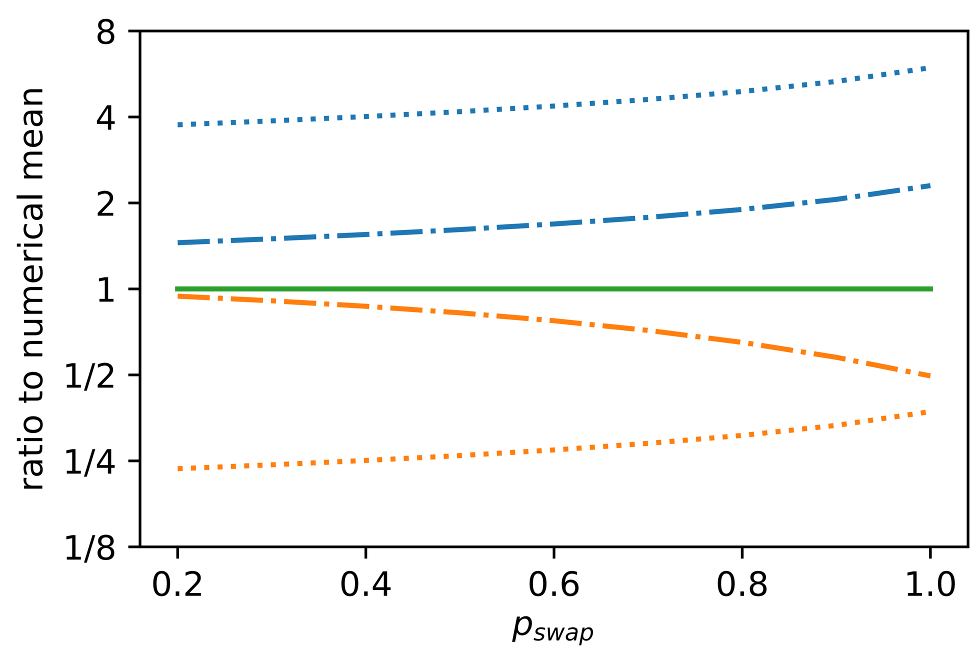
Proposition*: let T_{output} be the time until success of an entanglement swap with success probability p, applied to two states which are produced with iid times T_{input} (also a random variable. If T_{input} is continuous and NBU, then:*

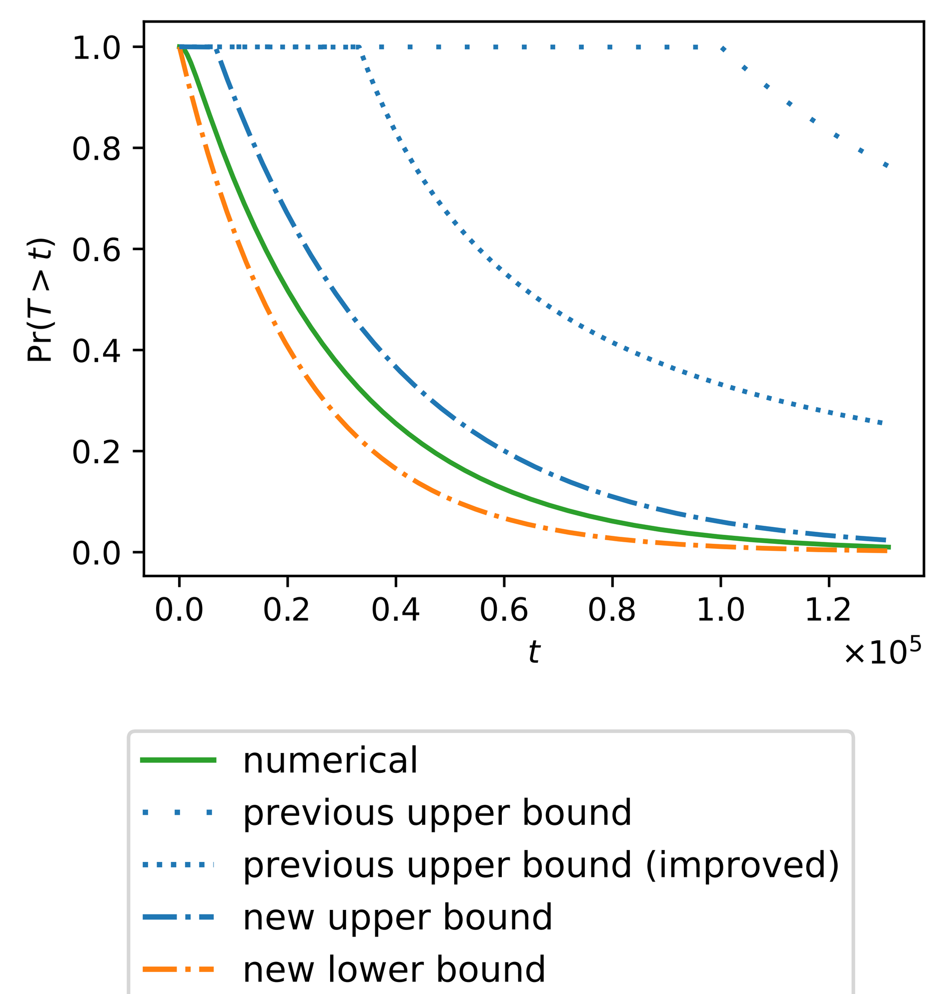
1. *T_{output} is NBU*
2. *$mean(T_{output}) \leq \frac{3T_{input}}{2p}$*
3. *$Pr(T_{output} > t) \leq exp\left(p - \frac{2pt}{3mean(T_{input})}\right)$ (and also a lower bound)*

And we have similar statements for ≥2 input states.

Application to the famous nested BDCZ scheme







For the mean delivery time, our bounds imply that very-often-used **3-over-2 approximation** is **effectively an upper bound**:

$$mean(T) \leq \left(\frac{3}{2\, p_{swap}}\right)^{\#nesting\ levels} \cdot mean(elementary\ link\ GENERATE\ delivery\ time)$$

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QUANTUM INTERNET ALLIANCE

Quantum Software Consortium

BDCZ scheme: H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 81, 5932–5935 (1998)